Dynamics of star clusters containing stellar mass black holes: 4. Formation and Interaction of Binaries

July 28, 2017
Bonn

Hyung Mok Lee
Seoul National University
Binaries in Star Clusters

- Large fraction of stars are in the form of binaries
- Binaries can be also be formed via tidal capture, gravitational wave capture or three-body processes
- Binary orbits can be significantly affected by the interaction with other objects in dense environments
- The dynamical properties of the binaries are determined by the ‘hardness’ of the binaries
- Hardness is defined as the binding energy in units of the average kinetic energy of surrounding stars, i.e.,

\[ x \equiv \frac{Gm^2}{2a} \frac{2}{3m\sigma^2/2} \]
Tidal capture binary formation

- When two stars approach each other tidal forces lead to the excitation of oscillation
- Oscillation energy is extracted from the orbital energy
- If the amount of dissipated energy is greater than the positive binding energy, the stars become bound
- Cross section (Lee & Ostriker 1987):
  \[ \sigma_{cap} = 7R_\star^2 \left( \frac{v_\infty}{v_\star} \right)^{-2.18} \]
  where \( v_\star \) is the escape velocity from the stellar surface and \( v_\infty \) is the relative velocity at infinity
- These binaries are very tight. After circularization the orbital radius would be
  \[ a_c < 2.1 \left( \frac{v_\infty}{v_\star} \right)^{-0.19} R_\star \sim 4R_\star \]
- It corresponds to hardness of \( \sim 1000 \)
Gravitational Wave Capture

• If the gravitational radiation energy during close encounter is greater than the binding energy two compact stars can become bound i.e., \((\Delta E)_GR > \frac{1}{2} \mu v^2_{\infty}\)

• The encounter stars with hyperbolic orbit, but the orbit is close to the parabolic one near the pericenter where most of the gravitation waves are radiated.

• Thus we may assume that the orbit is parabolic which can be specified by angular momentum only: parabolic approximation

• The resulting cross section:

\[
\sigma_{cap} = 17 \frac{G^2 m_1 m_2 \eta^{-5/7}}{c^{10/7} v_{\infty}^{18/7}}
\]

\[
\eta \equiv \frac{m_1 m_2}{(m_1 + m_2)^2}
\]
Properties of captured binaries

- **Tight:** 
  \[ a \sim 0.05 \frac{Gm}{\sigma^2} \]

- **Very eccentric:** 
  \[ 1 - e \sim 10^{-4} \left( \frac{\sigma}{75 \text{km/s}} \right)^{10/7} \]

- **Merge very quickly:**

 Hong & Lee 2015
Binaries formed by three-body processes

- If three stars are in a small volume, they form a temporary triple system which is rather unstable.
- One star gets ejected, leaving the other two bound.
- The formation rate:
  - Rate of strong encounters per volume:
  - Probability of finding another star within volume of sphere of \( r_s \): Volume \( \pi r_s^2 \) \( n \)
  - Therefore the binary formation rate per unit volume becomes
- Hardness is \( x > 3 \) at the time of formation

\[
\frac{dn_B}{dt} \sim n^3 \left( \frac{Gm}{v^9} \right)
\]

\[
n^2 (\pi r_s^2) v \sim \frac{G^2 m^2}{v^3} n^2
\]
Consequences of binary-single interactions

- Types of interactions: flyby, resonant, exchange
- Statistical results:
  - Soft binaries become softer
    → endothermic (i.e., heat sink)
  - Hard binaries become harder
    → exothermic (i.e., heat source)
- $\Delta E \sim 0.4|E_{bin}|$ per resonant interactions (i.e., interactions with $p \sim a$).
- $2/3$ goes to KE of binaries for equal mass case
Hardness of Binaries

• It is useful to define hardness parameter $x$, as a binding energy of binaries normalized by the average kinetic energy of the background.

\[ x \equiv \frac{Gm^2/2a}{3m_\ast \sigma^2/2}; \quad m_\ast: \text{stellar mass} \]

• Binaries get ejected from the cluster if the kick energy exceeds the escape energy:

\[ \Delta E > \frac{1}{2} (2m) v_{esc}^2 \]

• The above condition for ejection can be re-expressed as

\[ x_{crit} \sim 5 \left( \frac{m}{m_\ast} \right) \left( \frac{v_{esc}^2}{\sigma^2} \right), \quad \text{typically} \quad \left( \frac{v_{esc}^2}{\sigma^2} \right) \sim 12 \]

• Depending on $(m/m_\ast)$, $x_{crit}$ could vary significantly.
Evolution of binaries in e-a space
Binaries as heating source

• Soft binaries become resolved quickly

• However, hard binaries become harder, releasing energy, as a result of the interactions with other stars.

• The endothermic nature of binary-single interaction means heating

• Heating is highly localized in the central part

• Binaries stabilizes the core collapse, similar to the role of the nuclear reaction in the stellar core

• In practice the clusters expands due to the heating of binaries, after the core-collapse
Consequences of heating

- If the rate of energy conduction becomes smaller than the heating rate, the core collapse does not take place.

- In the absence of the radiative cooling, the system cannot be in static equilibrium, but expands because of the increase of energy, i.e.

\[ E \approx -0.2 \frac{GM^2}{r_h} \]

and \( r_h \) must increase if \( E \) increases.
Treatment of Binaries formation in Fokker-Planck Framework

- Binary formation = source term for binary, but loss term for singles

\[ \left( \frac{\partial}{\partial t} \right)_k f_t(v) = - \int_{v'} f_t(v) \sigma_k(|v - v'|)|v - v'|d^3v'. \]

- Source term for binary: compute the energy expectation value and dispersion, and generate the distribution function
Simpler treatment: adding heating term

- Local heating rate:
  \[ \frac{dE}{dt} \text{ as a function of local density and velocity} \]

- Simple estimate: formation rate \( \times \) energy release per binary

- Amount of energy release = \( \Delta E \) until ejection

- Fokker-Planck coefficient:
  \[ H_E = \frac{\int \frac{dE}{dt} vr^2 dr}{\int vr^2 dr} \]
Example of solutions: heating by three-binaries
Numerical Simulation of BH Binary Formation and Evolution

- Direct N-body simulations
- Spherical, non-rotating models in static tidal field
- Composed of ordinary and degenerate stars
  - Ordinary stars: \(0.7 \, M_☉\) (actual average mass could be lower)
  - \(1.4 \, M_☉\), representing NS
  - \(10 \sim 40 \, M_☉\), representing BH
    - Single component
    - Two component
    - Continuous mass spectrum
- \(N = 5,000 \sim 50,000\)
- No Primordial binaries (cf, Banerjee 2017 for the effect of primordial binaries)
Some Concerns of the Simulations

- $N \ll$ actual number of stars in GC
  - We try to look for properties that are independent of $N$
- No stellar evolution
  - Stellar evolution is important only in the very early phase of evolution ($\sim 10^7$ - $10^8$ years)
- No initial BH binaries
  - Soft binaries easily disrupted
  - Hard binaries behave same way as dynamical binaries, and therefore our results provide conservative lower limits
- No mass function for normal stars
- No direct capture is included
  - Ratio between three-body processes and direct capture depends strongly on $N$
  - Direct capture will be investigated separately with Fokker-Planck code
Example of N-body Run

BH mass = 10 M\text{sun}
NS mass = 1.4 M\text{sun}
Normal Stars = 0.7 M\text{sun}

Red: BH
Blue: NS
Green: Normal
Multiple interaction leads to very tight binaries and binaries get ejected

\[ x \equiv \frac{G m_1 m_2}{2a} \cdot \frac{1}{3 m_* \sigma^2 / 2} \]

\[ \frac{1}{t_r(0)} : 4.29 \]

binary was ejected here!
Properties of ejected binaries

- Ejection occurs as the binaries become very hard, consistent with the predictions
- But actual distribution of $x$ is broad
- The relative velocity to the cluster is $> v_e$. 

\[
x = \frac{G m_1 m_2 / a}{3 < m_a^2 > \sigma^2 / 2}
\]
How can we use these results?

• The distribution of orbital separation depends only on the velocity dispersion

• The eccentricity distribution is almost invariant of any parameters (i.e., thermal distribution)

• Consequently ‘merging fraction’ depends on the central velocity dispersion

Bae, Kim & Lee 2014
Models with two-mass BH: 10 & 20 $M_\odot$

- Higher mass BHs form binaries first and the lower mass ones follow.
- Time gap between formation and ejection is a few $t_{rh}$, but it could be smaller for realistic systems.

Park et al., (2017)
Time gap between formation and ejection

- As $N$ grows, $\Delta t$ becomes smaller.
- There is a long tail of large $\Delta t$
- Those with large $\Delta t$ would merge in the cluster, rather than get ejected.
Long lived binaries are very hard ones.
Orbital Evolution of the Long-Lived Binary

- Experiences many weak encounters
- Composed of lower mass BHs
Efficiency of Binary Formation with Mass

- Higher mass BHs form binaries more efficiently
- ‘Mass function’ obtained by GW observation would be skewed toward higher mass.

Park et al, (2017)
Continuous BH mass function

- BH mass function depends on metallicity

- Biased formation of higher mass BH binaries also seen here

Park et al., (2017)

BH mass function by Belczynski et al. (2016)
Mass ratios of BH binaries

- In two-component case, binaries with mixed masses (i.e., 10 - 20 $M_\odot$) are very rare
- Similar trend is seen in models with continuous BH mass function
  - Binaries with $m_2/m_1 > 2$ would be extremely rare
  - Conflict with GW151226?
Galactic Globular Clusters

Assuming $x = x_{\text{crit}}$, $f(e) = 2e$

Median $V_{\text{esc}} = 20$ km/sec

Fraction of merger binaries within Hubble time
Estimation of merger rate

• Assume 0.45 % NS (but 10% of this remains) and 0.18% BH by number in GC

• 15% of these objects escape in the form of compact binaries

• We used 142 clusters in the catalogue by Harris (2010) with mass and velocity dispersion http://physwww.mcmaster.ca/~harris/Databases.html
  • We computed the number of binaries whose merging time is shorter than Hubble time for each cluster and add them up

• Number density of globular clusters $n_{GC} = 8.7 \, h^3 \, \text{Mpc}^{-3}$ and $h=0.72$

• Results
  • $0.1 - 1 \, \text{yr}^{-1}$ within advanced detector range
  • $\sim 10 \, \text{yr}^{-1} \, \text{Gpc}^{-3}$ for BH-BH merger (note LIGO estimation: $12 \sim 210 \, \text{yr}^{-1} \, \text{Gpc}^{-3}$)
  • Very small number of NS-BH binaries
Uncertainties

- Initial Mass function could have been top heavy for in low metallicity environments
- Clusters with relatively large $\sigma$ could have produced direct capture binaries
- Many GCs have been already disrupted due to
  - Galactic tidal field
  - Tidal Shocks
  - Dynamical friction

Gnedin & Ostriker 1997

IMPRESS Blackboard Lecture, July 28, 2017

Hyung Mok Lee
Planned Improvements

• More careful assessment of dependence on $N$
• Effects of the mass function of the ordinary stars
  • We assumed 0.7 $M_{\text{Sun}}$, but more realistic mean mass is much lower
• The rotation of the clusters could accelerate the dynamical
• Consideration of captured binaries.
Summary

• Black hole binaries can be formed by dynamical processes in globular clusters efficiently
• Most of them are formed by three-body processes.
• Some of them will merge in Hubble time after getting ejected
  • Mass ratios are likely to be less than 2
  • Massive BHs have higher chances in forming binaries
  • Lower limit for the merger rate is estimated to be $\sim 10 \, yr^{-1} \, Gpc^{-3}$
  • Some binaries could have been merged within the cluster ($<10\%$). They are mostly composed of lower mass BHs.
• Substantial uncertainties in the estimated rate exist, but actual rate could be higher