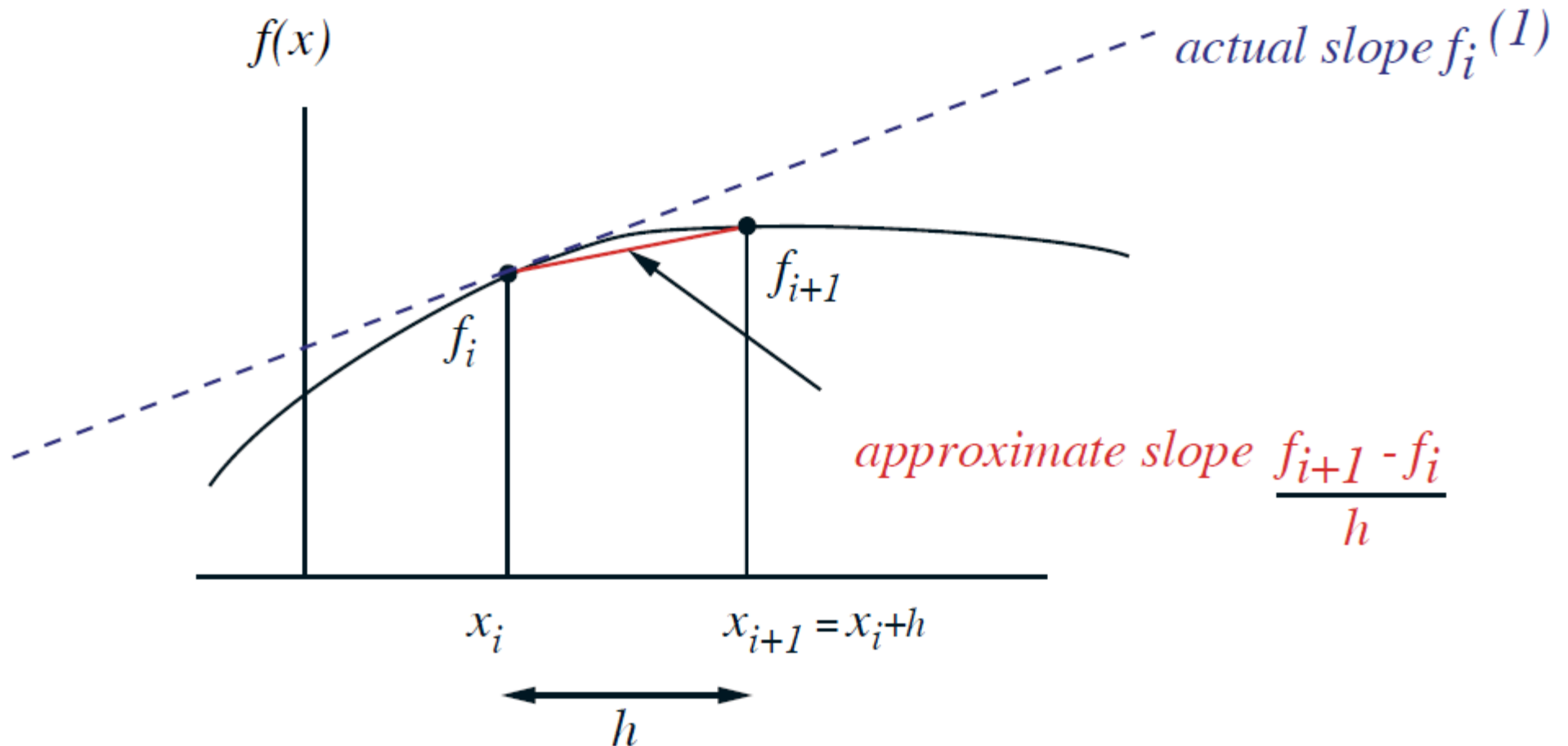IMPRS - BBL
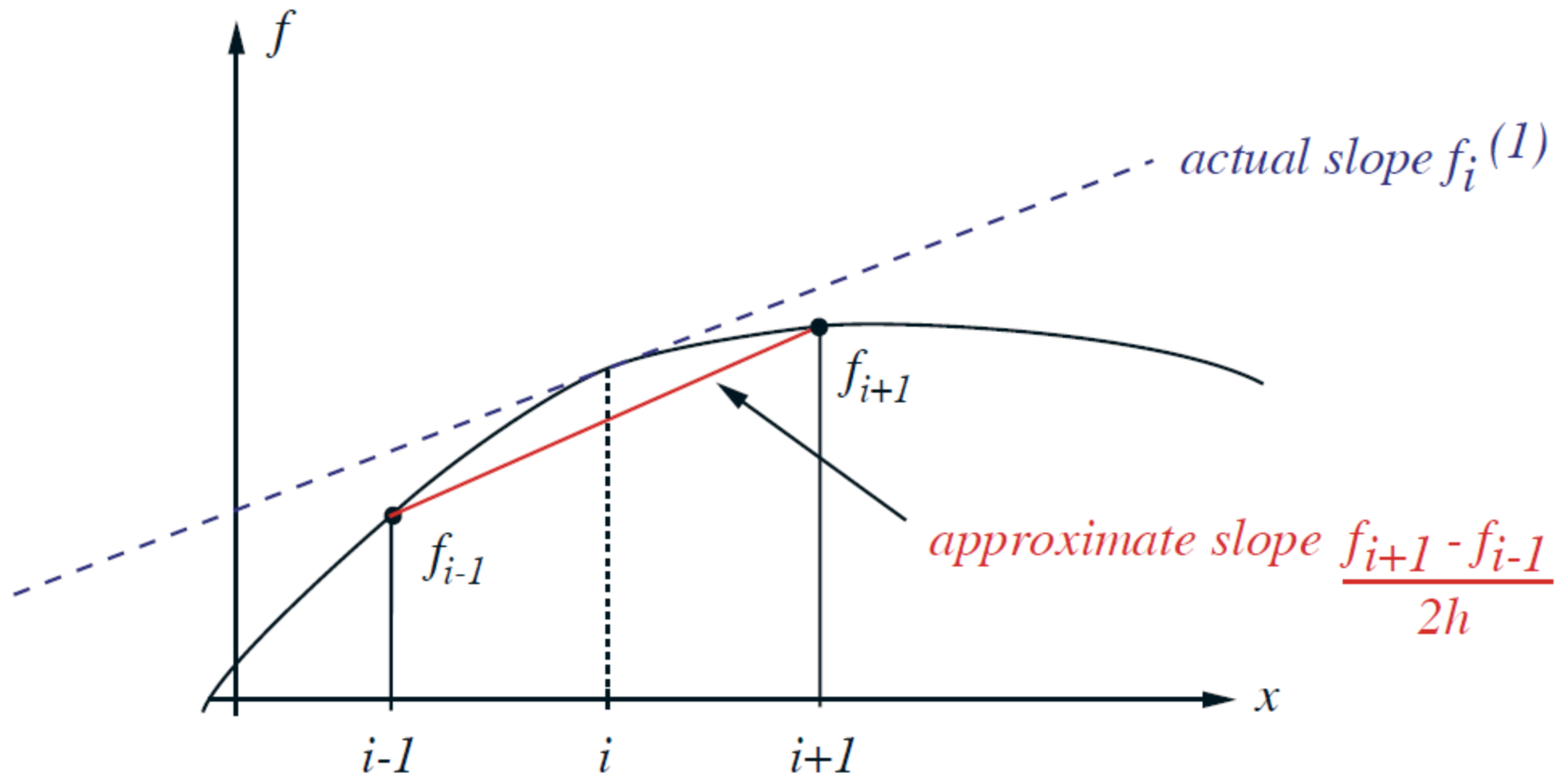
Numerical methods

Lecture 6

Michael Marks

Topics:

Day 1: Linear algebraic equations
Day 2: Inter- and Extrapolation
Day 3: Integration
Day 4: Random numbers and distribution functions
Day 5: Root finding, Minimization and Maximization
Day 6: Differentiation

First order forward derivative

Second order central derivative

1ˢᵗ derivative approximations

Forward difference approximations:

🔴 $f_i^{(1)} = \dfrac{f_{i+1} - f_i}{h} + E$, $E \cong -\dfrac{1}{2} h f_i^{(2)}$

⚪ $f_i^{(1)} = \dfrac{-f_{i+2} + 4f_{i+1} - 3f_i}{2h} + E$, $E \cong \dfrac{1}{3} h^2 f_i^{(3)}$

$f_i^{(1)} = \dfrac{2f_{i+3} - 9f_{i+2} + 18f_{i+1} - 11f_i}{6h} + E$, $E \cong -\dfrac{1}{4} h^3 f_i^{(4)}$

Backward difference approximations:

🔴 $f_i^{(1)} = \dfrac{f_i - f_{i-1}}{h} + E$, $E \cong \dfrac{1}{2} h f_i^{(2)}$

$f_i^{(1)} = \dfrac{3f_i - 4f_{i-1} + f_{i-2}}{2h} + E$, $E \cong \dfrac{1}{3} h^2 f_i^{(3)}$

$f_i^{(1)} = \dfrac{11f_i - 18f_{i-1} + 9f_{i-2} - 2f_{i-3}}{6h} + E$, $E \cong \dfrac{1}{4} h^3 f_i^{(4)}$

Central difference approximations:

🔴 $f_i^{(1)} = \dfrac{f_{i+1} - f_{i-1}}{2h} + E$, $E \cong -\dfrac{1}{6} h^2 f_i^{(3)}$

$f_i^{(1)} = \dfrac{-f_{i+2} + 8f_{i+1} - 8f_{i-1} + f_{i-2}}{12h} + E$, $E \cong \dfrac{1}{30} h^4 f_i^{(5)}$

🔴 derived in lecture

2nd derivative approximations

Forward difference approximations

- $f_i^{(2)} = \dfrac{f_{i+2} - 2f_{i+1} + f_i}{h^2} + E$, $E \cong -hf_i^{(3)}$

$f_i^{(2)} = \dfrac{-f_{i+3} + 4f_{i+2} - 5f_{i+1} + 2f_i}{h^2} + E$, $E \cong \dfrac{11}{12}h^2 f_i^{(4)}$

Backward difference approximations:

$f_i^{(2)} = \dfrac{f_i - 2f_{i-1} + f_{i-2}}{h^2} + E$, $E \cong hf_i^{(3)}$

$f_i^{(2)} = \dfrac{2f_i - 5f_{i-1} + 4f_{i-2} - f_{i-3}}{h^2} + E$, $E \cong \dfrac{11}{12}h^2 f_i^{(4)}$

Central difference approximations:

$f_i^{(2)} = \dfrac{f_{i+1} - 2f_i + f_{i-1}}{h^2} + E$, $E \cong -\dfrac{1}{12}h^2 f_i^{(4)}$

$f_i^{(2)} = \dfrac{-f_{i+2} + 16f_{i+1} - 30f_i + 16f_{i-1} - f_{i-2}}{12h^2} + E$, $E \cong \dfrac{1}{90}h^4 f_i^{(6)}$

● derived in lecture