Dynamics of star clusters containing stellar mass black holes: 4. Formation and Interaction of Binaries

July 28, 2017
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Binaries in Star Clusters

- Large fraction of stars are in the form of binaries
- Binaries can be also be formed via tidal capture, gravitational wav capture or three-body processes
- Binary orbits can be significantly affected by the interaction with other objects in dense environments
- The dynamical properties of the binaries are determined by the 'hardness' of the binaries
- Hardness is defined as the binding energy in units of the average kinetic energy of surrounding stars, i.e.,

$$x \equiv \frac{Gm^2/2a}{3m\sigma^2/2}$$

Tidal capture binary formation

- When two stars approach each other tidal forces lead to the excitation of oscillation
- Oscillation energy is extracted from the orbital energy
- If the amount of dissipated energy is greater than the positive binding energy, the stars become bound
- Cross section (Lee & Ostriker 1987):

$$\sigma_{cap} = 7R_*^2(v_\infty/v_*)^{-2.18}$$

where v_* is the escape velocity from the stellar surface and v_∞ is the relative velocity at infinity

- These binaries are very tight. After circularization the orbital radius would be

$$a_c < 2.1(v_\infty/v_*)^{-0.19}R_* \sim \ 4R_*$$

- It corresponds to hardness of ~1000

Gravitational Wave Capture

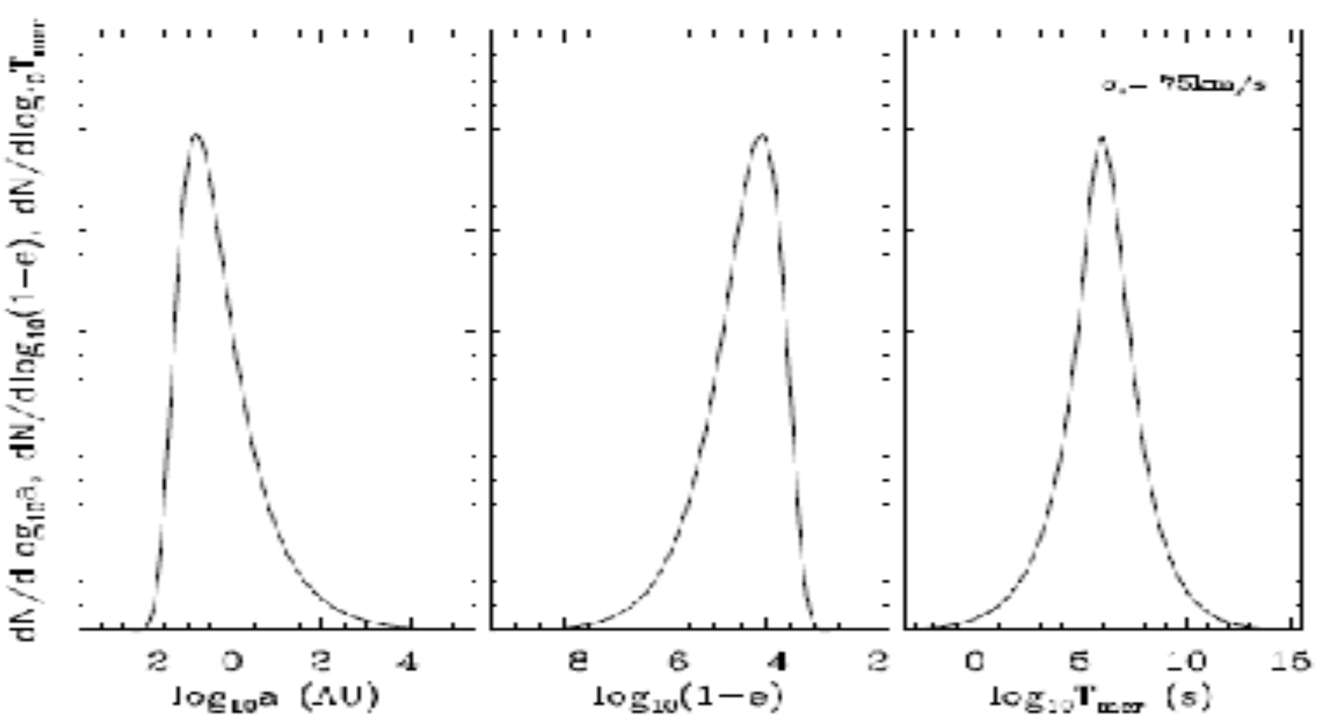
- If the gravitational radiation energy during close encounter is greater than the binding energy two compact stars can become bound i.e., $(\Delta E)_{GR} > \frac{1}{2}\mu v_\infty^2$
- The encounter stars with hyperbolic orbit, but the orbit is close to the parabolic one near the pericenter where most of the gravitation waves are radiated.
- Thus we may assume that the orbit is parabolic which can be specified by angular momentum only: parabolic approximation
- The resulting cross section:

$$\sigma_{cap} = 17 \frac{G^2 m_1 m_2 \eta^{-5/7}}{c^{10/7} v_\infty^{18/7}}$$

$$\eta \equiv \frac{m_1 m_2}{(m_1 + m_2)^2}$$

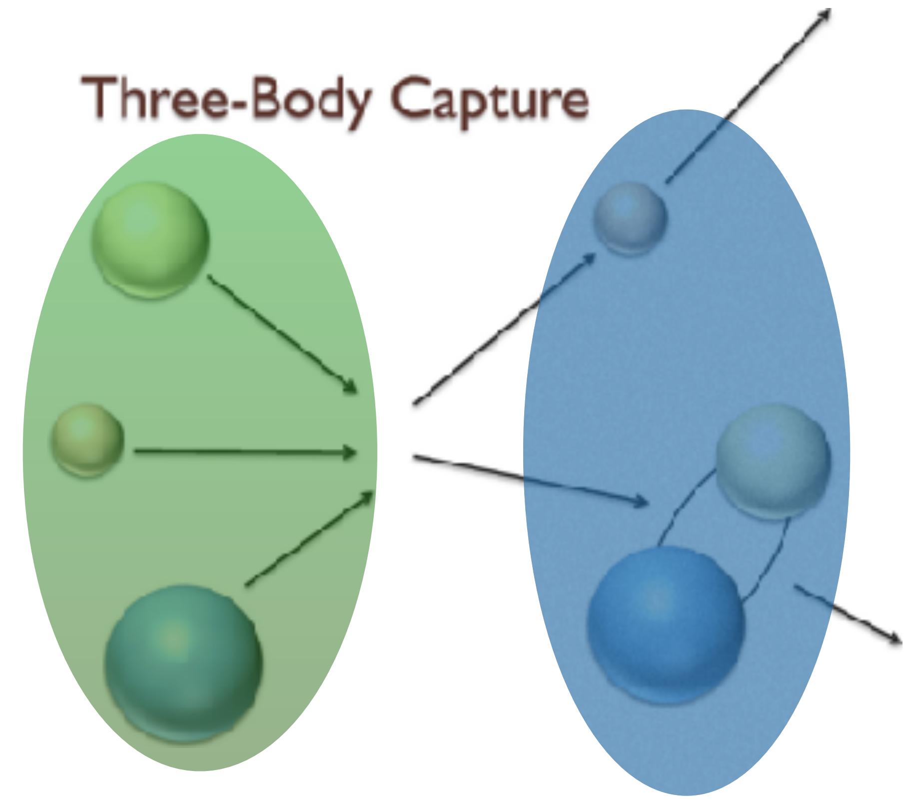
Properties of captured binaries

- Tight: $a \sim 0.05 \dfrac{Gm}{\sigma^2}$
- Very eccentric: $1 - e \sim 10^{-4} \left(\dfrac{\sigma}{75 \mathrm{km/s}} \right)^{10/7}$
- Merge very quickly:



Hong & Lee 2015

Binaries formed by three-body processes



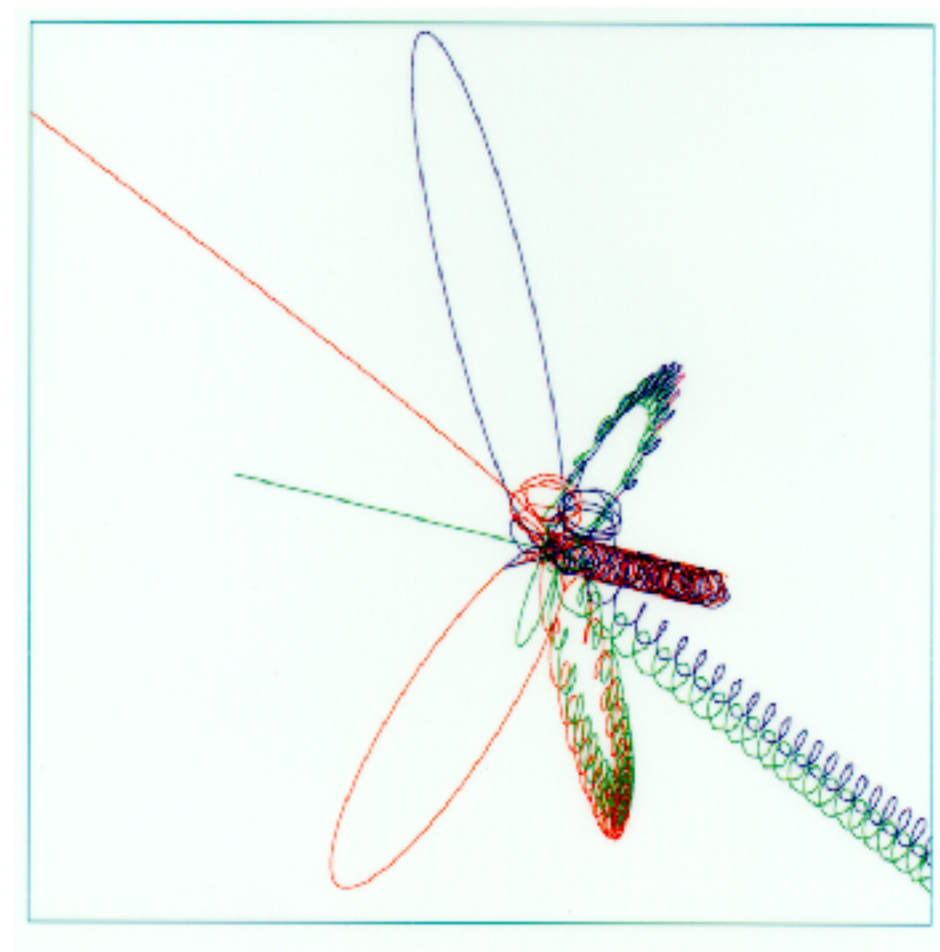
- If three stars are in a small volume, they form a temporary triple system which is rather unstable
- One star gets ejected, leaving the other two bound.
- The formation rate:
  - Rate of strong encounters per volume:
  - Probability of finding another star within volume of sphere of r_s: Volume x n
  - Therefore the binary formation rate per unit volume becomes
- Hardness is x>3 at the time of formation

$$n^2(\pi r_s^2)v \sim \frac{G^2m^2}{v^3}n^2$$

$$\frac{dn_B}{dt}_{3B} \sim n^3\frac{(Gm)^5}{v^9}$$

Consequences of binary-single interactions

- Types of interactions: flyby, resonant, exchange
- Statistical results:
  - Soft binaries become softe
  - → endothermic (i.e., heat sink)
  - Hard binaries become harder
  - → exothermic (i.e, heat source)
- $\Delta E \sim 0.4|E_{bin}|$ per resonant interactions (i.e., interactions with p~a).
- 2/3 goes to KE of binaries for equal mass case



Hardness of Binaries

- It is useful to define hardness parameter x, as a binding energy of binaries normalized by the average kinetic energy of the background.

$$x \equiv \frac{Gm^2/2a}{3m_*\sigma^2/2}; \quad m_* : \text{stellar mass}$$

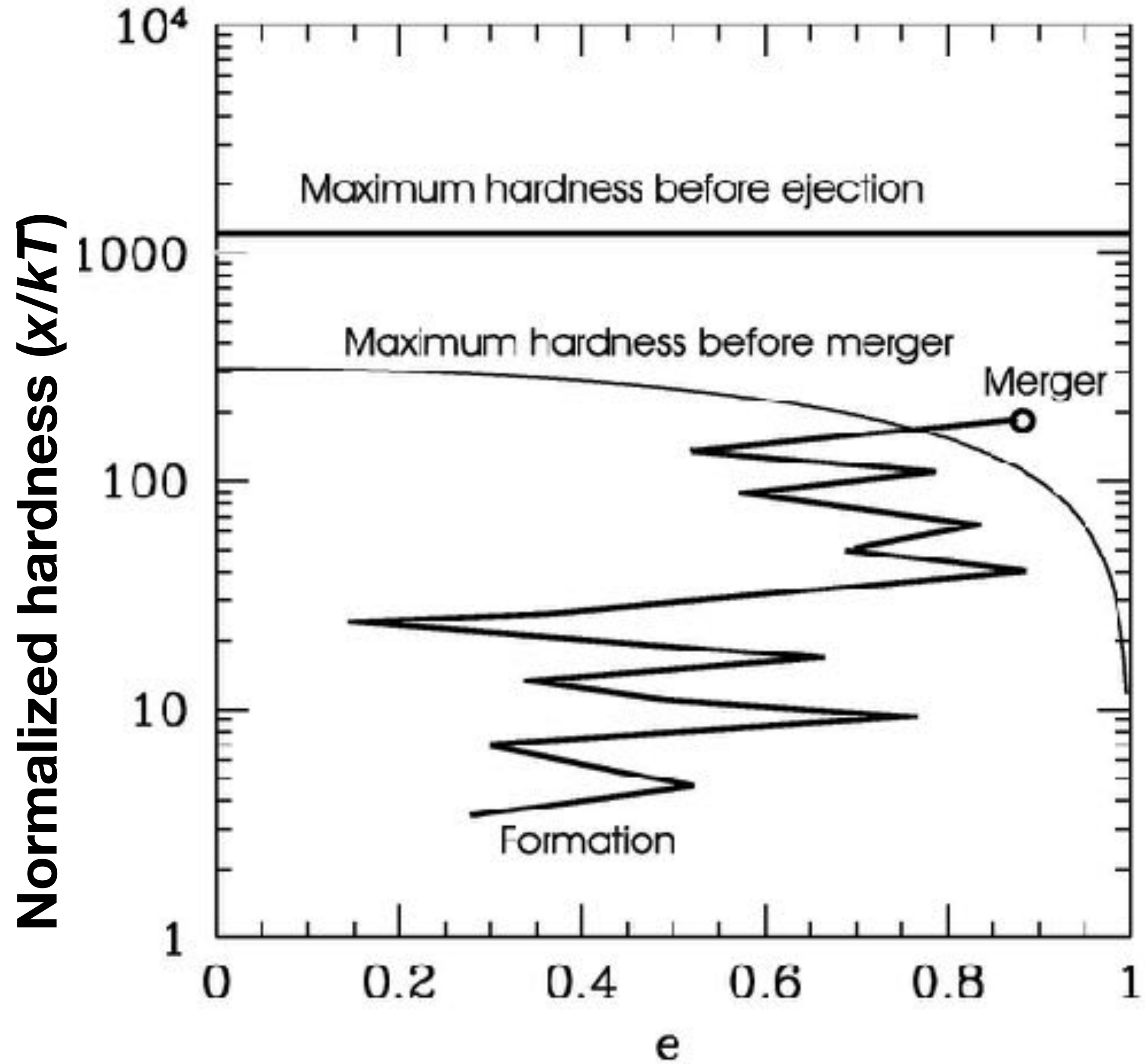
- Binaries get ejected from the cluster if the kick energy exceeds the escape energy: $\Delta E > \frac{1}{2}(2m)v_{esc}^2$

- The above condition for ejection can be re-expressed as

$$x_{crit} \sim 5\left(\frac{m}{m_*}\right)\left(\frac{v_{esc}^2}{\sigma^2}\right), \quad \text{typically} \quad \left(\frac{v_{esc}^2}{\sigma^2}\right) \sim 12$$

- Depending on (m/m_*), x_{crit} could vary significantly.

Evolution of binaries in e-a space



Binaries as heating source

- Soft binaries become resolved quickly
- However, hard binaries become harder, releasing energy, as a result of the interactions with other stars.
- The endothermic nature of binary-single interaction means *heating*
- Heating is highly localized in the central part
- Binaries stabilizes the core collapse, similar to the role of the nuclear reaction in the stellar core
- In practice the clusters expands due to the heating of binaries, after the core-collapse

Consequences of heating

- If the rate of energy conduction becomes smaller than the heating rate, the core collapse does not take place

- In the absence of the radiative cooling, the system cannot be in static equilibrium, but expands because of the increase of energy, i.e.

$$E \approx -0.2 \frac{GM^2}{r_h}$$

and r_h must increase if E increases.

Treatment of Binaries formation in Fokker-Planck Framework

- Binary formation = source term for binary, but loss term for singles

$$\left(\frac{\partial}{\partial t}\right)_k f_i(\mathbf{v}) = -\int_{\mathbf{v}'} f_i(\mathbf{v})\sigma_k(|\mathbf{v}-\mathbf{v}'|)|\mathbf{v}-\mathbf{v}'|d^3\mathbf{v}'.$$

- Source term for binary: compute the energy expectation value and dispersion, and generate the distribution function

Simpler treatment: adding heating term

- Local heating rate:

dE/dt as a function of local density and velocity

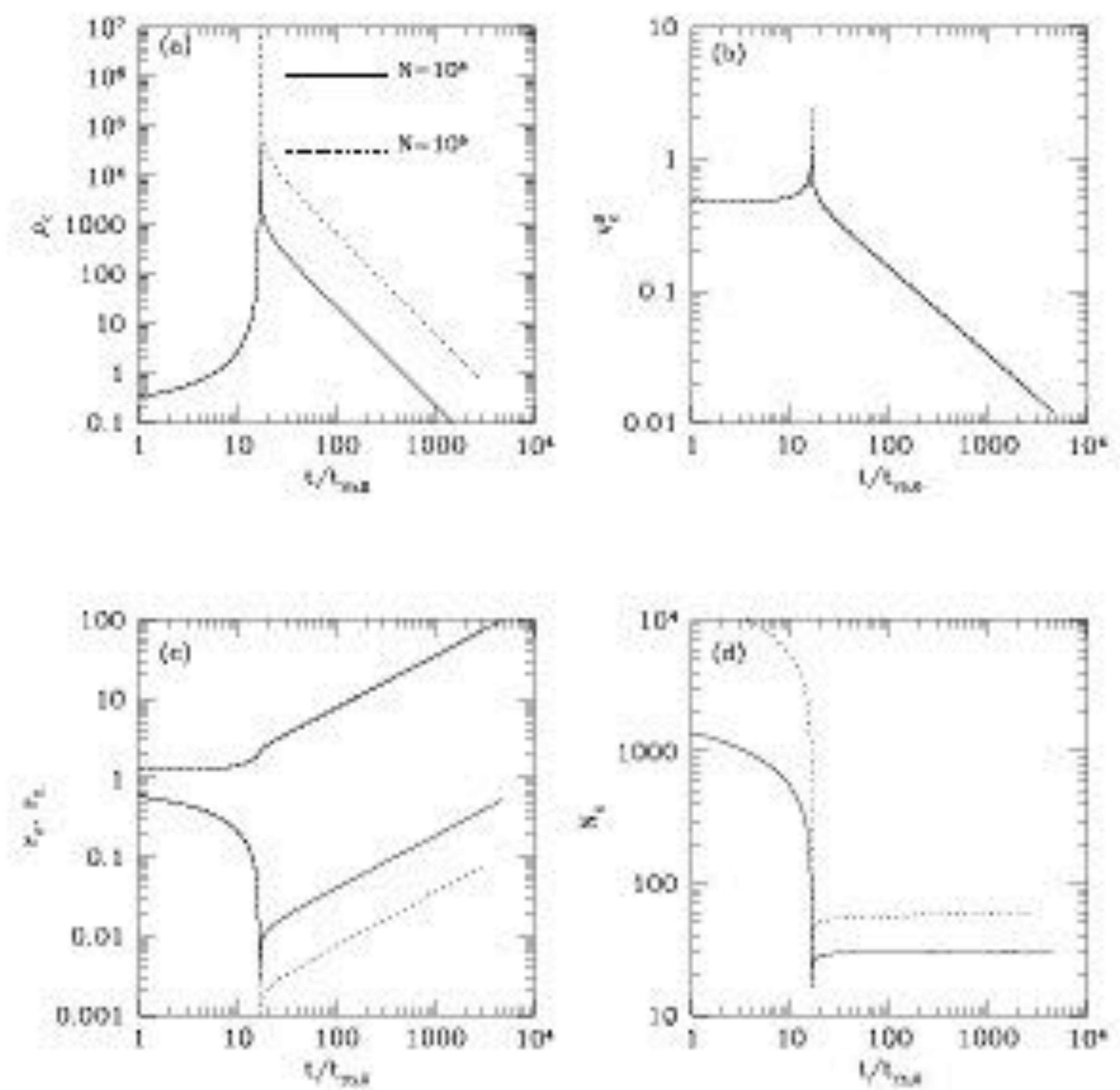
- Simple estimate: formation rate x energy release per binary

- Amount of energy release= ΔE until ejection

- Fokker-Planck coefficient:

$$H_E = \frac{\int \frac{dE}{dt} v r^2 dr}{\int v r^2 dr}$$

Example of solutions: heating by three-binaries



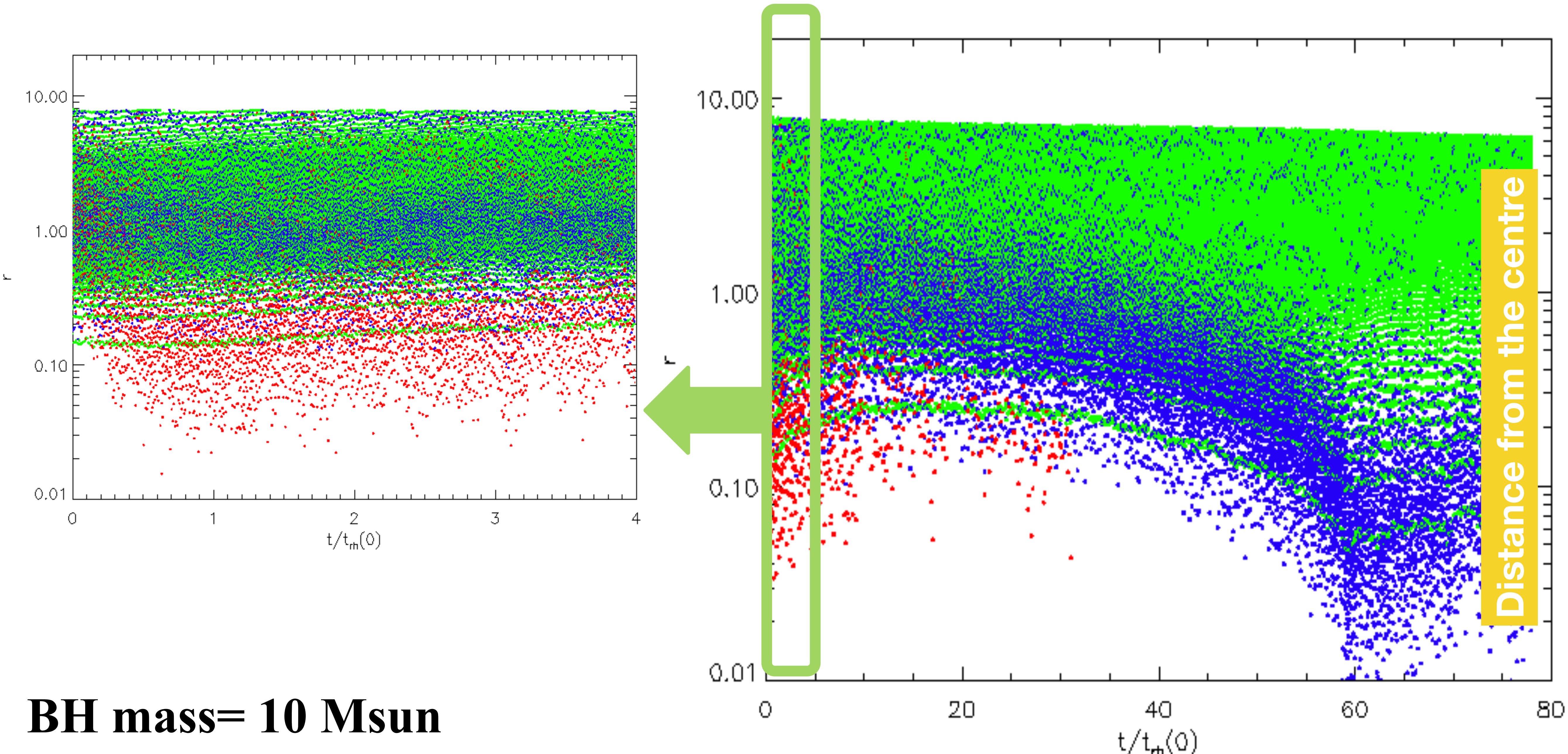
Numerical Simulation of BH Binary Formation and Evolution

- Direct N-body simulations
- Spherical, non-rotating models in static tidal field
- Composed of ordinary and degenerate stars
  - Ordinary stars: 0.7 $M_\odot$ (actual average mass could be lower)
  - 1.4 $M_\odot$, representing NS
  - 10 ~ 40 $M_\odot$, representing BH
    - Single component
    - Two component
    - Continuous mass spectrum
- N=5,000 ~ 50,000
- No Primordial binaries (cf, Banerjee 2017 for the effect of primordial binaries)

Some Concerns of the Simulations

- N<< actual number of stars in GC
  - We try to look for properties that are independent of N
- No stellar evolution
  - Stellar evolution is important only in the very early phase of evolution (~10^7 - 10^8 years)
- No initial BH binaries
  - Soft binaries easily disrupted
  - Hard binaries behave same way as dynamical binaries, and therefore our results provide conservative lower limits
- No mass function for normal stars
- No direct capture is included
  - Ratio between three-body processes and direct capture depends strongly on N
  - Direct capture will be investigated separately with Fokker-Planck code

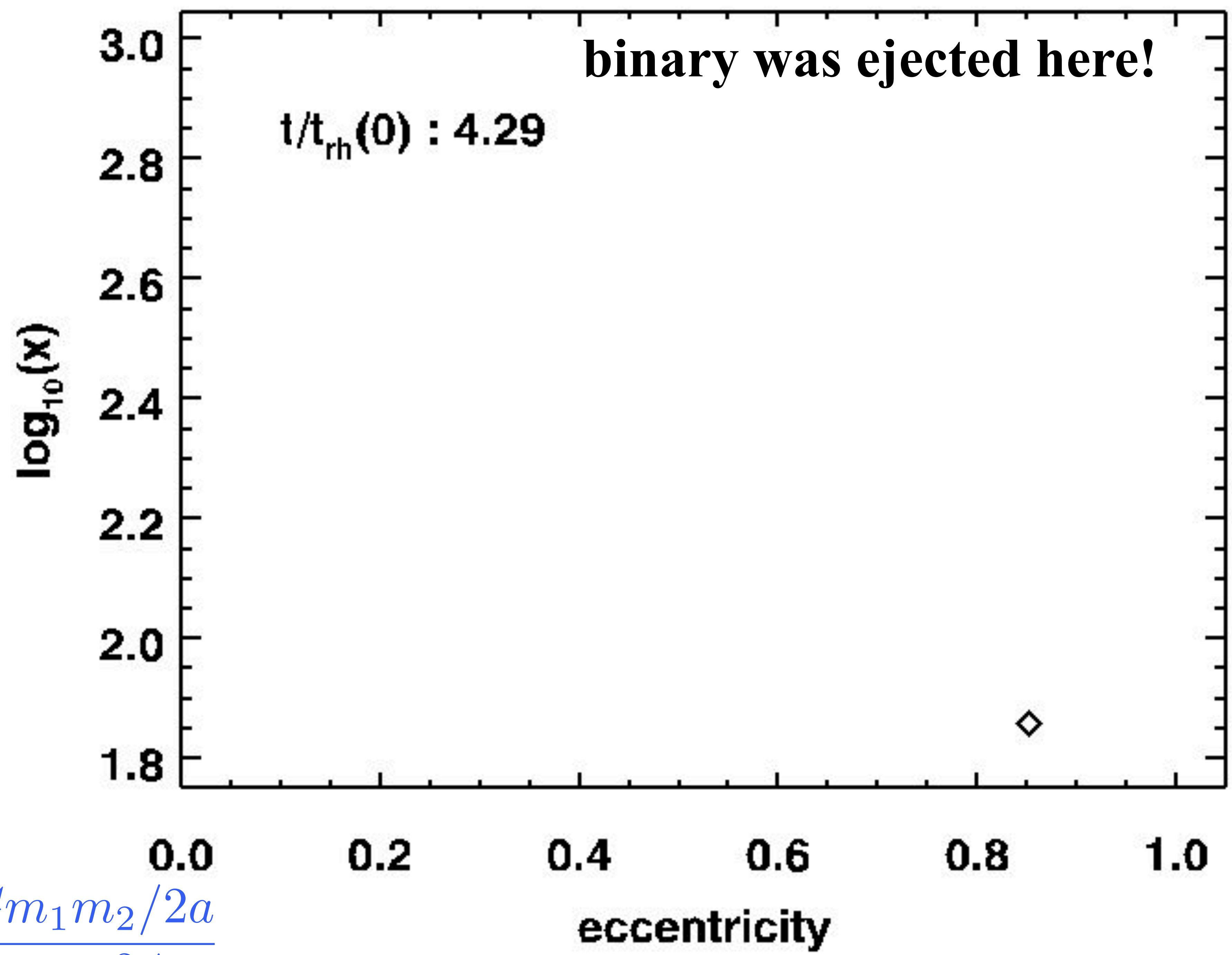
Example of N-body Run



BH mass= 10 Msun
NS mass = 1.4 Msun
Normal Stars = 0.7 Msun

Red: BH
Blue: NS
Green: Normal

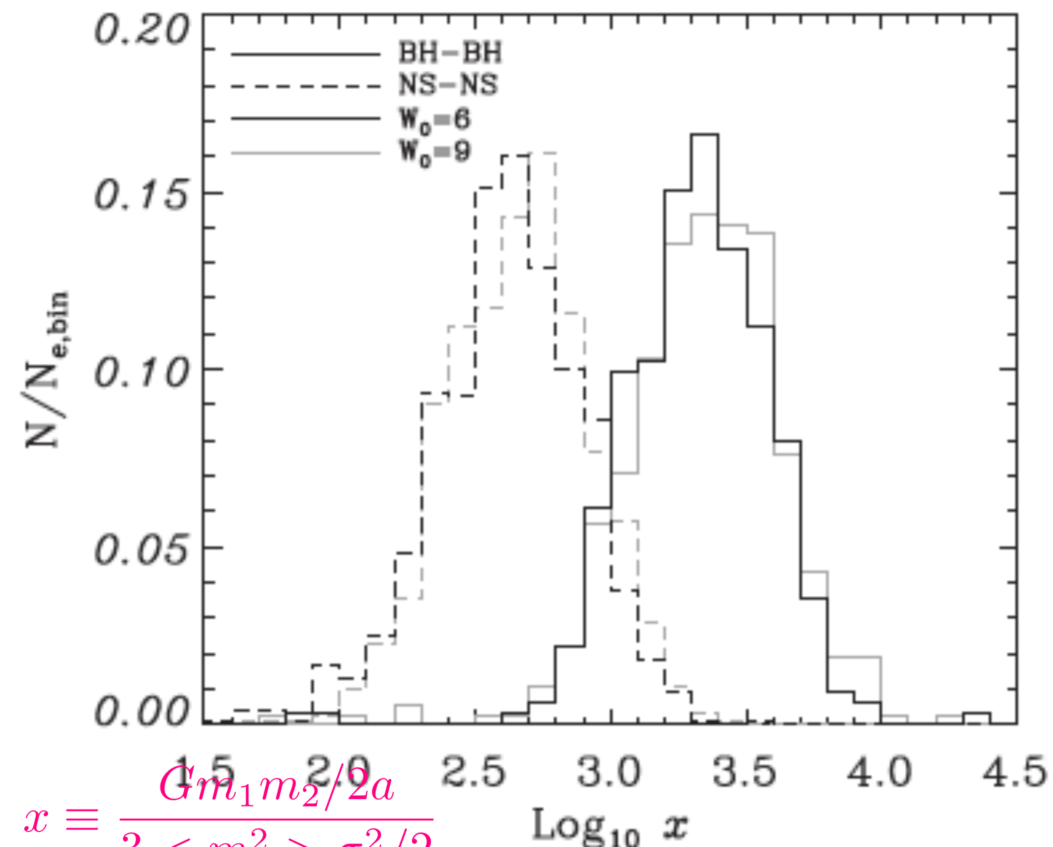
Multiple interaction leads to very tight binaries and binaries get ejected



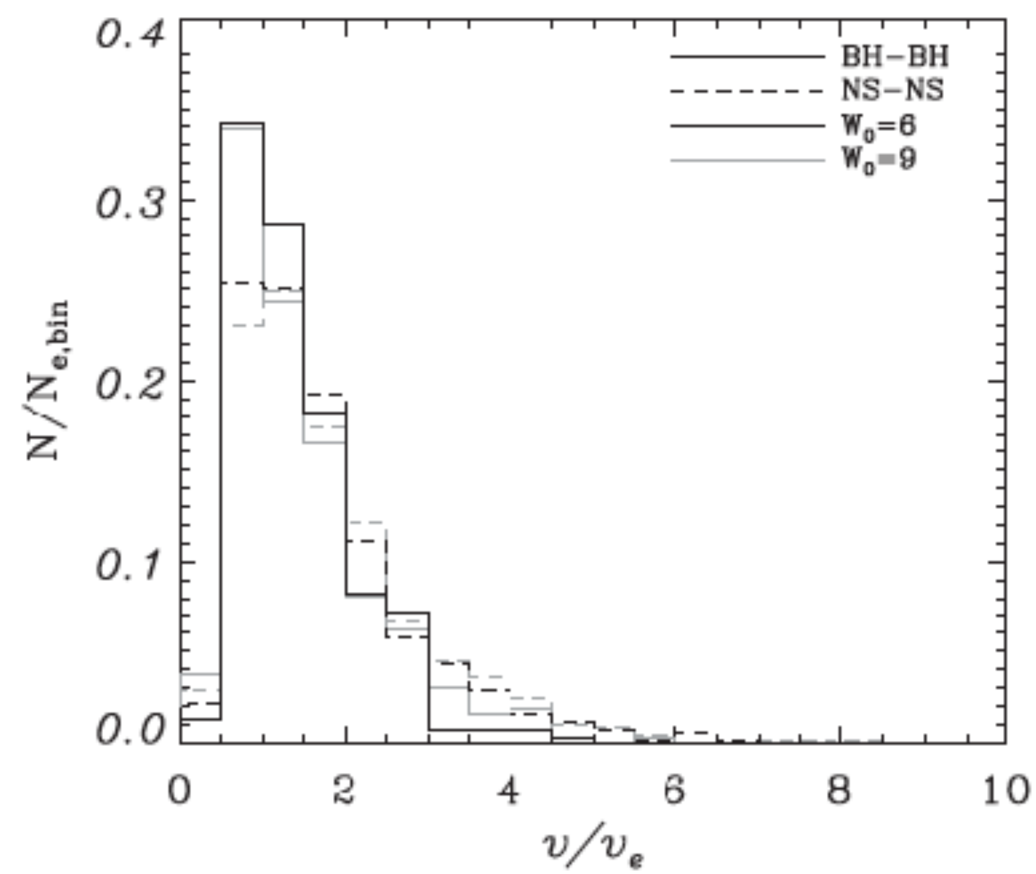
$$x \equiv \frac{Gm_1m_2/2a}{3m_*\sigma^2/2}$$

Properties of ejected binaries

- Ejection occurs as the binaries become very hard, consistent with the predictions
  - But actual distribution of x is broad
- The relative velocity to the cluster is $> v_e$.

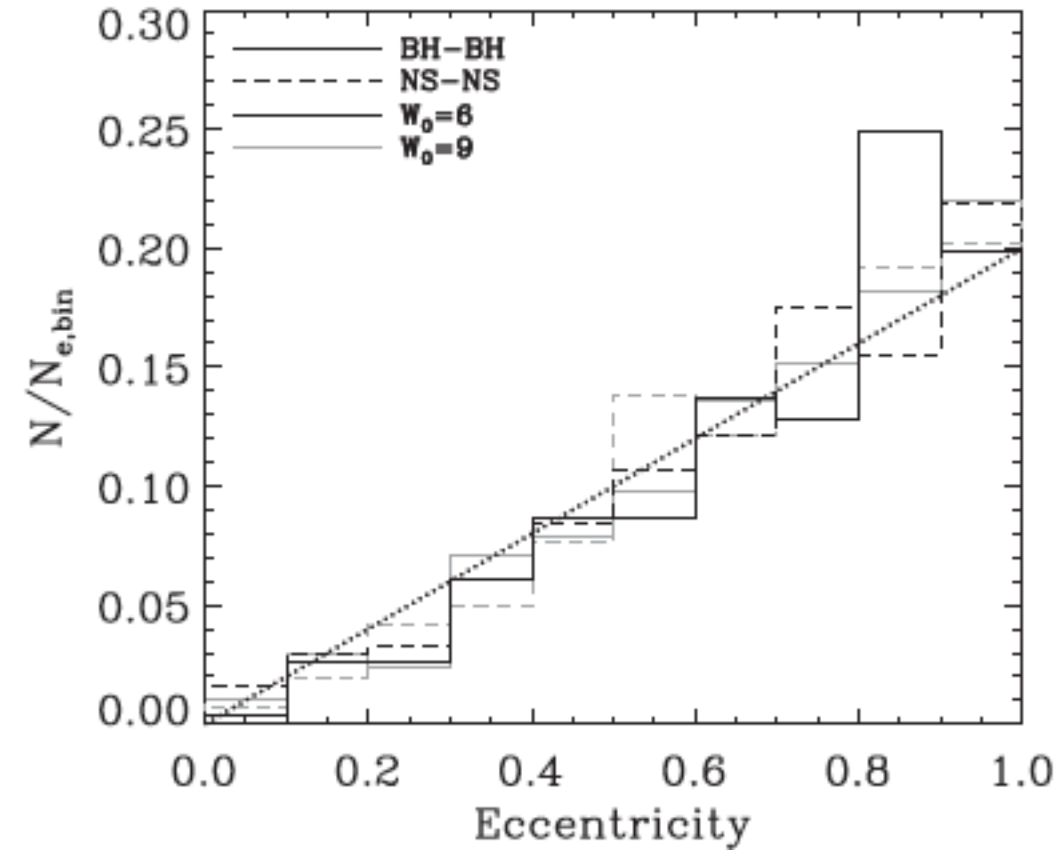


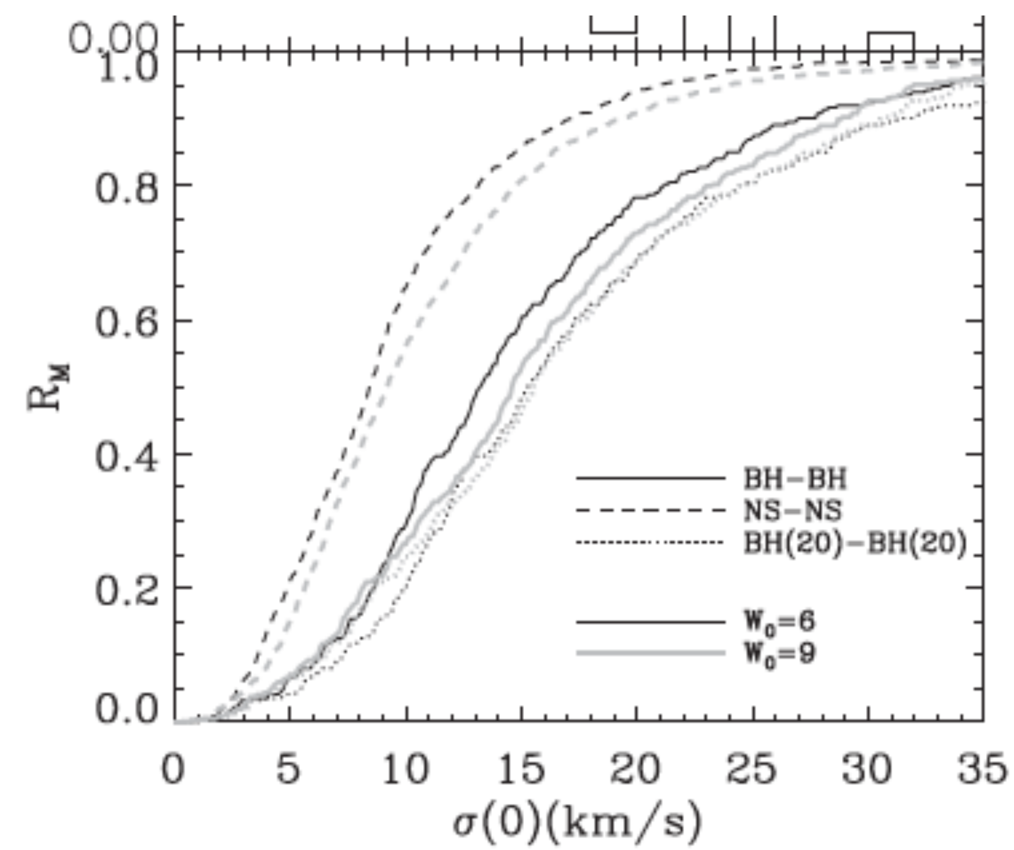
$$x \equiv \frac{Gm_1m_2/2a}{3 < m_*^2 > \sigma^2/2}$$



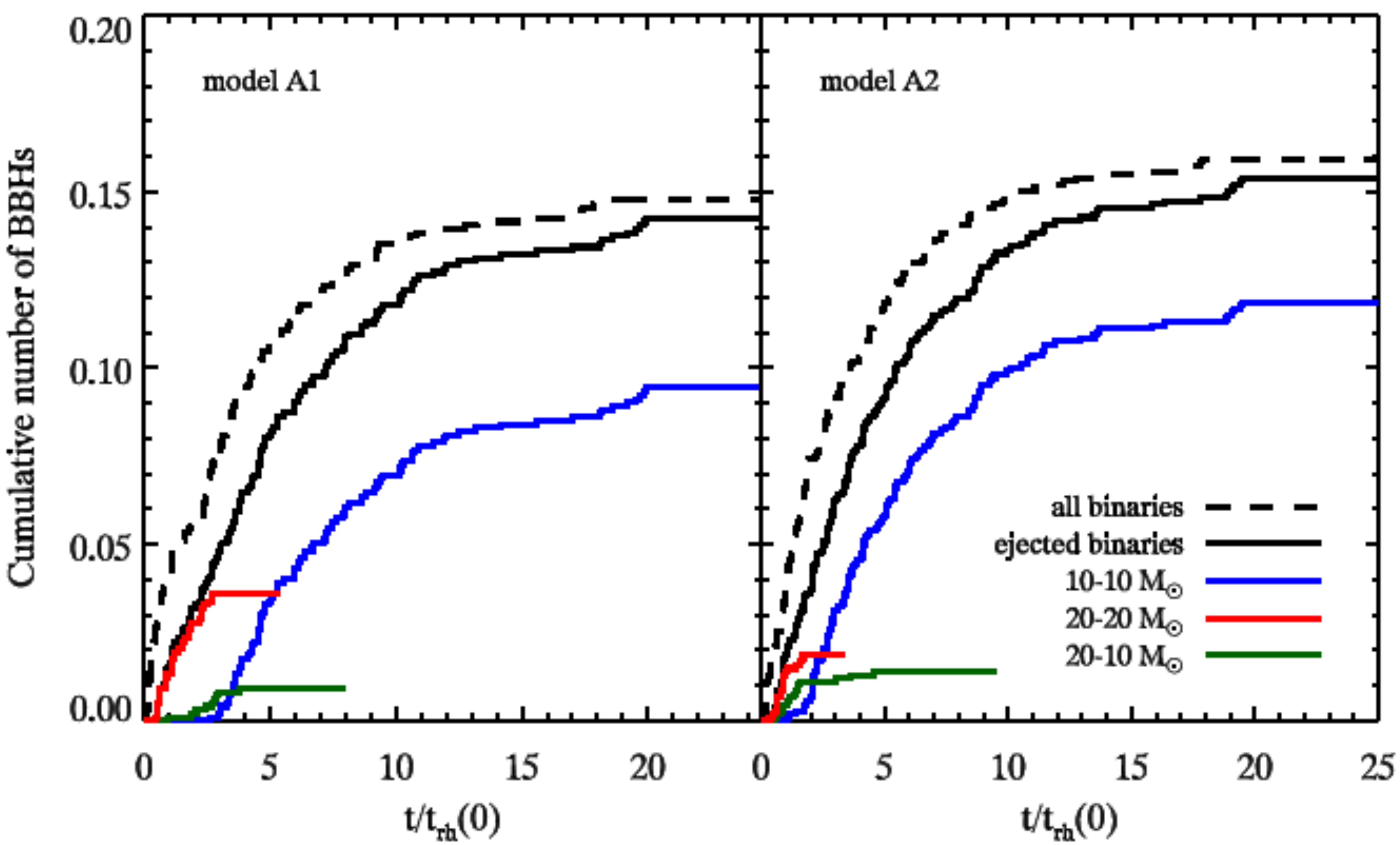
How can we use these results?

- The distribution of orbital separation depends only on the velocity dispersion
- The eccentricity distribution is almost invariant of any parameters (i.e., thermal distribution)
- Consequently 'merging fraction' depends on the central velocity dispersion





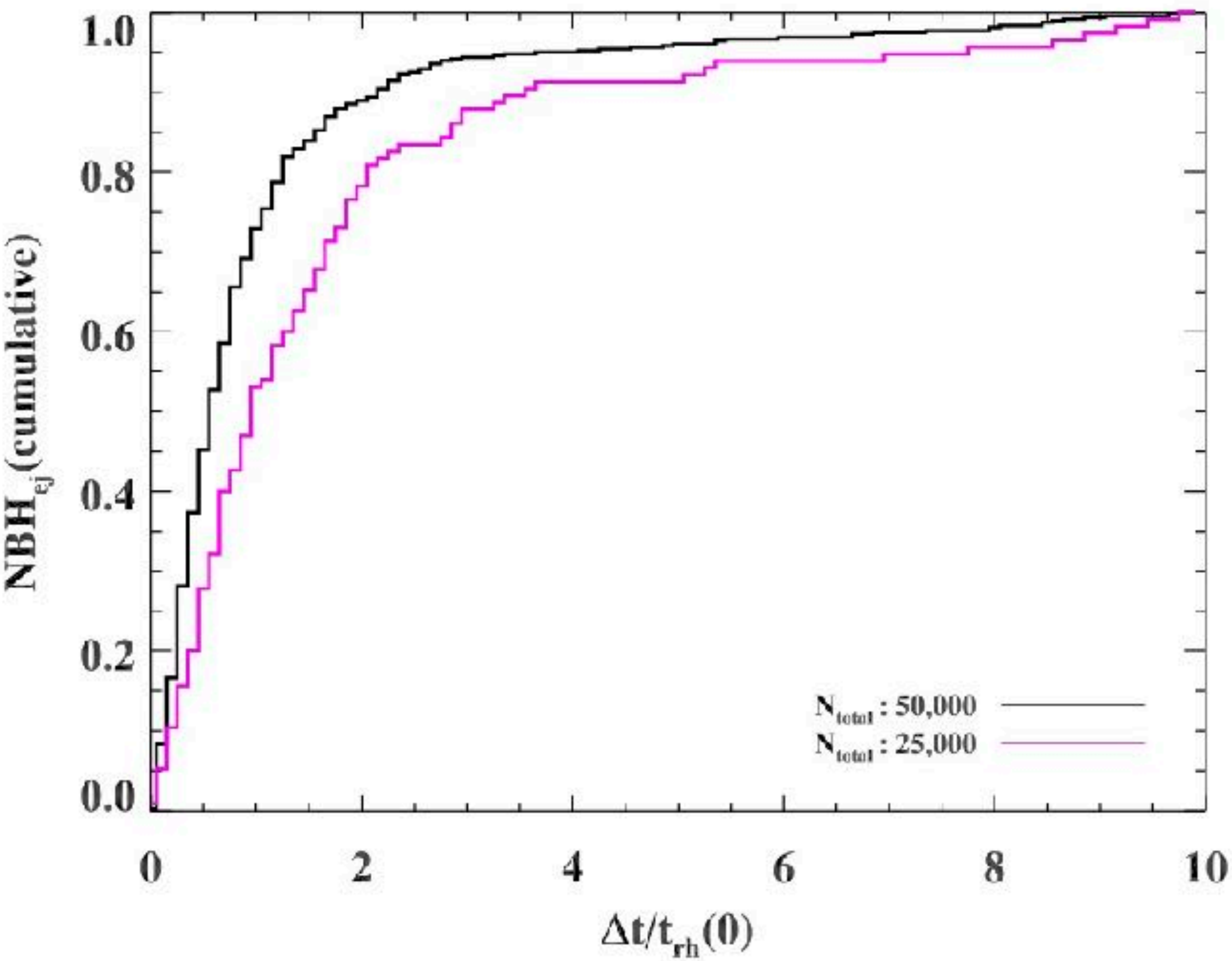
Models with two-mass BH: 10 & 20 M⊙



Park et al., (2017)

- Higher mass BHs form binaries first and the lower mass ones follow
- Time gap between formation and ejection is a few t_{rh}, but it could be smaller for realistic systems

Time gap between formation and ejection



- As N grows, Δt becomes smaller.
- There is a long tail of large Δt
- Those with large Δt would merge in the cluster, rather than get ejected.

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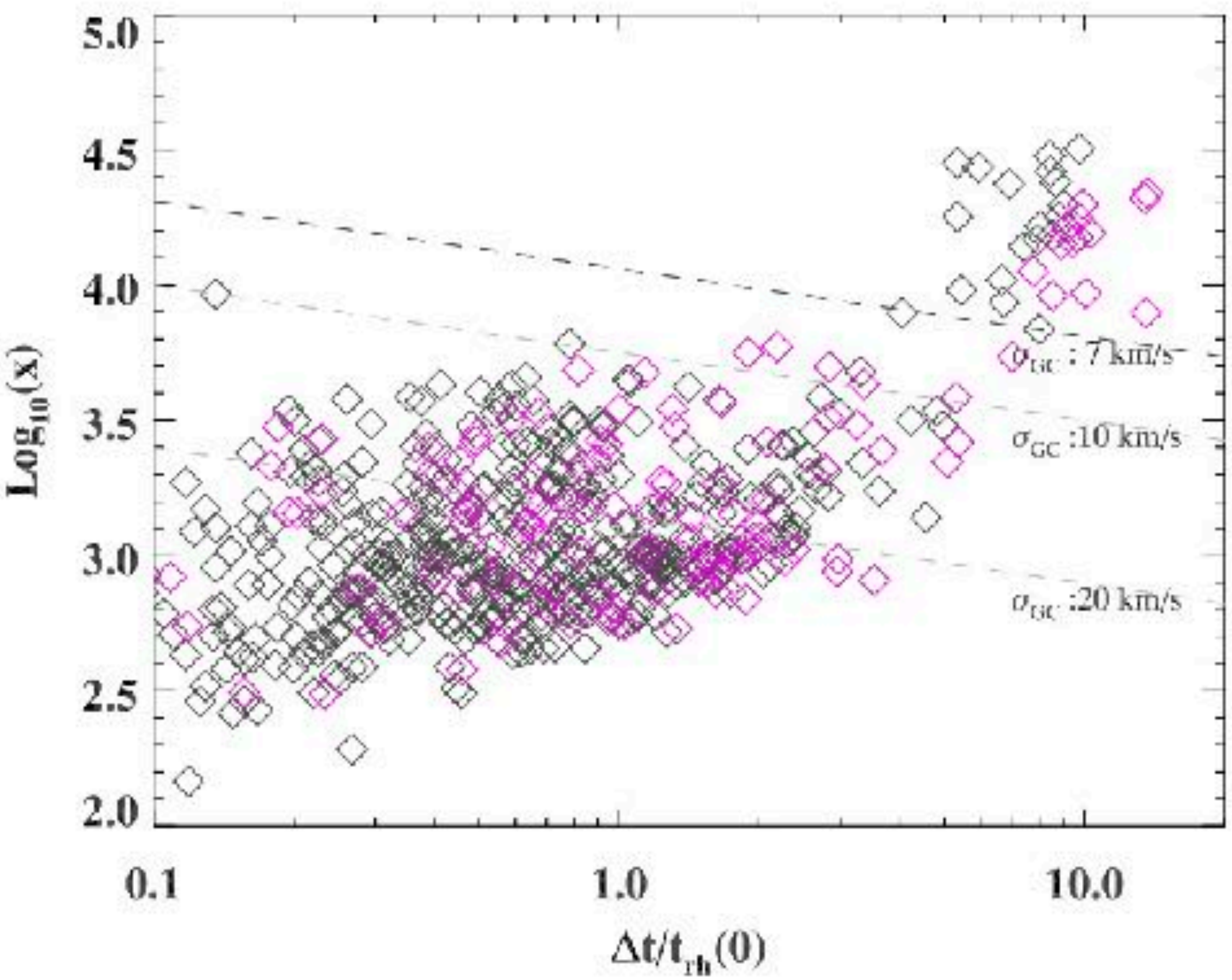
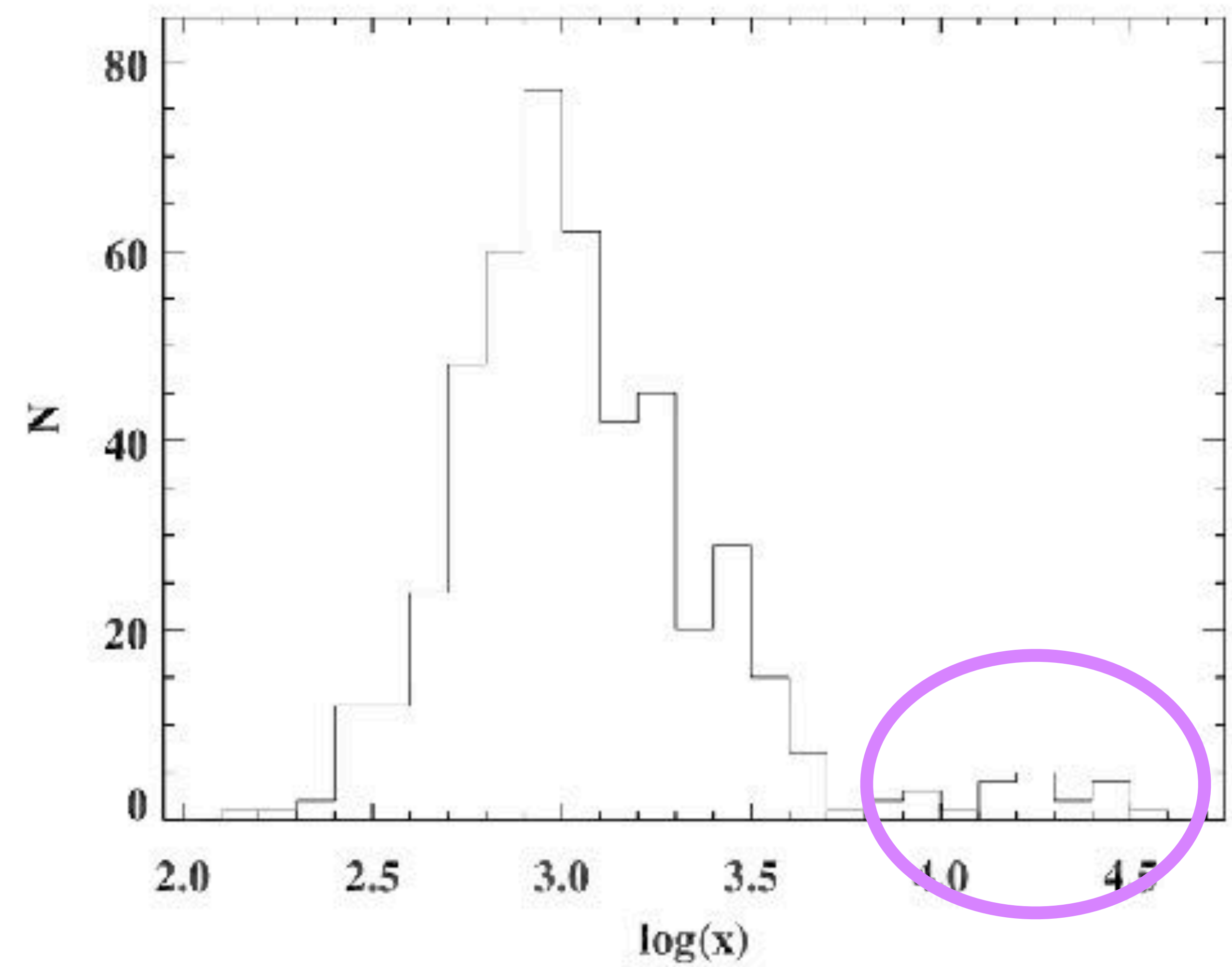
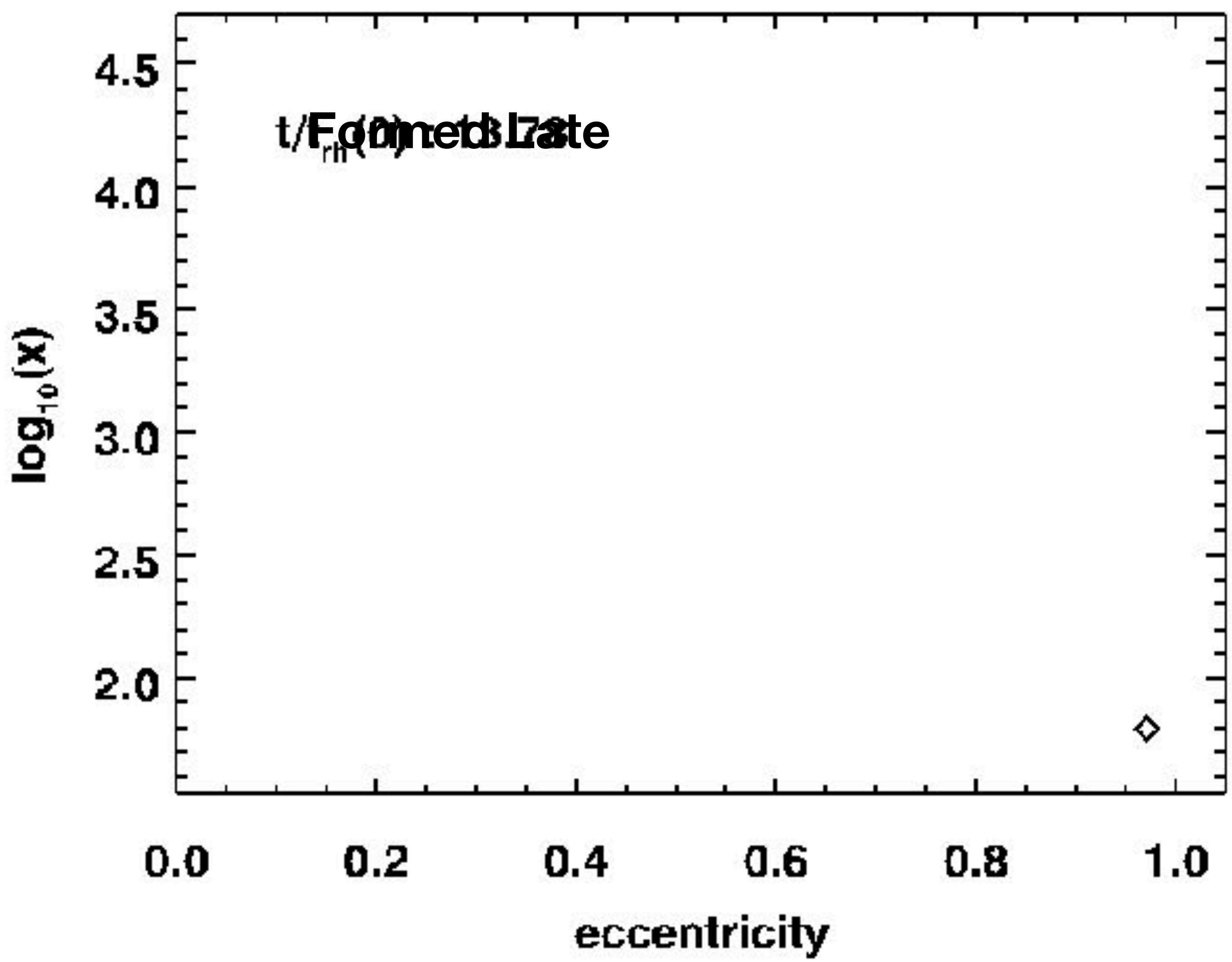
Long lived binaries are very hard ones



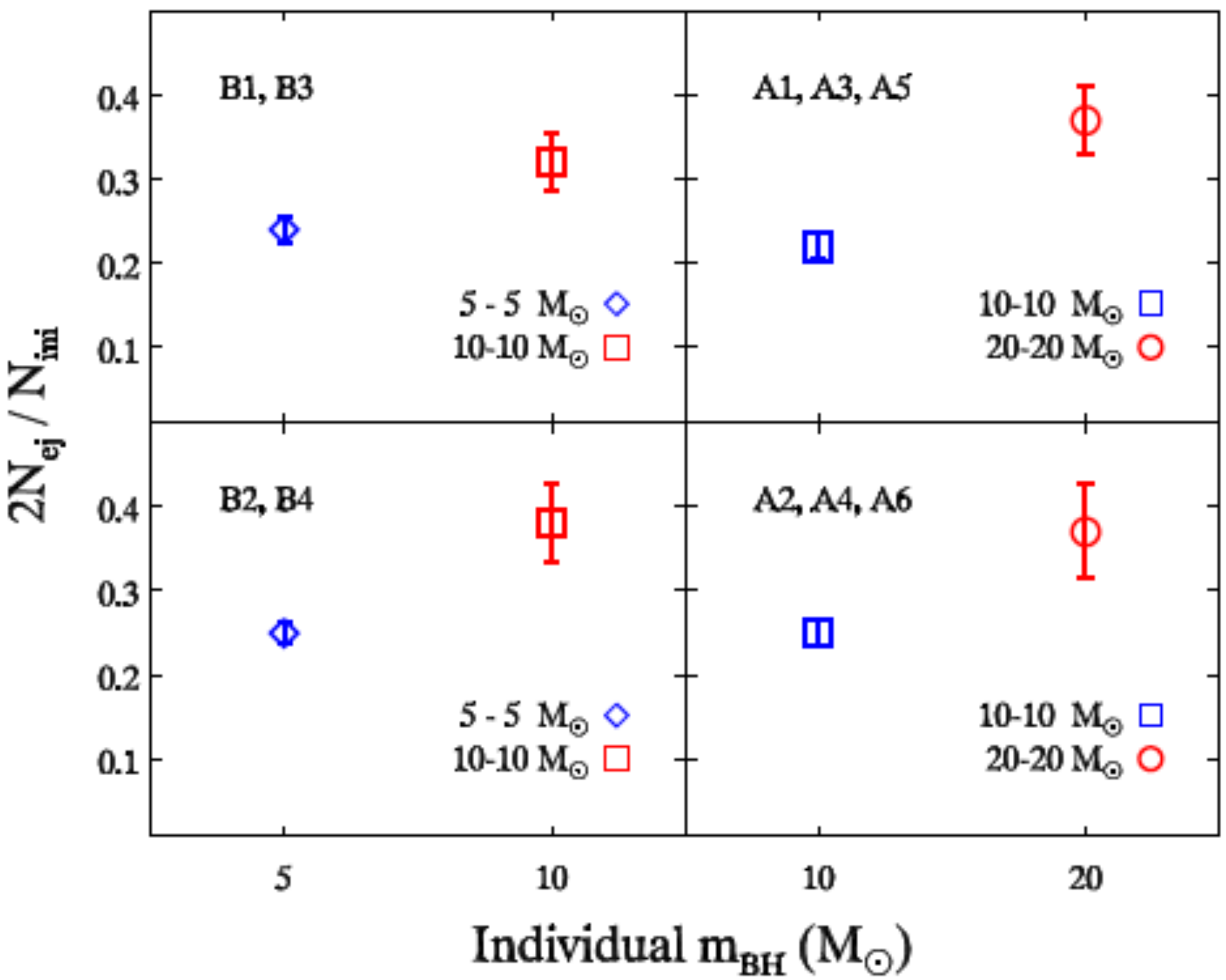


Orbital Evolution of the Long-Lived Binary

- Experiences many weak encounters
- Composed of lower mass BHs



Efficiency of Binary Formation with Mass

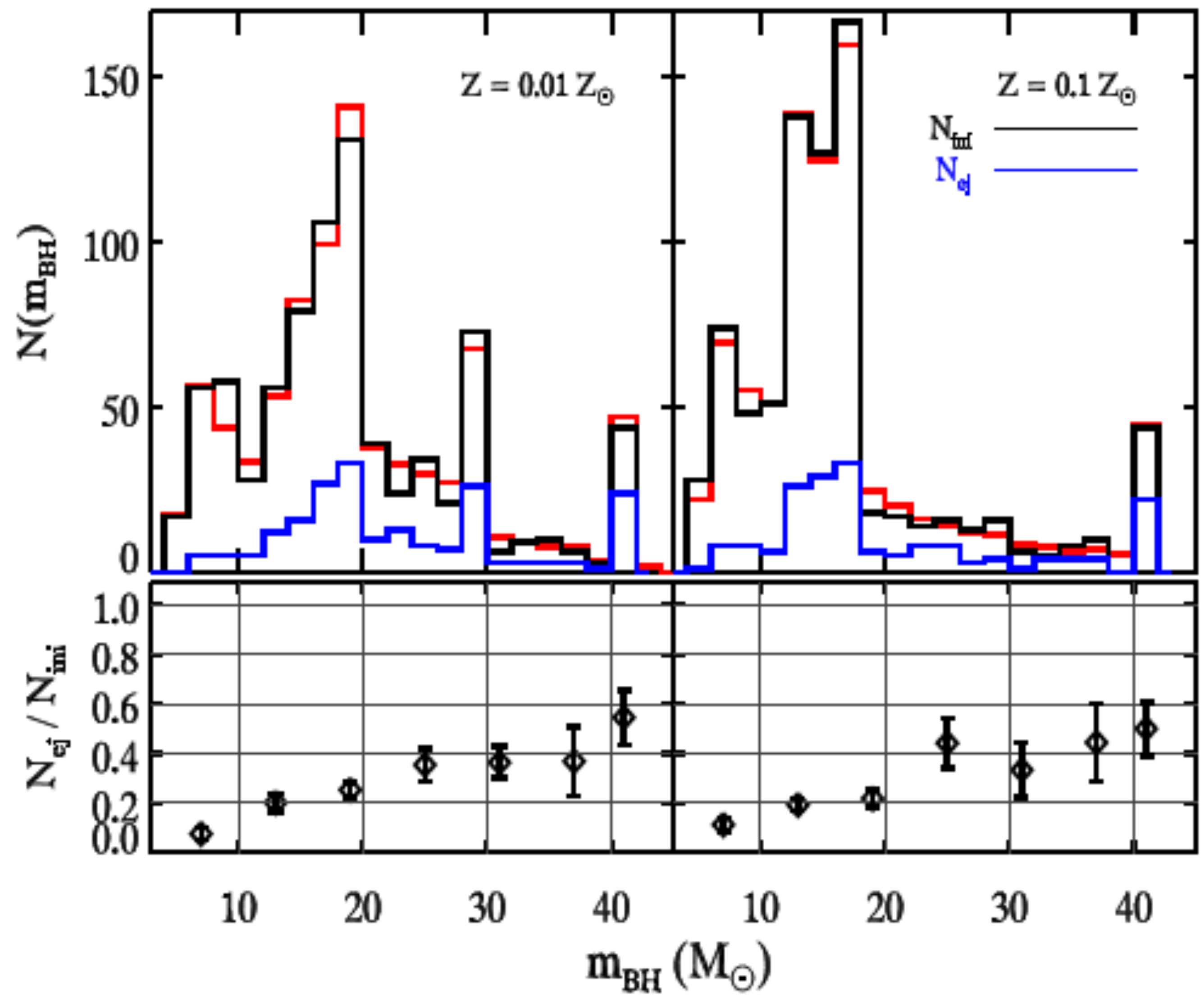


Park et al, (2017)

- Higher mass BHs form binaries more efficiently
- ‘Mass function’ obtained by GW observation would be skewed toward higher mass.

Continuous BH mass function

- BH mass function depends on metallicity
- Biased formation of higher mass BH binaries also seen here

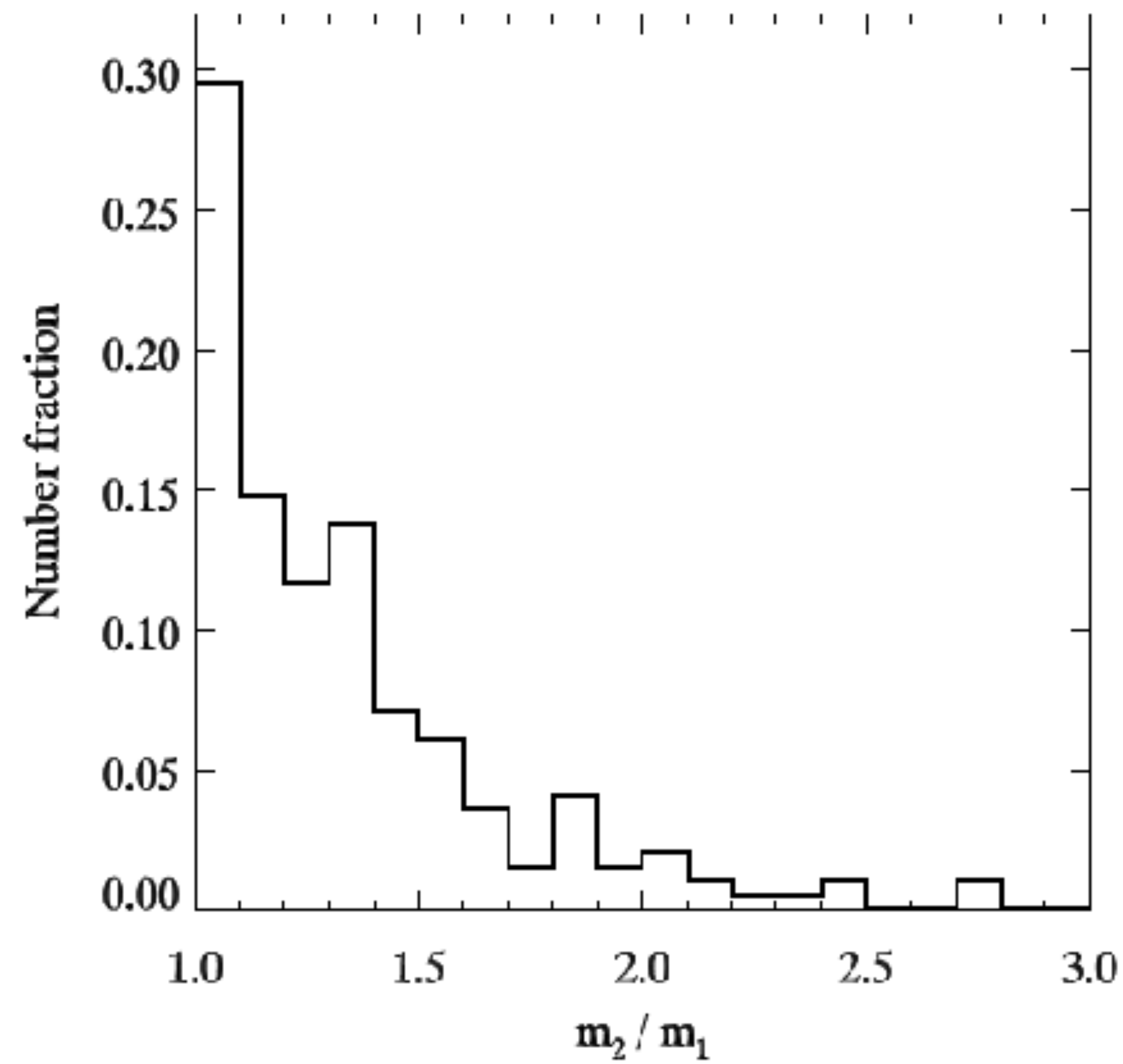


Park et al., (2017)

BH mass function by Belczynski et al. (2016)

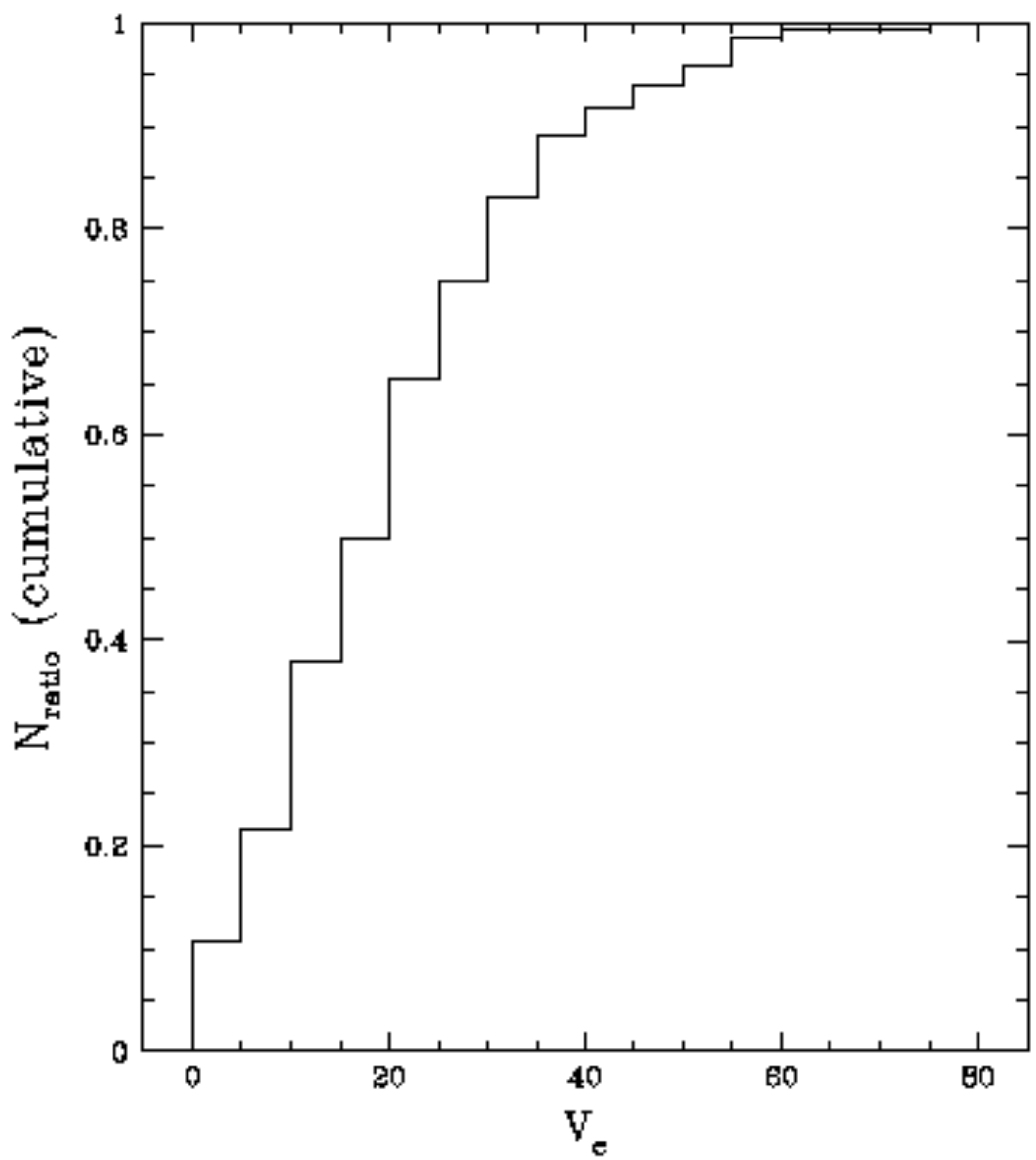
Mass ratios of BH binaries

- In two-component case, binaries with mixed masses (i.e., 10 - 20 $M_{\odot}$) are very rare
- Similar trend is seen in models with continuous BH mass function
  - Binaries with $m_2/m_1 > 2$ would be extremely rare
  - Conflict with GW151226?

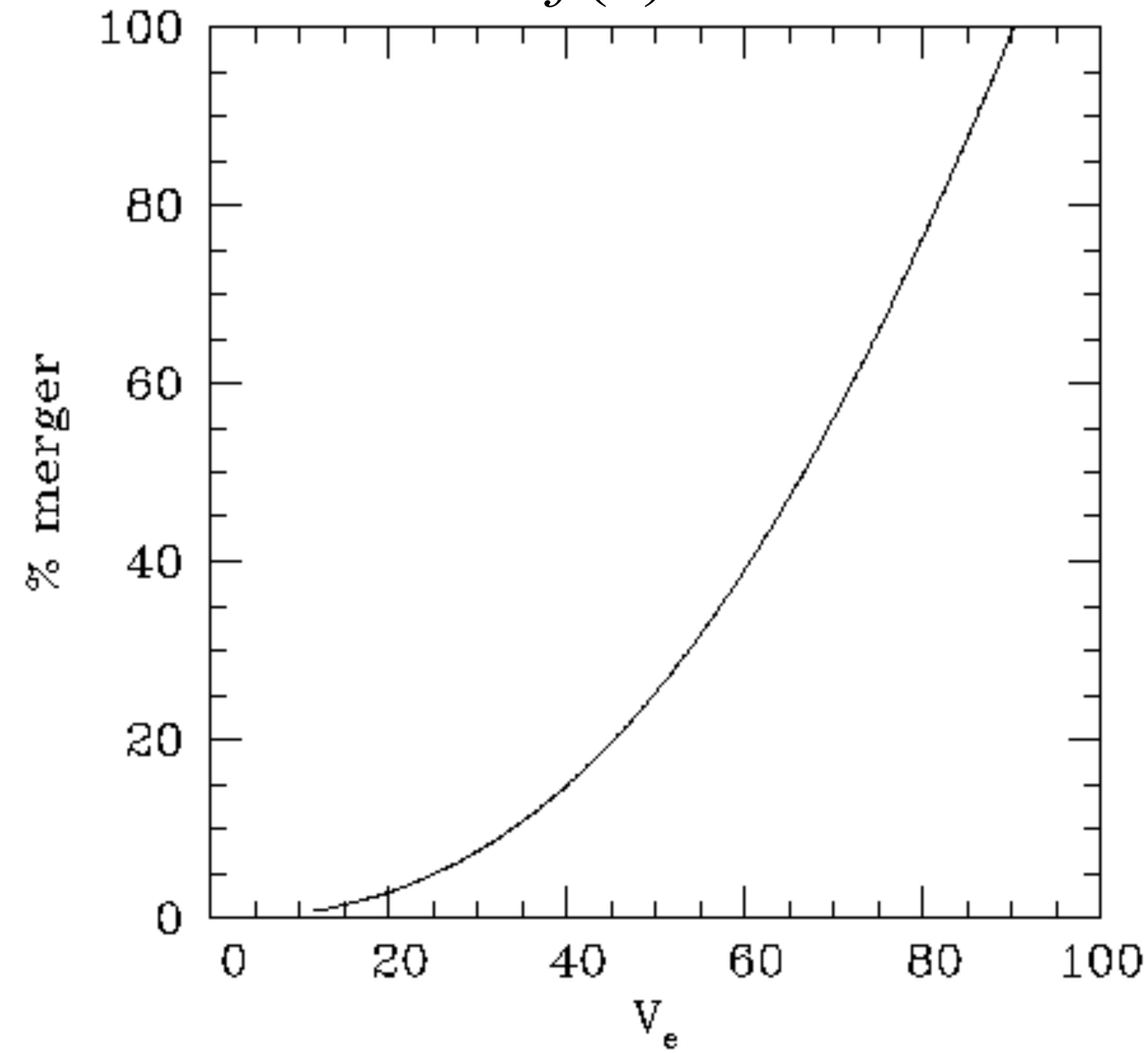


Galactic Globular Clusters

Assuming $x = x_{crit}$,
$f(e) = 2e$



Median V_{esc} = 20 km/sec



Fraction of merger binaries within Hubble time

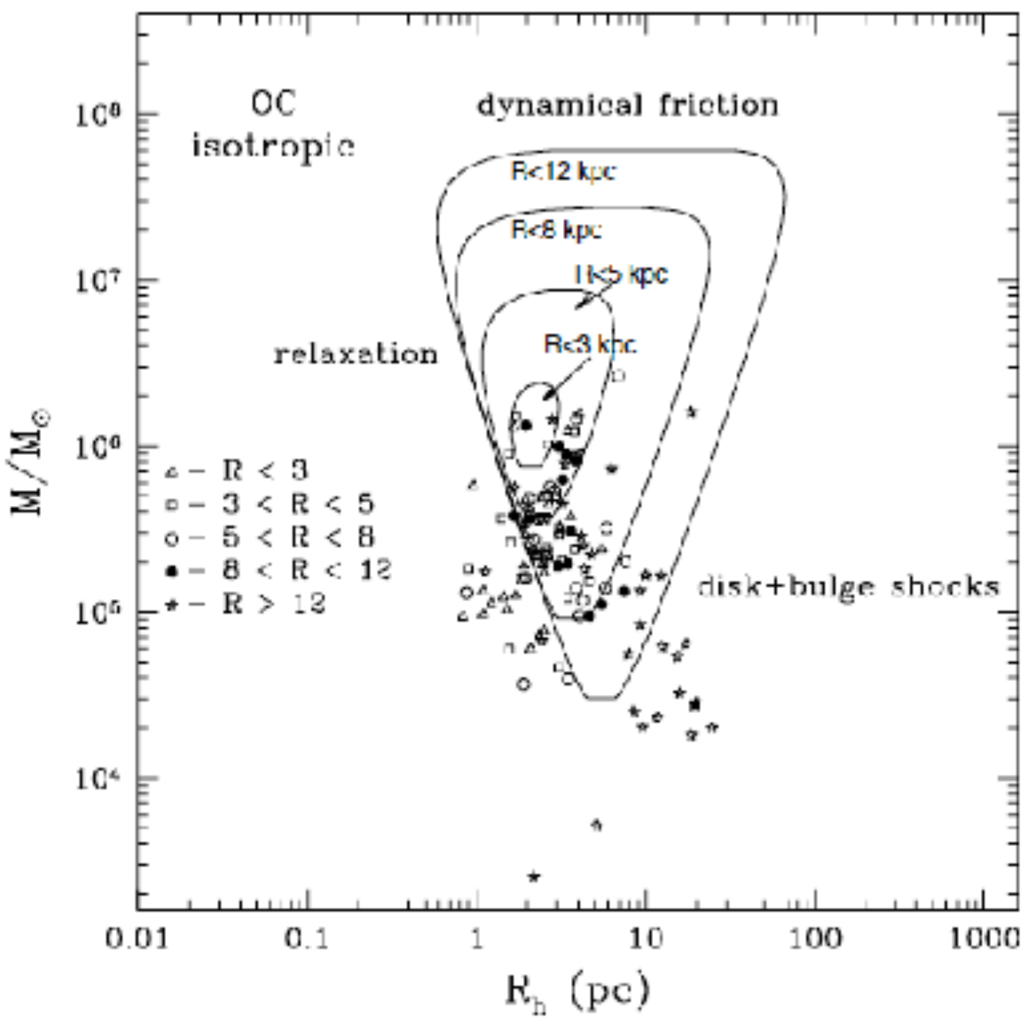
Estimation of merger rate

- Assume 0.45 % NS (but 10% of this remains) and 0.18% BH by number in GC
- 15% of these objects escape in the form of compact binaries
- We used 142 clusters in the catalogue by Harris (2010) with mass and velocity dispersion http://physwww.mcmaster.ca/~harris/Databases.html
  - We computed the number of binaries whose merging time is shorter than Hubble time for each cluster and add them up
- Number density of globular clusters $n_{GC} = 8.7\ h^3\ \mathrm{Mpc}^{-3}$ and h=0.72,
- Results
  - 0.1 - 1 yr^{-1} within advanced detector range
  - ~10 $\mathrm{yr}^{-1}\ \mathrm{Gpc}^{-3}$ for BH-BH merger (note LIGO estimation: 12 ~ 210 $\mathrm{yr}^{-1}\ \mathrm{Gpc}^{-3}$)
  - Very small number of NS-BH binaries

Uncertainties

- Initial Mass function could have been top heavy for in low metallicity environments
- Clusters with relatively large σ could have produced direct capture binaries
- Many GCs have been already disrupted due to
  - Galactic tidal field
  - Tidal Shocks
  - Dynamical friction

Gnedin & Ostriker 1997



Planned Improvements

- More careful assessment of dependence on N
- Effects of the mass function of the ordinary stars
  - We assumed 0.7 Msun, but more realistic mean mass is much lower
- The rotation of the clusters could accelerate the dynamical
- Consideration of captured binaries.

Summary

- Black hole binaries can be formed by dynamical processes in globular clusters efficiently
- Most of them are formed by three-body processes.
- Some of them will merge in Hubble time after getting ejected
  - Mass ratios are likely to be less than 2
  - Massive BHs have higher chances in forming binaries
  - Lower limit for the merger rate is estimated to be $\sim 10\ \mathrm{yr}^{-1}\ \mathrm{Gpc}^{-3}$
  - Some binaries could have been merged within the cluster (<10%). They are mostly composed of lower mass BHs.
- Substantial uncertainties in the estimated rate exist, but actual rate could be higher