Principles of Interferometry

Hans-Rainer Klöckner
IMPRS  Black Board Lectures 2014
acknowledgement

- Mike Garrett lectures
- Uli Klein lectures
- Adam Deller NRAO Summer School lectures
- WIKI – for technical stuff
Lecture 3

- radio astronomical system
- heterodyne receivers
- low-noise amplifiers
- system noise performance
- data sampling/representation
- Fourier transformation
a basic system

relate the voltages measured at the receiver system to the antenna temperature

\[ S_\nu = \frac{2k}{A_{eff}} T_A \]

alternating current (AC)

direct current (DC)

detector input power \( \sim 10^{-5} \text{ W} \)

\[ P = k \cdot \Delta \nu \cdot T_{sys} \]

\( T_{sys} = 20 \text{ K}, \Delta \nu \text{ 50 MHz} \)

\( P = 1.4 \times 10^{-14} \text{ W} \)

\( \sim 10^8 \text{ amplification / gain} \)
heterodyne receiver

after all it's just listening to radio

the most used setup

$$T_1 \text{ needs cooling}$$
low noise amplifier

need to stay in the linear regime
A typical receiver tries to down-convert the “sky signal” or “Radio Frequency” (or RF) to a lower, “Intermediate Frequency” (or IF) signal.

The reasons for doing this include: (i) signal losses (e.g. in cables) typically go as frequency^2; (ii) it is much easier to manipulate the signal (e.g. amplify, filter, delay, sample/process/digitise it) at lower frequencies.

We use so-called “heterodyne” systems to mix the RF signal with a pure, monochromatic frequency tone, known as a Local Oscillator (or LO).

Consider an RF signal in a band centred on frequency $v_{RF}$, and an LO with frequency $v_{LO}$, these can be represented as two sine waves with angular frequencies $w$ and $w_o$:

$$v_{IF} = v_{RF}v_{LO} \sim \sin(\omega t)\sin(\omega_0 t) = \frac{1}{2}(\cos(\omega - \omega_0)t + \cos(\omega + \omega_0)t)$$

- Difference frequency
- Sum frequency
The higher frequency component ("sum frequency" $v_{RF}+v_{LO}$) is usually removed by a filter that is included in the LO electronics. Hence the process of down-conversion, takes a band with centre frequency $v_{RF}$ and converts it to a lower (difference) frequency, $v_{RF}-v_{LO}$.

The mixer signal products preserves the noise characteristics of the input RF (sky) signal, but they contain an arbitrary phase-shift due to the unknown phase of the LO.

Usually there will be several mixers and frequency conversions in a receiver system. Eventually one edge of the frequency band reaches 0 Hz, known as a "base-band" or "video" signal.

At high frequencies (e.g. millimetre wavelengths), down-conversion occurs before amplification.

USB = upper side band

LSB = lower side band
low noise amplifier

we have covered that already
bandpass filter
low noise amplifier

we have covered that already
Since radio astronomy signals have the characteristics of white noise, the voltage induced in the receiver output alternates positively and negatively about zero volts. Any measurement of the Voltage expectation value or time average will read zero (e.g. hooking up a receiver to a DC voltmeter will not measure any signal).

What is needed is a non-linear device \( V_{\text{out}} = AV_{\text{in}}^2 \) that will only measure the passage of the signal in one preferred direction (either positive or negative) i.e. we must incorporate a semiconductor diode into our measuring system.
integrator

capacity needs time $\tau$ to charge
reads out the capacity
signal processing tools 1d

Convolution

Convolution theorem

\[(f \ast g)(t) \overset{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau\]

Fourier Transformation

\[F_f(t) = \int f(\nu) e^{2\pi i\nu t} d\nu\]

\[F_f(\nu) = \int f(t) e^{-2\pi i\nu t} dt\]

\[F(f \ast g) = F(f) F(g)\]
heterodyne receiver

Fourier transformation
convolution theorem in action
The higher frequency component ("sum frequency" $v_{RF} + v_{LO}$) is usually removed by a filter that is included in the LO electronics. Hence the process of down-conversion, takes a band with centre frequency $v_{RF}$ and converts it to a lower (difference) frequency, $v_{RF} - v_{LO}$.

The mixer signal products preserves the noise characteristics of the input RF (sky) signal, but they contain an arbitrary phase-shift due to the unknown phase of the LO. Usually there will be several mixers and frequency conversions in a receiver system. Eventually one edge of the frequency band reaches 0 Hz, known as a "base-band" or "video" signal.

At high frequencies (e.g. millimetre wavelengths), down-conversion occurs before amplification.

\[ \text{Power}(\tau) \sim |U^2(\tau)| \]

Continuum measurement

\[ \text{Power}(\nu) \sim |U^2(\nu)| \]

Line measurement

Continuum measurement

Line measurement
how to get \( P(\nu) = |U^2(\nu)| \)
- approach 1 -

Theoretically

\[
\tilde{U}(\nu) = \int_{-\infty}^{+\infty} U(t) e^{-i2\pi \nu t} dt
\]

\[
P(\nu) = |\tilde{U}(\nu)|^2
\]

Hardware

filters split signal into channels
feed each channel into detector
FFT spectrometer

FFTS :: 1.5 GHz bandwidth Board

- Instantaneous bandwidth: 0.1 – 1.8 GHz
- Spectral resolution @ 1.5 GHz: 212 kHz
- Calibration- and aging free digital processing

FPGA – Field-Programmable Gate Array
recap what we measure in a single dish

Observing time
Integration time \( \tau \)
Base Band spectrum in \( \nu \) per integration time \( \tau \)

Fourier Transformation

\[ F(f) = \int f(\nu) e^{-2\pi i \nu \tau} d\nu \]

\[ F_f(\tau) = \int f(\nu) e^{2\pi i \nu \tau} d\nu \]

note usually the integration time will be defined as \( t \)
how to get $P(\nu) = |U^2(\nu)|$

- approach 2 -

Theoretically

auto correlation function

$$R(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} U(t) U(t + \tau) \, dt$$

$$P(\nu) = \int_{-\infty}^{+\infty} R(\tau) e^{-i 2\pi \nu \tau} \, d\tau$$

Convolution

$$(f \ast g)(t) \overset{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau) g(t - \tau) \, d\tau$$
how to get $P(\nu) = |U^2(\nu)|$

- approach 2 -

old style hardware to shift the data
signal processing

clipper - quantisation 1 bit (0 or 1)
signal processing
signal processing

Sensitivity loss

- 8 bit: 0.1%
- 4 bit: 1.3%
- 2 bit: 12%
- 1 bit: 36%
Square-law detectors are not used so very often these days. The receiver produces a varying analogue output voltage that is usually digitised and stored for further (offline) processing. How often must be sample the signal?

Consider the following sine wave:

If we sample once per cycle time (period) we would consider the signal to have a constant amplitude.

If we sample twice per cycle time (period) we get a saw-tooth wave that is becoming a good approximation to a sinusoid.

For lossless digitisation we must sample the signal at least twice per cycle time.

Nyquist’s sampling theorem states that for a limited bandwidth signal with maximum frequency $f_{\text{max}}$, the equally spaced sampling frequency $f_s$ must be greater than twice the maximum frequency $f_{\text{max}}$, i.e. $f_s > 2 f_{\text{max}}$ in order for the signal to be uniquely reconstructed without aliasing.

The frequency $2f_{\text{max}}$ is called the Nyquist sampling rate.

e.g. If a reciever system provides a baseband signal of 20 MHz, the signal must be sampled $40E6$ times per second.
how to get $P(\nu) = |U^2(\nu)|$
auto correlation

digital data
auto correlation cross

using the signal from different antennas we build an interferometer
Correlator

Correlator platform overview

Development effort required

Reuse-ability

Correlator capacity per hardware $$

CPU

GPU

FPGA

ASIC
Young's slit experiment

solid line
unresolved
dashed line
resolved
aperture synthesis

mix the signal from all the telescope that they are in phase