IMPRS - BBL

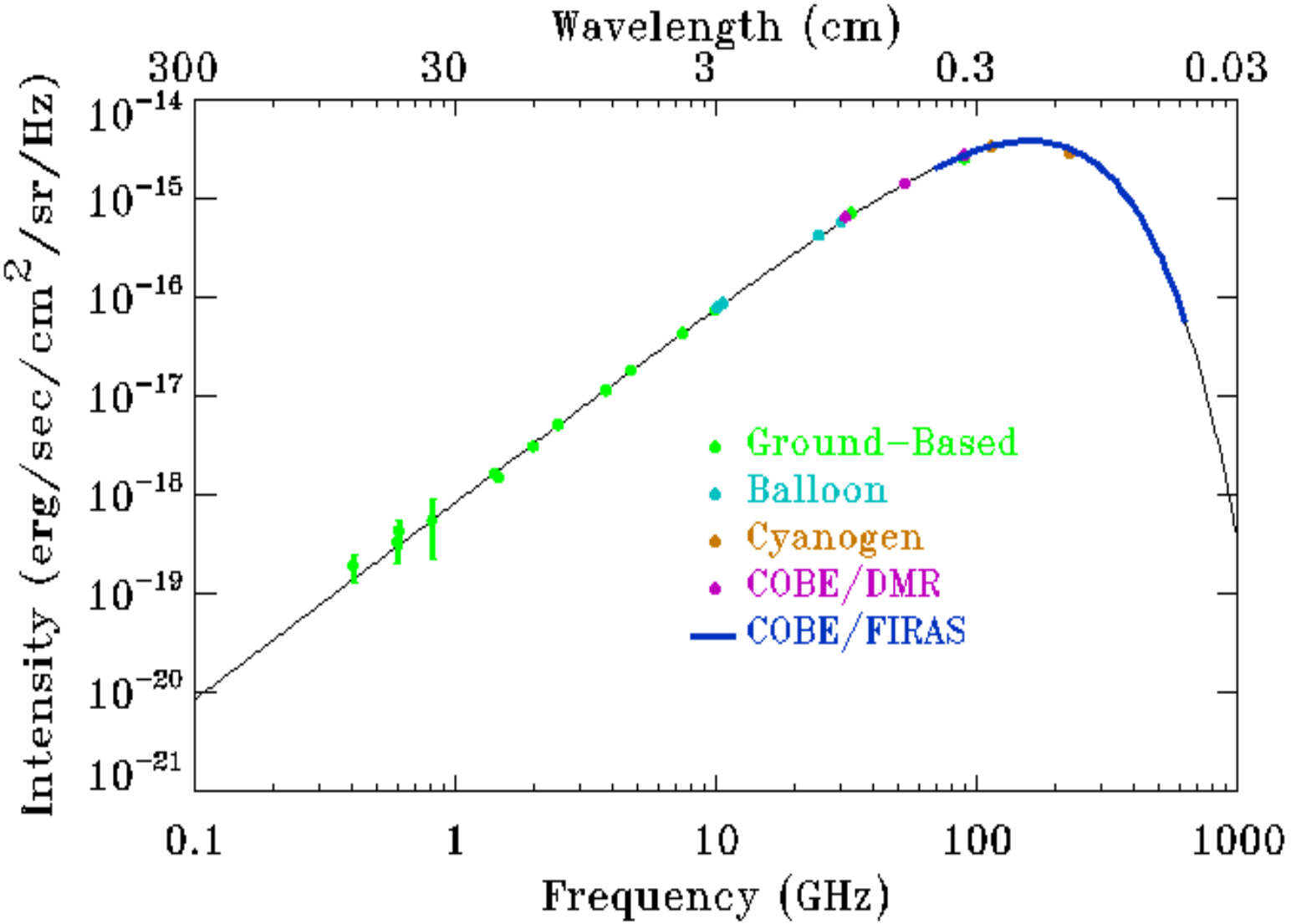
Numerical methods

Lecture 2

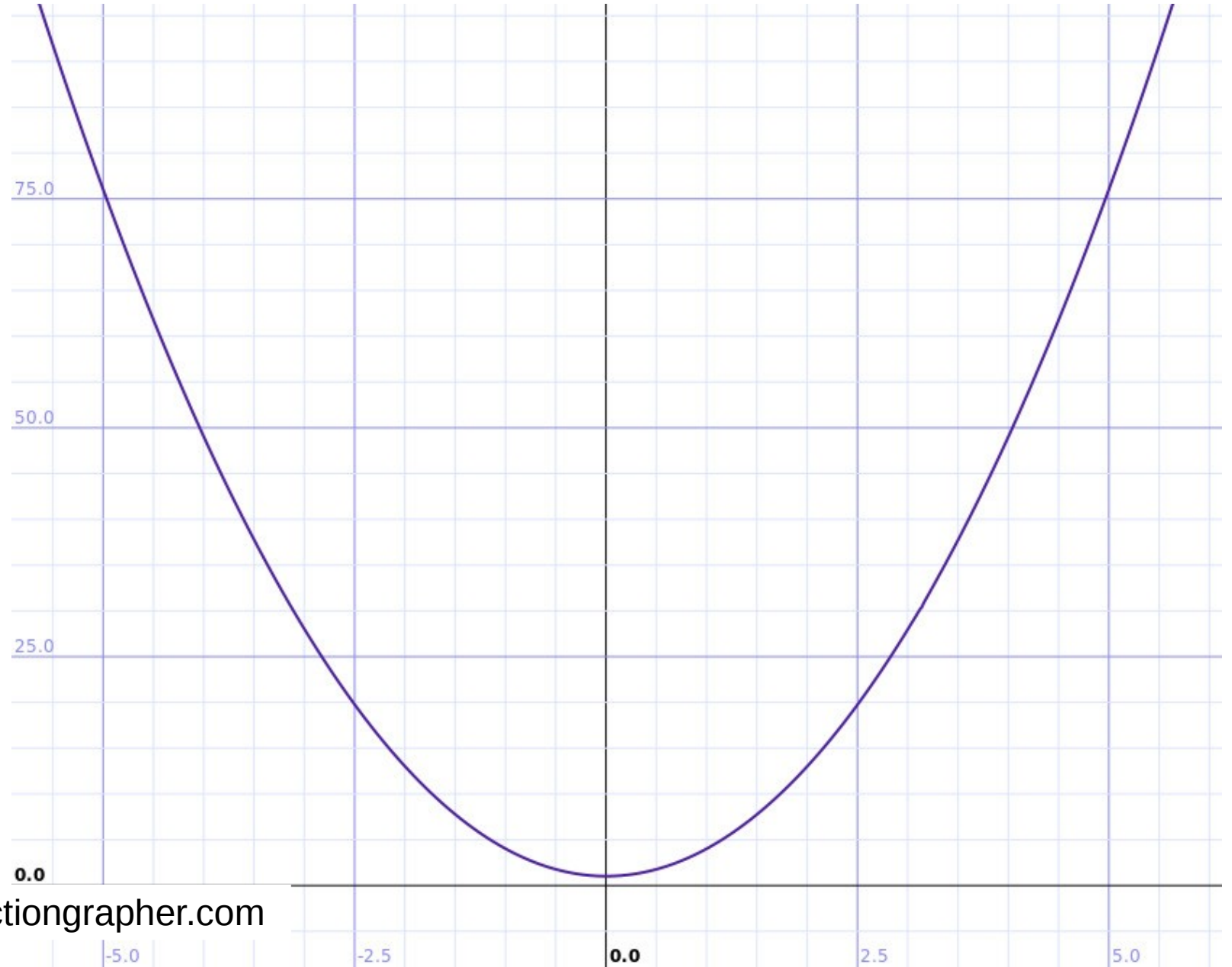
Michael Marks

Topics:

Day 1: Linear algebraic equations
Day 2: Inter- and Extrapolation
Day 3: Integration
Day 4: Random numbers and distribution functions
Day 5: Root finding, Minimization and Maximization
Day 6: Differentiation


Wavelength (cm)
300
30
3
0.3
0.03
Intensity (erg/sec/cm^2/sr/Hz)
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
0.1
1
10
100
1000
Frequency (GHz)
Ground−Based
Balloon
Cyanogen
COBE/DMR
COBE/FIRAS

$$f(x) = 3x^2 + \frac{1}{\pi^4} \ln\left[(\pi - x)^2\right] + 1$$



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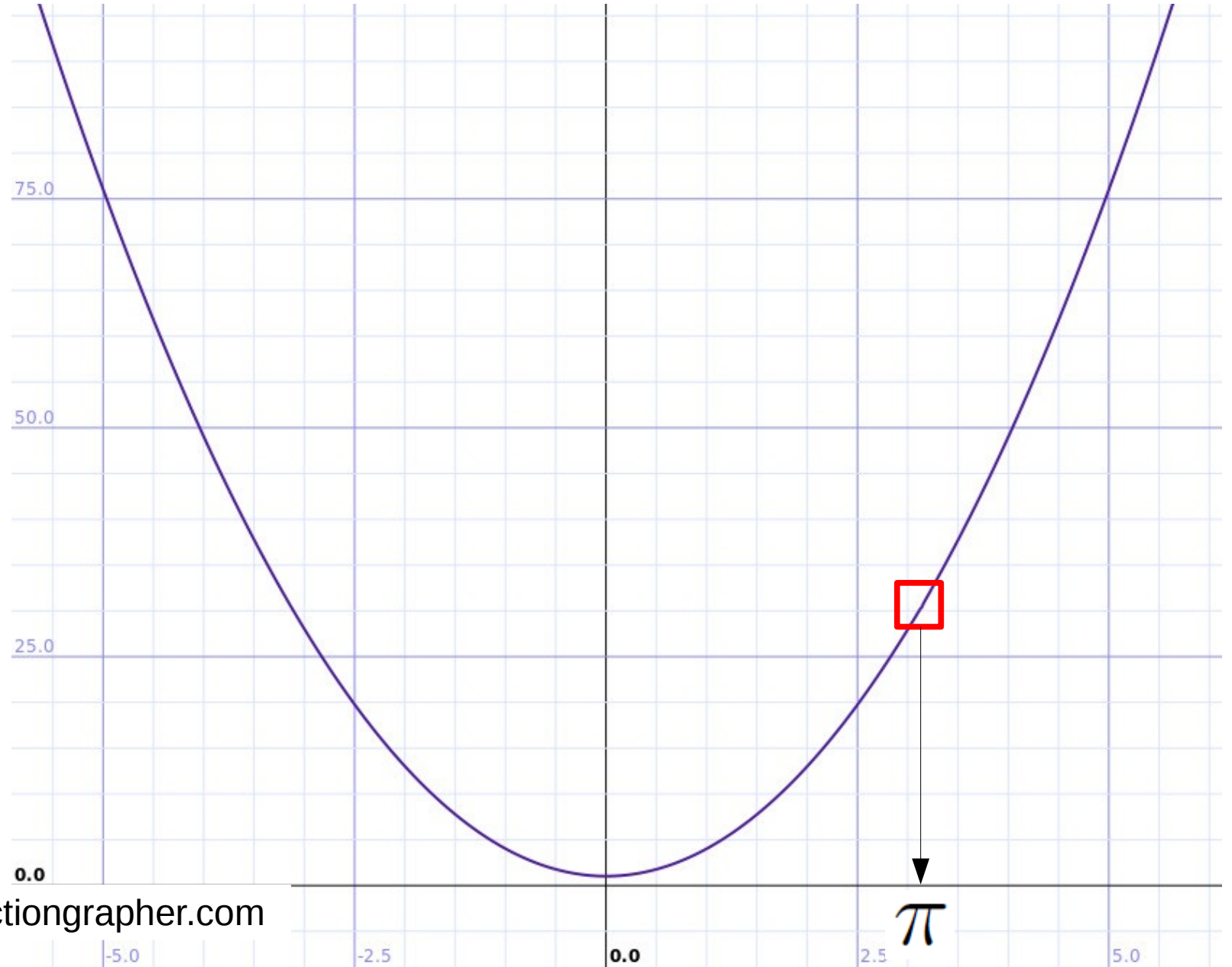

onlinefunctiongrapher.com

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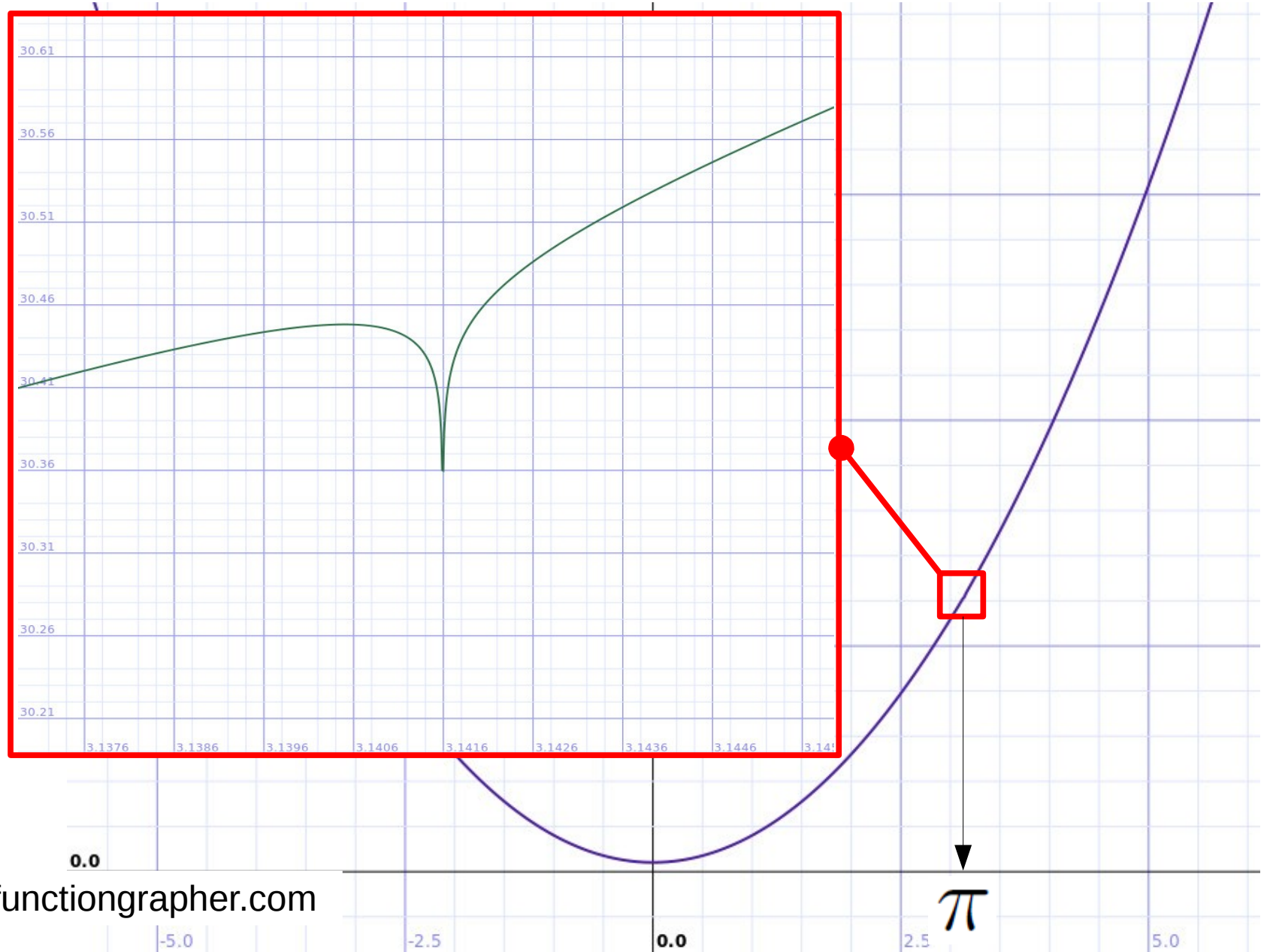

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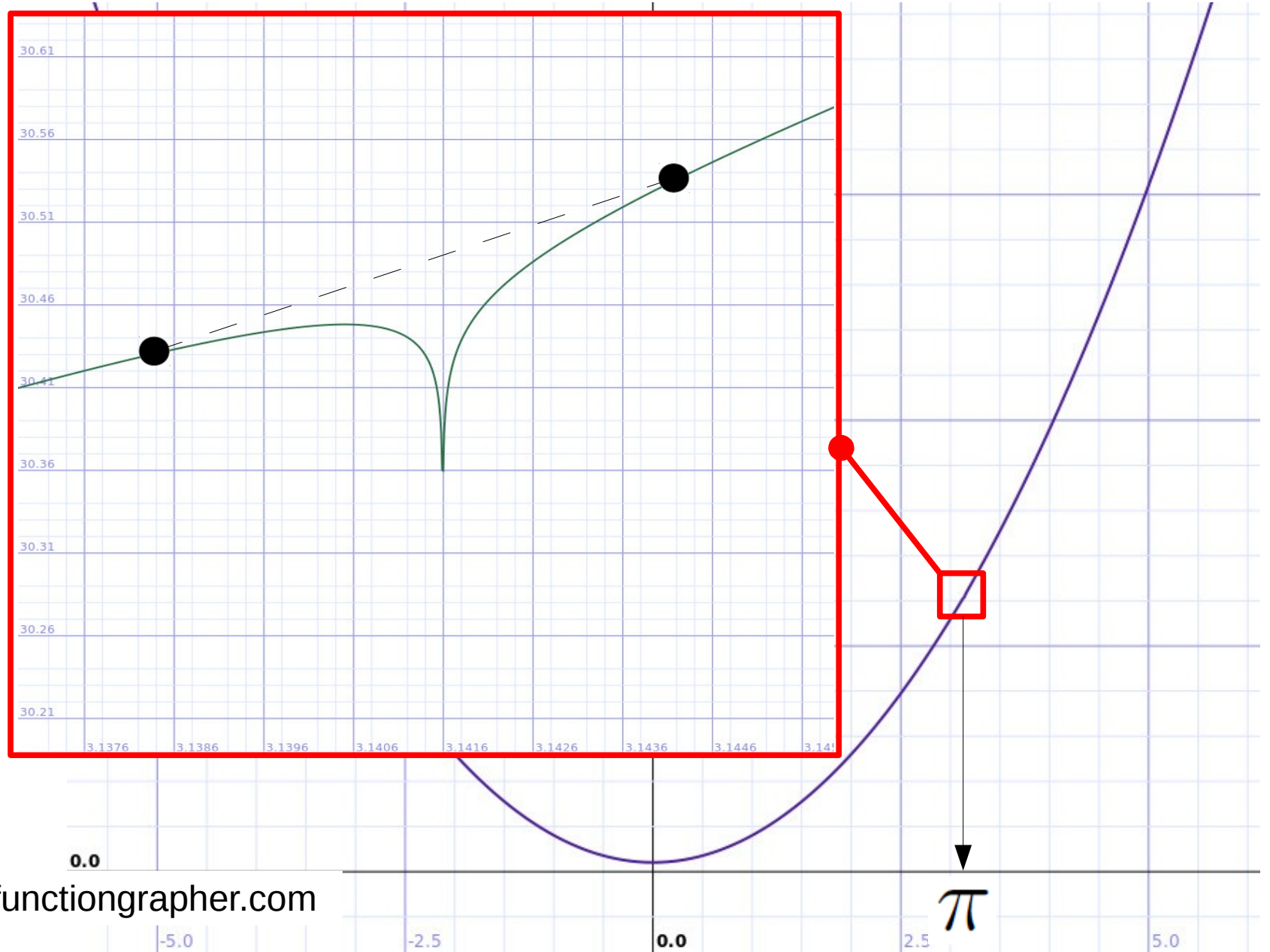
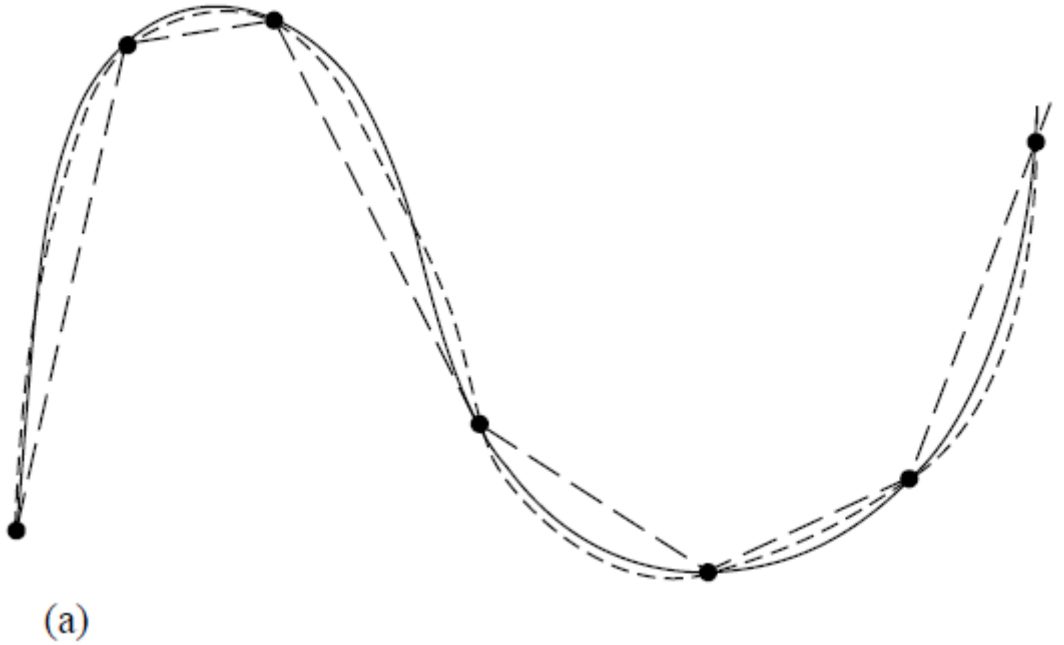
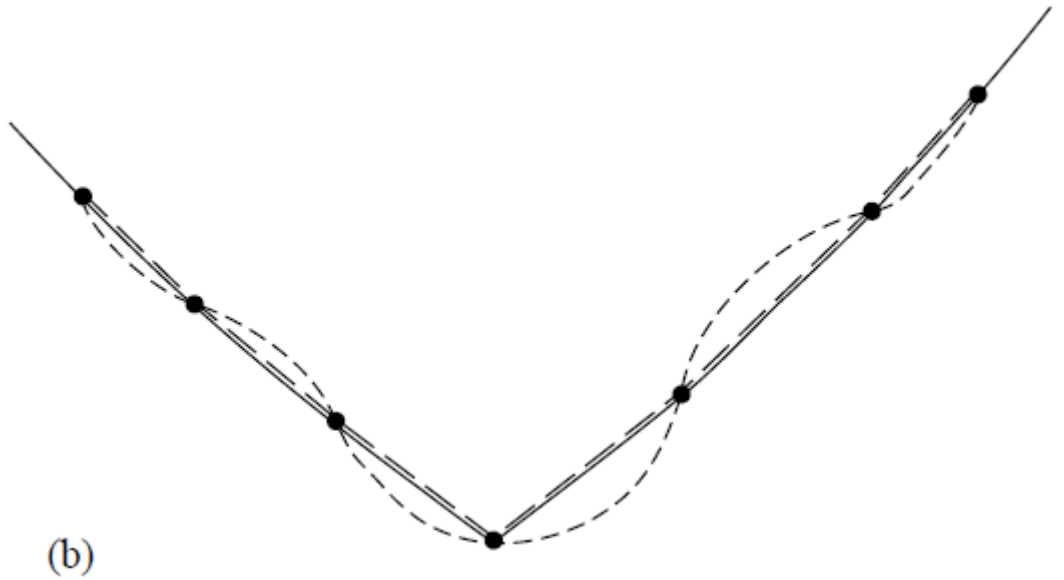


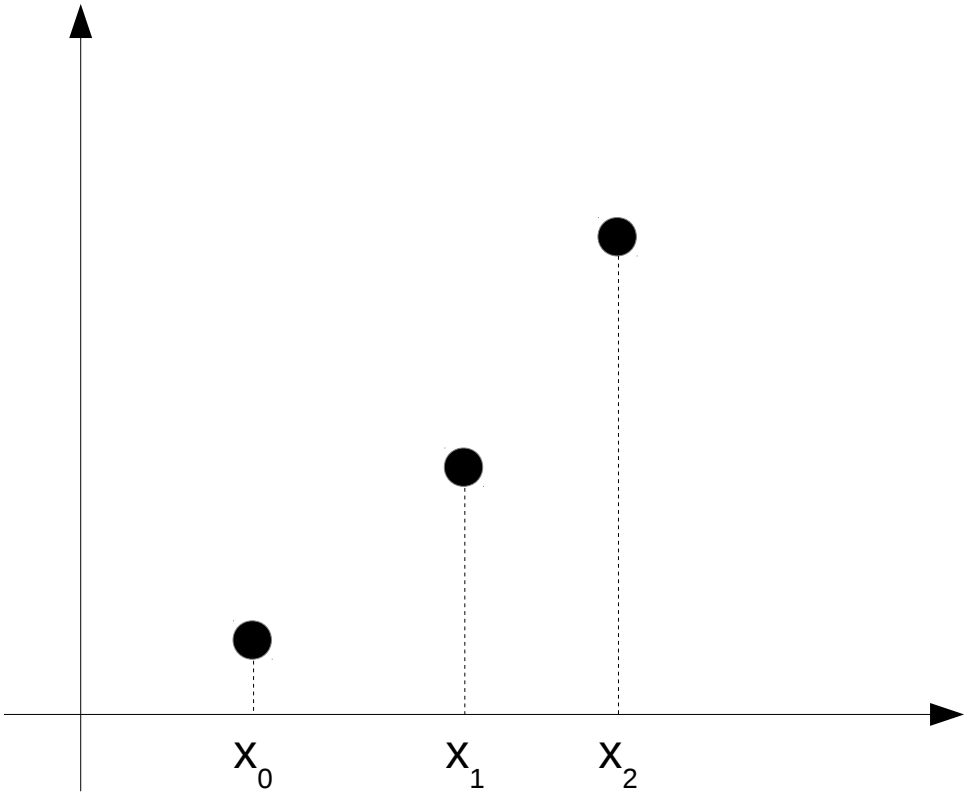




Numerical Recipes (Press et al. 1992)

Neville-Aitken algorithm

Example: Find polynomial of 2nd order to interpolate 3 points



Neville-Aitken algorithm

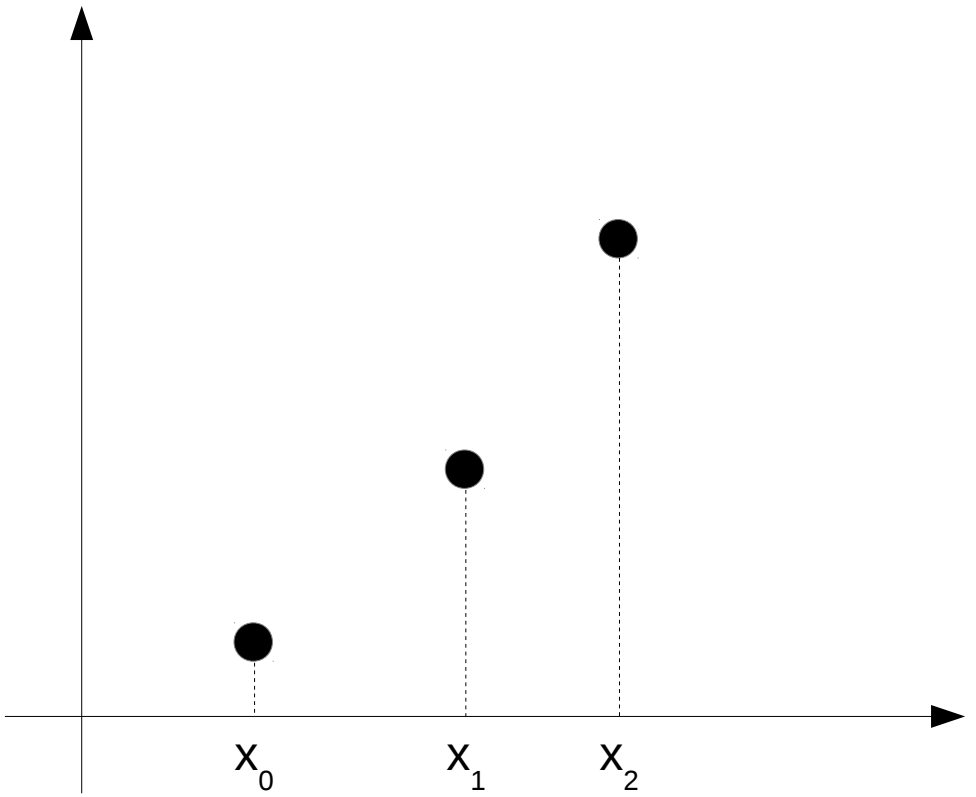
Example: Find polynomial of 2nd order to interpolate 3 points

P_0

P_1

P_2

0^{th} order polynomials
(N=1 points)



Neville-Aitken algorithm

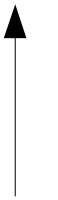
Example: Find polynomial of 2nd order to interpolate 3 points

Level m=1

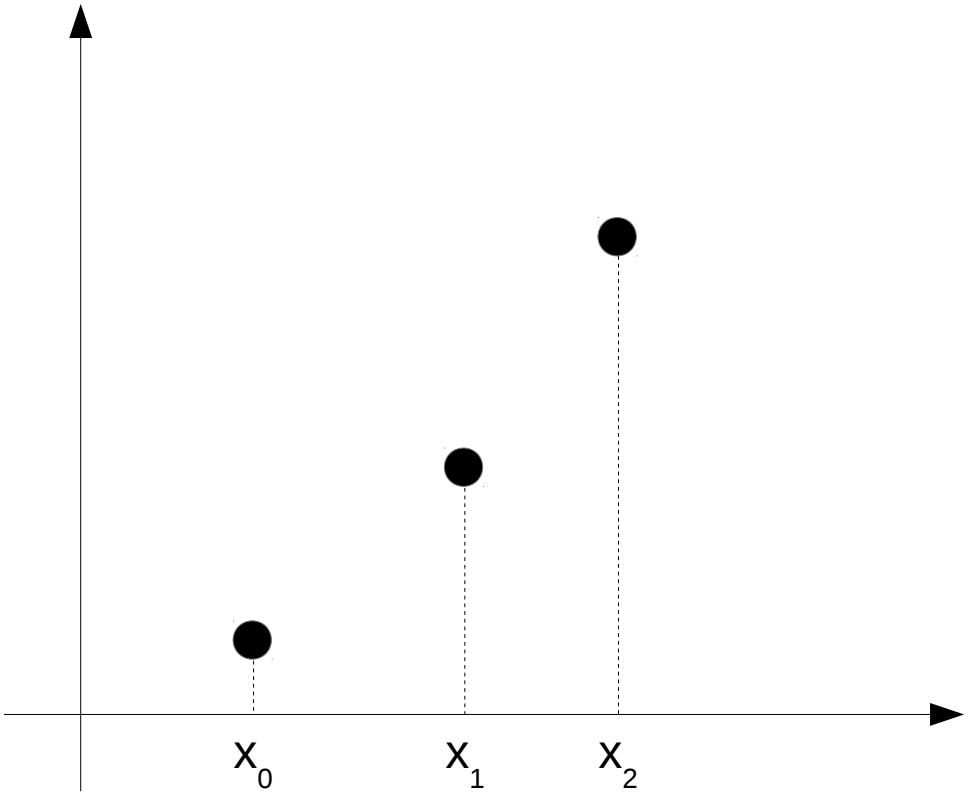
P_0

P_1

P_2



0^{th} order polynomials
(N=1 points)



Neville-Aitken algorithm

Example: Find polynomial of 2nd order to interpolate 3 points

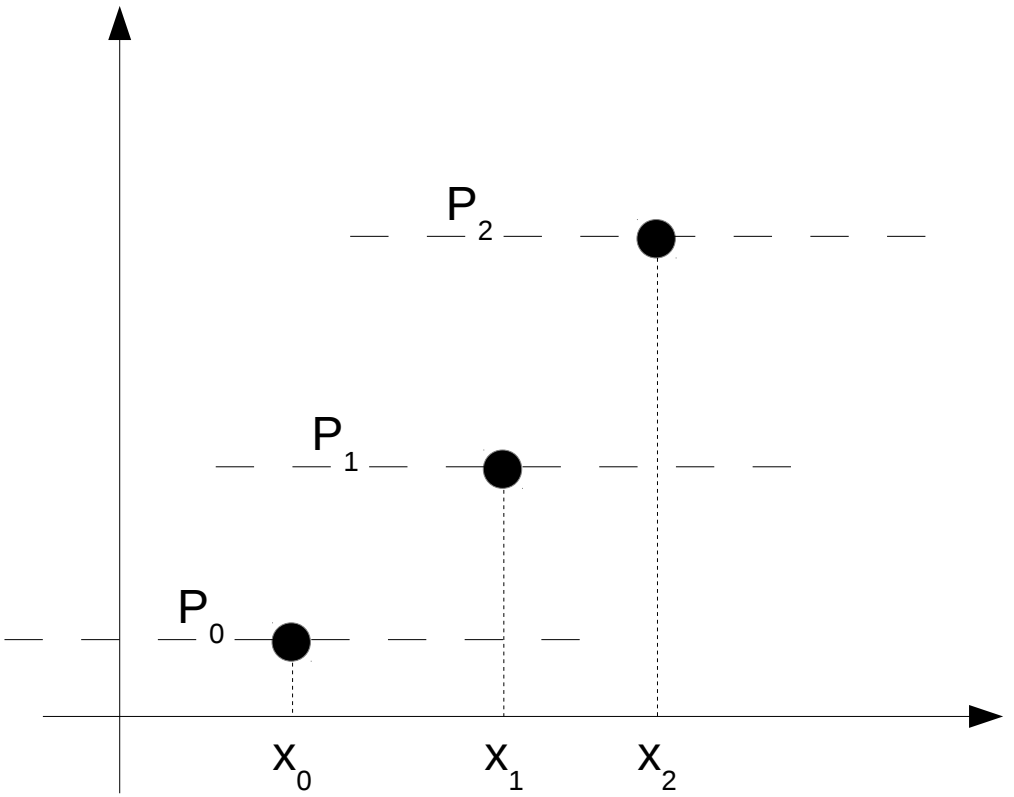
Level m=1

$f(x_0)=P_0$

$f(x_1)=P_1$

$f(x_2)=P_2$

0^{th} order polynomial
(N=1 points)



Neville-Aitken algorithm

Example: Find polynomial of 2nd order to interpolate 3 points

Level m=1 2

$f(x_0)=P_0$

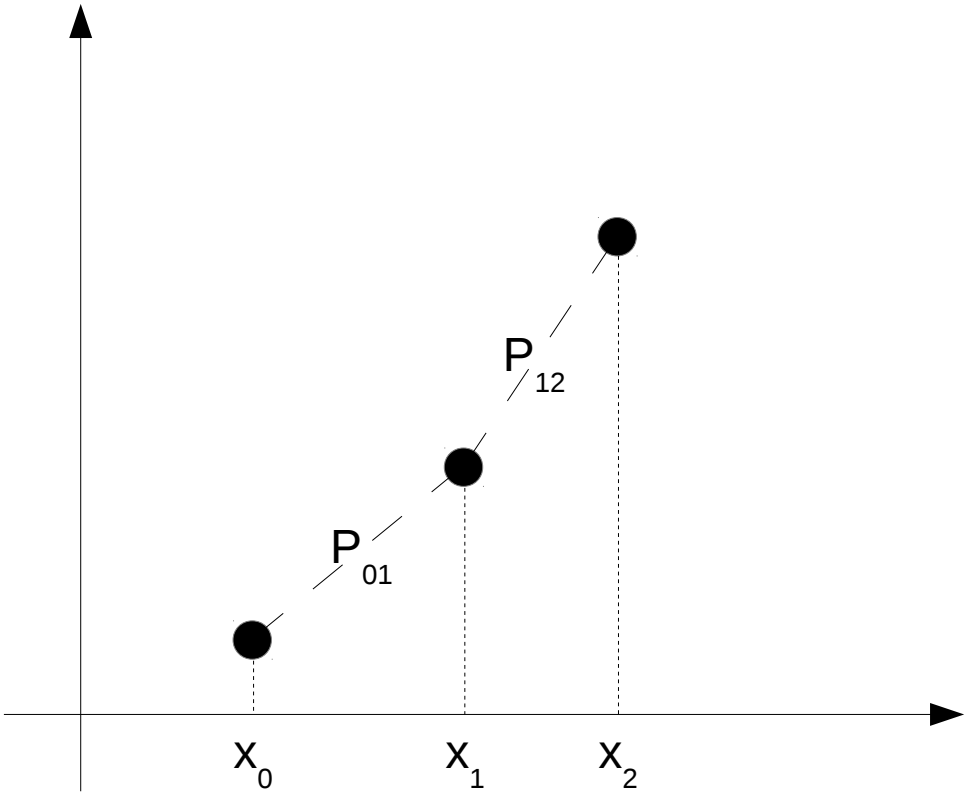
P_{01}

$f(x_1)=P_1$

P_{12}

$f(x_2)=P_2$

1^{st} order polynomial
(N=2 points)



Neville-Aitken algorithm

Example: Find polynomial of 2nd order to interpolate 3 points

Level m=1 2 3

$f(x_0)=P_0$

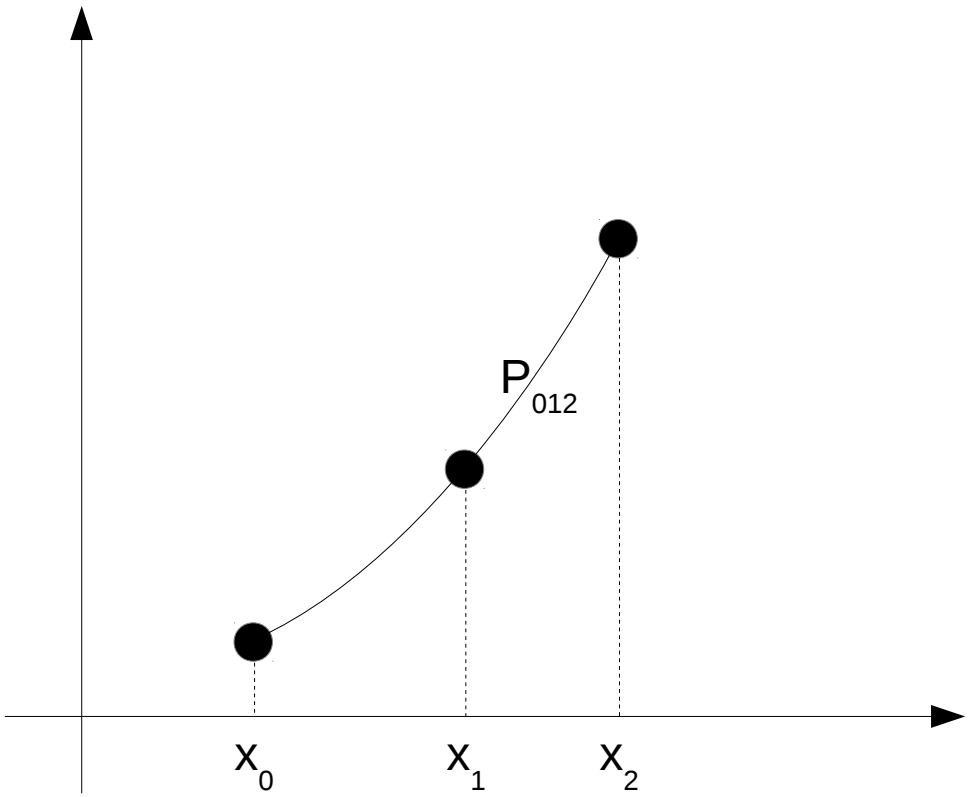
P_{01}

$f(x_1)=P_1$ P_{012}

P_{12}

$f(x_2)=P_2$

2^{nd} order polynomial
(N=3 points)



Bulirsch-Stoer algorithm

Recurrence relation:

$$R_{i(i+1)\ldots(i+m)} = R_{(i+1)\ldots(i+m)} + \frac{R_{(i+1)\ldots(i+m)} - R_{i\ldots(i+m-1)}}{\left(\frac{x-x_i}{x-x_{i+m}}\right)\left(1 - \frac{R_{(i+1)\ldots(i+m)} - R_{i\ldots(i+m-1)}}{R_{(i+1)\ldots(i+m)} - R_{(i+1)\ldots(i+m-1)}}\right) - 1}$$

Error estimate in step i:

$$C_{m,i} \equiv R_{i\ldots(i+m)} - R_{i\ldots(i+m-1)}$$

$$D_{m,i} \equiv R_{i\ldots(i+m)} - R_{(i+1)\ldots(i+m)}$$

Reccurence:

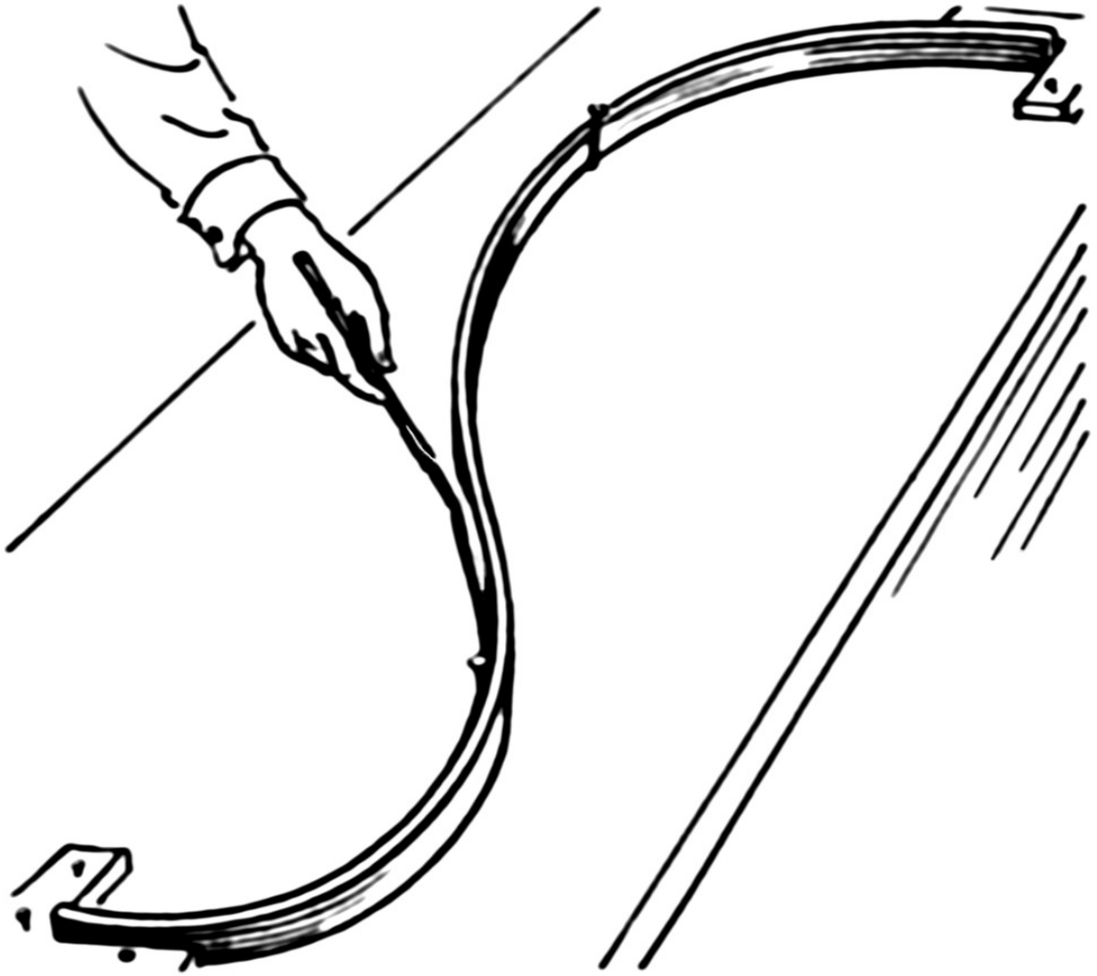
$$D_{m+1,i} = \frac{C_{m,i+1}(C_{m,i+1} - D_{m,i})}{\left(\frac{x-x_i}{x-x_{i+m+1}}\right) D_{m,i} - C_{m,i+1}}$$

$$C_{m+1,i} = \frac{\left(\frac{x-x_i}{x-x_{i+m+1}}\right) D_{m,i}(C_{m,i+1} - D_{m,i})}{\left(\frac{x-x_i}{x-x_{i+m+1}}\right) D_{m,i} - C_{m,i+1}}$$

Numerical Recipes (Press et al. 1992)

Splines

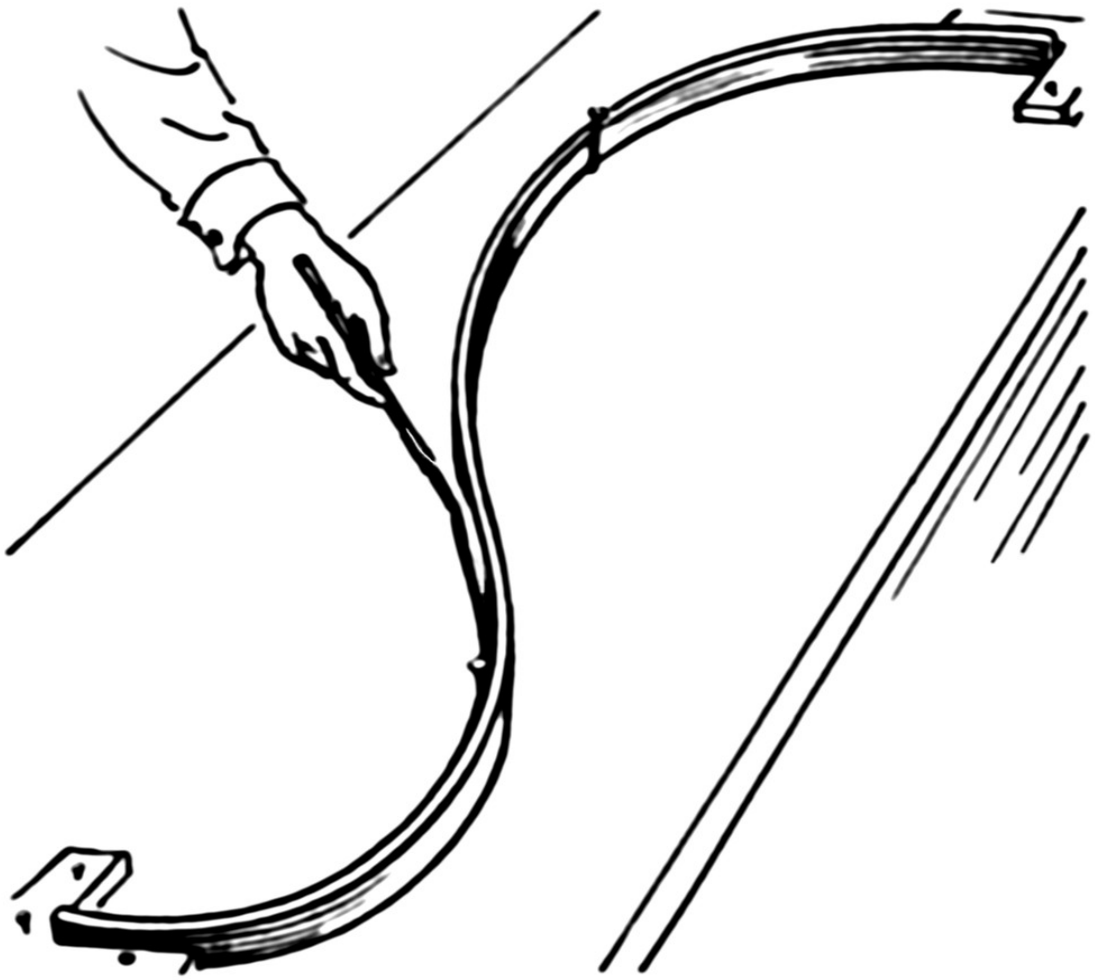
Used in ship construction



Source: Wikipedia

Splines

Used in ship construction



Has „natural“ curvature (in this shape the tension energy / total curvature is minimal)

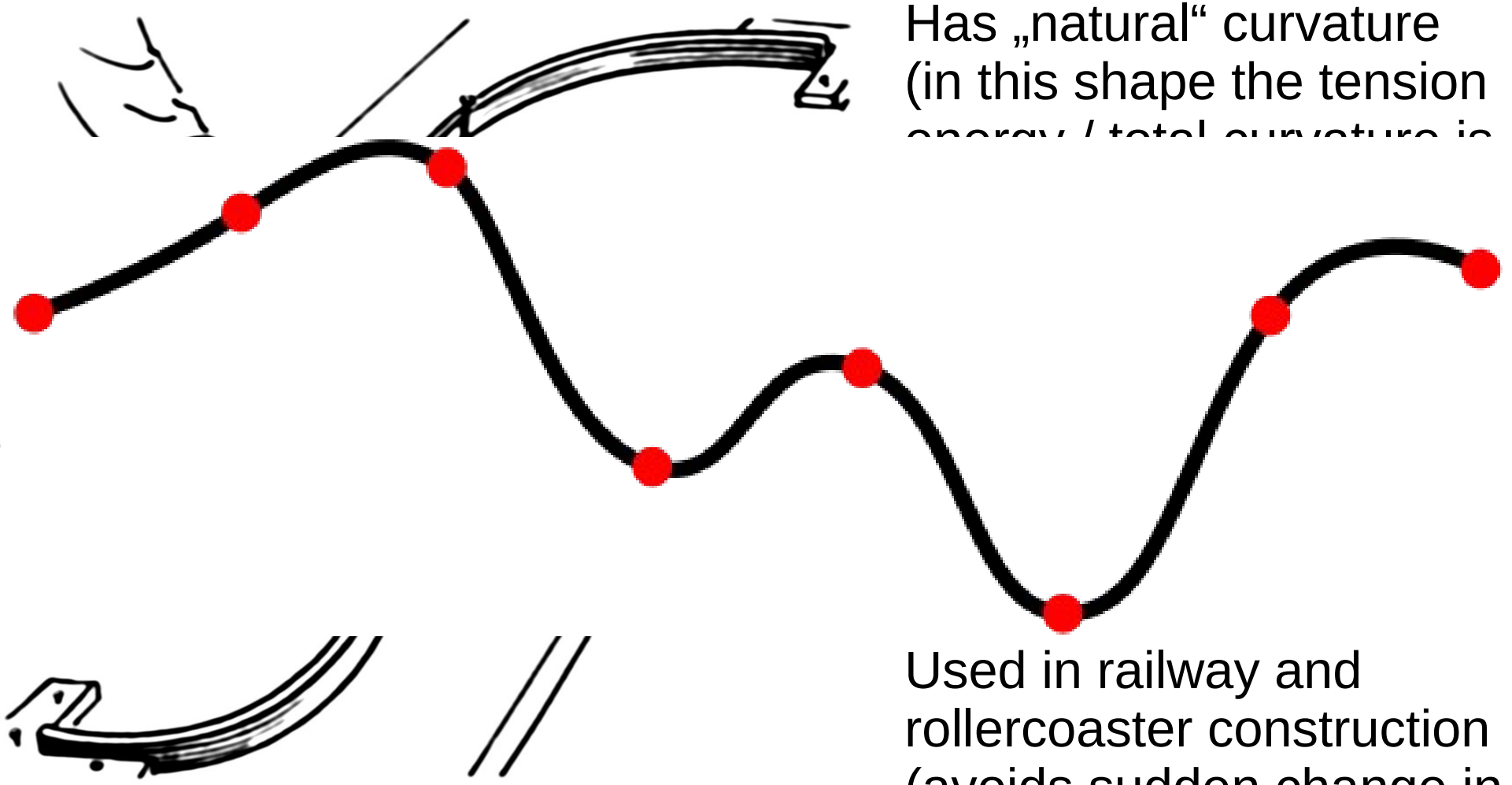
No abrupt changes in curvature
→ 1st derivative smooth
→ 2nd derivative continous (no „jumps“)

Used in railway and rollercoaster construction (avoids sudden change in acceleration).

Source: Wikipedia

Splines

Used in ship construction



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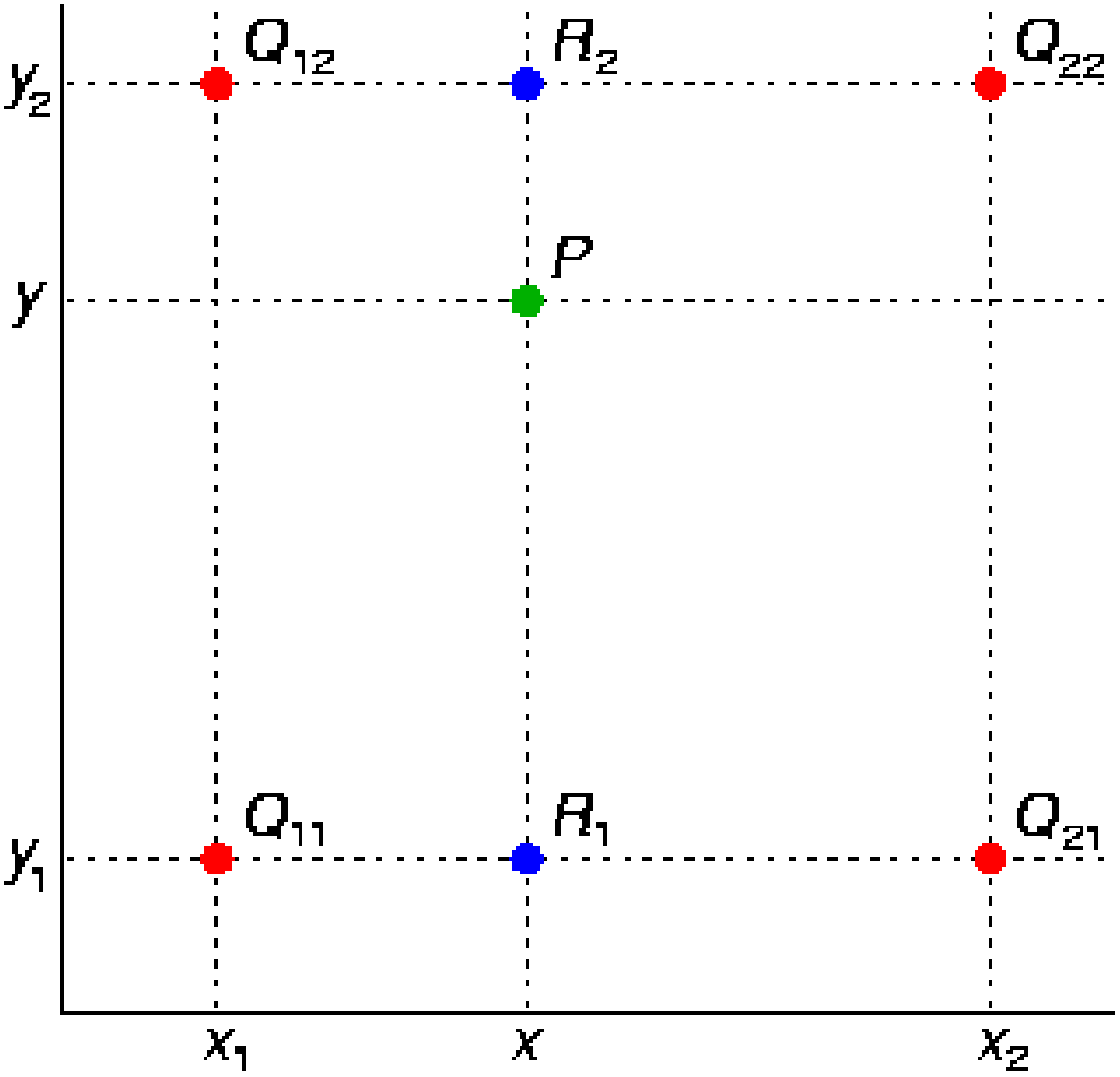
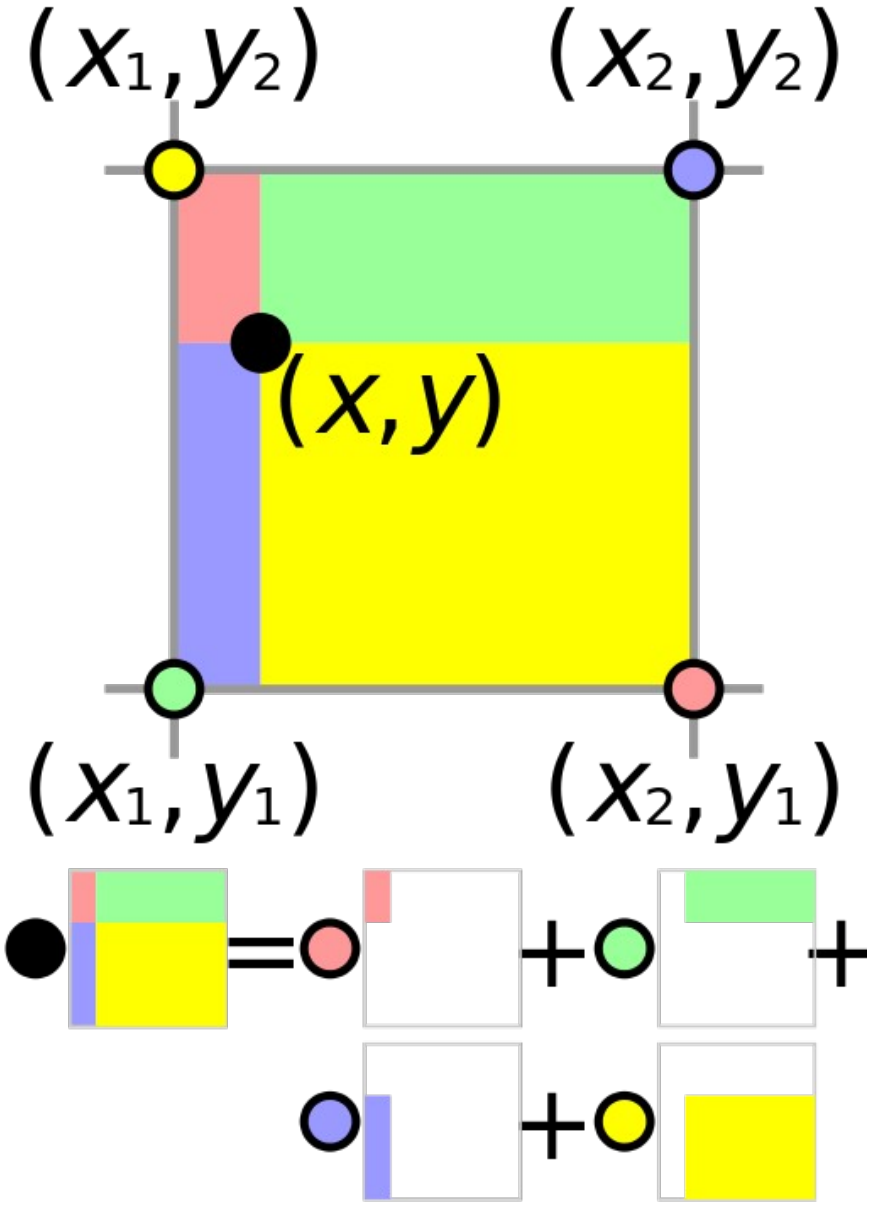
Bi-linear Interpolation



Bi-linear Interpolation



Excercise

Given the following 3 data points

x_i	1.23	3.45	5.67
$f(x_i)$	0.91	1.52	-0.73

write a program which can interpolate / extrapolate these three values at an arbitrary value x specified by the user.

Use Neville's algorithm to calculate an estimate of the functionvalues f(2) (interpolation; res.: 1.45), and f(7) (extrapolation; result: -3.45). Make sure that your program returns the values $f(x_i)$ when you pass $f(x_i)$. Estimate the error you are making.

Questions:
- Which degree does the interpolating polynomial need to have?
- Is it a good idea to calculate, e.g., f(30)?