Energy extraction from black holes and the jet formation

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Outline

• Lecture 1
  – Introduction
  – Classes of relativistic jets
  – Observations relevant to jet physics (apparent motion, polarization, Faraday rotation)
  – Accretion disks and their magnetic field
  – Unipolar inductors and energy extraction

• Lecture 2
  – Magnetohydrodynamic modeling of jets
  – Bulk acceleration
  – Collimation
  – The role of the outflow environment
  – Jet kinematics
M87 Jet in April 2010
(VLBA 2GHz)

Relative Right Ascension (mas)

Relative Declination (mas)

Core 43GHz
50 Rs

HST-1 2GHz

10^4 Rs (6 pc)

(Hada+)
collimation at \sim 100 Schwarzschild radii
Black Holes at all scales

18 September 2013, Ioannina

(Asada & Nakamura 2011)

Viewing angle $i = 14^\circ$

$M_{BH} = 6.6 \times 10^9 M_{\odot}$

Jet radius $r \ [r_s]$

Parabolic

$z \propto r^{1.73 \pm 0.05}$

Conical

$z \propto r^{0.96 \pm 0.1}$

ISCO

Bondi radius

VLBA at 43 GHz
VLBA at 15 GHz
EVN at 1.6 GHz
MERLIN at 1.6 GHz
Edge-brightened region

Hada et al

\( z \propto r^{1.76 \pm 0.03} \)

Good agreement

Jet radius (\( R_s \))

Distance from jet origin (\( R_s \))
The Quasar 3C345

Zoom in the Jet-Origin with Space-VLBI

VLBI-Observation of the Plasma-Jet

Plasma Jet

super-massive Black-Hole

observed Jet Origin

Plasma Components

observed Jet Origin

Observation:
Jens Klare et al.

10 Light-Years

35 Light-Years

(credit: Klare+)
The plasma components move with superluminal apparent speeds

They travel on curved trajectories

The trajectories differ from one component to the other
microquasars

scale-down of quasars

speed $\sim 0.9 - 0.99c$
(scale = 1000 AU, $V_\infty = \text{a few} \ 100 \text{km/s}$)
Black Holes at all scales

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GRB prompt emission

Variability timescale $\delta t$
compact source $R < c \delta t \sim 1000$ km
huge optical depth for $\gamma\gamma \rightarrow e^+e^-$
compactness problem: how the photons escape?

relativistic motion $\gamma \gtrsim 100$

$R < \gamma^2 c \delta t$
blueshifted photon energy
beaming
optically thin
Jet speed

Superluminal Motion in the M87 Jet
• Superluminal apparent motion: $\beta_{\text{app}}$ is a lower limit of real $\gamma$

• If we know both $\beta_{\text{app}} = \frac{\beta \sin \theta_n}{1 - \beta \cos \theta_n}$ and $\delta \equiv \frac{1}{\gamma (1 - \beta \cos \theta_n)}$
we find $\theta_n(t_{\text{obs}})$, $\beta(t_{\text{obs}})$ and $v = c\beta$, $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

Rough estimates of $\delta$ from:

– comparison of radio and high energy emission (SSC)
e.g., for the C7 component of 3C 345 Unwin+ argue that $\delta$
changes from $\approx 12$ to $\approx 4$ ($t_{\text{obs}} = 1992 - 1993$) $\implies$
acceleration from $\gamma \sim 5$ to $\gamma \sim 10$ over $\sim 3 - 20$ pc from the
core ($\theta_n$ changes from $\approx 2$ to $\approx 10^\circ$)
Similarly Piner+2003 inferred an acceleration from $\gamma = 8$ at
$R < 5.8$pc to $\gamma = 13$ at $R \approx 17.4$pc in 3C 279

– variability timescale (compared to the light crossing time),
Jorstad, Marscher+
On the bulk acceleration

- More distant components have higher apparent speeds
- Brightness temperature increases with distance

(Lee and Lobanov)
• recent measurements of apparent velocity in M87, along and across the jet (Mertens and Lobanov)

• A more general argument on the acceleration (Sikora+):
  ✴ lack of bulk-Compton features $\rightarrow$ small ($\gamma < 5$) bulk Lorentz factor at $\lesssim 10^3 r_g$
  ✴ the $\gamma$ saturates at values $\sim$ a few 10 around the blazar zone ($10^3 - 10^4 r_g$)

So, relativistic AGN jets undergo the bulk of their acceleration on parsec scales ($\gg$ size of the central black hole)

• Sikora+ also argue that the protons are the dynamically important component in the outflow.
Polarization

(Marscher+2008 Nature)

helical motion and field rotate the EVPA as the blob moves

observed $E_{rad} \perp B_{rad}$ and $B_{rad}$ is $\parallel B_{\perp los}$

(modified if the jet is relativistic)
Faraday rotation

Faraday rotation – the plane of LP rotates when polarized EM wave passes through a magnetized plasma, due to different propagation velocities of the RCP and LCP components of the EM wave in the plasma.

If internal, there is also depolarization (fractional polarization depends on $\lambda$).

If external, the amount of rotation is proportional to the square of the observing wavelength, and the sign of the rotation is determined by the direction of the line of sight B field:

$$\chi = \chi_0 + (RM)\lambda^2$$

$$(RM) \propto \int n_e B_{||los} d\ell$$
Faraday RM gradients across the jet

(from Asada+)

helical field surrounding the emitting region
Theory: Hydro-Dynamics

Hard to explain bulk acceleration.

• In case $n_e \sim n_p$, $\gamma_{\text{max}} \sim kT_i/m_pc^2 \sim 1$ even with $T_i \sim 10^{12}K$

• If $n_e \neq n_p$, $\gamma_{\text{max}} \sim (n_e/n_p) \times (kT_i/m_pc^2)$ could be $\gg 1$

• With some heating source, $\gamma_{\text{max}} \gg 1$ is in principle possible

However, even in the last two cases, HD is unlikely to work because the HD acceleration saturates at distances comparable to the sonic surface where gravity is still important, i.e., very close to the disk surface (certainly at $\ll 10^3r_g$)
We need magnetic fields

- They extract energy (Poynting flux)
- Extract angular momentum
- Transfer energy and angular momentum to matter
- Explain relatively large-scale acceleration
- Collimate outflows and produce jets
- Needed for synchrotron emission
- Explain polarization and RM maps
$B$ field from advection, or dynamo, or cosmic battery (Poynting-Robertson drag).
Origin of magnetic fields?

\[
\frac{\partial B}{\partial t} = -c \nabla \times E \text{ and }
\]
\[
m_{e}n_{e}\frac{d\mathbf{v}_{e}}{dt} = -\nabla P_{e} - e n_{e} \left( E + \frac{\mathbf{v}_{e}}{c} \times \mathbf{B} \right) + (\ldots)(\mathbf{v} - \mathbf{v}_{e}), \text{ with }
\]
\[
\mathbf{v}_{e} = \mathbf{v} - \frac{\mathbf{J}}{en}, \text{ give (neglecting the electron's inertia)}
\]
\[
\frac{\partial B}{\partial t} \approx -c \nabla \times \left( -\frac{\mathbf{v}}{c} \times \mathbf{B} + \frac{\mathbf{J}}{en} \times \mathbf{B} - \frac{1}{en} \nabla P_{e} + \frac{\mathbf{J}}{\sigma} \right)
\]

- Biermann battery (for non-barotropic plasmas)
- $\alpha\omega$ dynamo
- Contopoulos and Kazanas battery

Advection by accretion? MRI?
A unipolar inductor

\[ J \times B_\phi \]

\[ \text{Poynting flux} \quad \frac{c}{4\pi} E B_\phi \]

is extracted (angular momentum as well)

The Faraday disk could be the rotating accretion disk, or the frame dragging if energy is extracted from the ergosphere of a rotating black hole (Blandford & Znajek mechanism)
How to model magnetized outflows?

★ as pure electromagnetic energy (force-free, magnetodynamics, electromagnetic outflows):
  – ignore matter inertia (reasonable near the origin)
  – this by assumption does not allow to study the transfer of energy form Poynting to kinetic
  – wave speed $= c \rightarrow$ no shocks
  – there may be some dissipation (e.g. reconnection) $\rightarrow$ radiation

★ as magneto-hydro-dynamic flow
  – the force-free case is included as the low inertia limit
  – MHD can also describe the back reaction from the matter to the field (this is important even in the superfast part of the regime where $\sigma \gg 1$)
**Magnetized outflows**

- Extracted energy per time $\dot{E}$ mainly in the form of Poynting flux (magnetic fields tap the rotational energy of the compact object or disk)
  \[
  \dot{E} = \frac{c}{4\pi} \frac{r\Omega}{c} B_p \ B_\phi \times (\text{area}) \approx \frac{c}{2} B^2 r^2
  \]

- Ejected mass per time $\dot{M}$

- The $\mu \equiv \dot{E} / \dot{M} c^2$ gives the maximum possible bulk Lorentz factor of the flow

- **Magnetohydrodynamics**: matter (velocity, density, pressure) + large scale electromagnetic field
Source of energy

If BZ, rotation of black hole, \( \dot{E} = 10^{42} a^2 (B/10^4 G)^2 M_8^2 \) erg/s

If extraction from disk, \( \dot{E} = b \frac{GM \dot{M}_a}{2r_i} \)

Thin disk physics \( \dot{M}_a = 2\pi r \times 2H \times \rho |V_r| \), \( H = \frac{c_s}{\Omega} \), \( c_s = \sqrt{\frac{P}{\rho}} \)

slow Mach number \( M_s = \frac{|V_r|}{c_s} \), plasma beta \( \beta = \frac{8\pi P}{B^2} \)

So, \( \frac{\dot{E}_{BZ}}{\dot{E}_{disk}} = 3 \times 10^{-3} \frac{a^2}{b \beta M_s} \left( \frac{r_i}{r_g} \right)^{-3/2} \)

E.g., for \( a = 1 \), \( r_i = r_g \) and JED with \( b \sim 0.5 \), \( \beta \sim M_s \sim 1 \), \( \dot{E}_{BZ} \) is only a few percent on \( \dot{E}_{disk} \) (Ferreira)

Other disk models?
MHD (Magneto-Hydro-Dynamics)

- matter: velocity $V$, density $\rho$, pressure $P$

- large scale electromagnetic field: $E$, $B$, $J$, $\delta$
The flow can be relativistic wrt

- **bulk velocity** \( V \approx c, \gamma \gg 1 \)

- **random motion in comoving frame** \( k_B T/m/c^2 \gg 1 \), or, \( P \gg \rho_0 c^2 \), where \( \rho_0 = \rho/\gamma \) the density in comoving frame

Define specific enthalpy
\[
\xi c^2 = \frac{\text{mass} \times c^2 + \text{internal energy} + P \times \text{volume}}{\text{mass}} = c^2 + \frac{1}{\Gamma-1} \frac{P}{\rho_0} + \frac{P}{\rho_0}, \text{ or,}
\]
\[
\xi = 1 + \frac{\Gamma}{\Gamma-1} \frac{P}{\rho_0 c^2} \gg 1
\]

- **gravity** \( r \sim GM/c^2 \)

Define lapse function \( h = \sqrt{1 - \frac{2GM}{c^2 r}} \)

In nonrelativistic flows, \( \gamma, \xi, h, \text{ all are } \approx 1 \)
• Ohm: \[ E + \frac{V}{c} \times B = 0 \]

• Maxwell:
\[ \nabla \cdot B = 0 = \nabla \times E + \frac{\partial B}{c \partial t}, \quad J = \frac{c}{4\pi} \nabla \times B - \frac{1}{4\pi} \frac{\partial E}{\partial t}, \quad \delta = \frac{1}{4\pi} \nabla \cdot E \]

• mass conservation:
\[ \frac{\partial (\gamma \rho_0)}{\partial t} + \nabla \cdot (\gamma \rho_0 \mathbf{V}) = 0 \]

• momentum:
\[ \gamma \rho_0 \left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) (\xi \gamma \mathbf{V}) = -\nabla P + \delta \mathbf{E} + \frac{J \times B}{c} - \gamma^2 \rho_0 \xi \frac{G \mathcal{M}}{r^2} \hat{r} \]

• energy:
\[ \left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \left( \frac{1}{\Gamma - 1} \frac{P}{\rho_0} \right) + P \left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \frac{1}{\rho_0} = \frac{q}{\rho_0} \]
On the energy equation

- no heating/cooling (adiabatic):
  \[
  \left( \frac{\partial}{\partial t} + V \cdot \nabla \right) \left( \frac{1}{\Gamma - 1} \frac{P}{\rho_0} \right) + P \left( \frac{\partial}{\partial t} + V \cdot \nabla \right) \frac{1}{\rho_0} = 0
  \]
  \[
  \Leftrightarrow (\partial/\partial t + V \cdot \nabla) \left( \frac{P}{\rho_0^\Gamma} \right) = 0 \text{ (entropy conservation)}
  \]

- with \( q \neq 0 \) two approaches:
  - polytropic: Give appropriate \( q \) such that the energy eq has a similar to the adiabatic form (in steady-state)
    \[
    \frac{q}{\rho_0} = \frac{\Gamma - \Gamma_{\text{eff}}}{\Gamma - 1} P V \cdot \nabla \frac{1}{\rho_0}
    \]
    like adiabatic with \( \frac{2}{\Gamma_{\text{eff}} - 1} \) degrees of freedom
  - non-polytropic: ignore the energy equation, use it only a posteriori to find \( q \) (after solving for the dynamics)

  In essence, the pressure is also eliminated in this method, by replacing the poloidal momentum eq with \( \nabla \times \nabla P = 0 \) (e.g., in nonrelativistic steady-state, with
  \[
  \nabla \times \left[ -\rho (V \cdot \nabla) V + J \times B / c - \rho G M / r^2 \right] = 0
  \]
Assumptions

- ideal MHD
  zero resistivity: \( 0 = E_{co} = \gamma \left( E + \frac{V}{c} \times B \right) - (\gamma - 1) \left( E \cdot \frac{V}{V} \right) \frac{V}{V} \)
  \( \Leftrightarrow \) \( E = -\frac{V}{c} \times B \)

- one fluid approximation
  \( V_+ \approx V_- \), or, \( J \ll \frac{\rho}{m} |e| V \) with \( J = n_+ q_+ V_+ + n_- q_- V_- \)
  quasi-neutrality \( \delta = n_+ q_+ + n_- q_- \ll \frac{\rho}{m} |e| \)
  \( n_+ \approx n_- \approx \rho/m_p \) for \( e^- p \) plasma

- further assume steady-state \( \partial / \partial t = 0 \)

- axisymmetry \( \partial / \partial \phi = 0 \) in cylindrical \((z, \varpi, \phi)\) or spherical \((r, \theta, \phi)\) coordinates
Integrals of motion

From $\nabla \cdot B = 0 \Rightarrow \nabla \cdot B_p = 0$

$$B_p = \frac{\nabla A \times \hat{\phi}}{\Omega}, \text{ or, } B_p = \nabla \times \left( \frac{A \hat{\phi}}{\Omega} \right)$$

$$A = \frac{1}{2\pi} \iint B_p \cdot dS$$

From $\nabla \times E = 0$, $E = -\nabla \Phi$

Because of axisymmetry $E_\phi = 0$.

Combining with Ohm’s law $(E = -V \times B/c)$ we find $V_p \parallel B_p$. 
Because $V_p \parallel B_p$ we can write

$$V = \frac{Ψ_A}{4πγρ₀} B + ωΩ\hat{ϕ}, \quad \frac{Ψ_A}{4πγρ₀} = \frac{V_p}{B_p}.$$  

$$\left( V_φ = \frac{B_φ}{B_p} V_p + ωΩ \right)$$

The $Ω$ and $Ψ_A$ are constants of motion, $Ω = Ω(A)$, $Ψ_A = Ψ_A(A)$.

- $Ω$ = angular velocity at the base
- $Ψ_A$ = mass-to-magnetic flux ratio

The electric field $E = -V \times B/c = - (ωΩ/c)\hat{ϕ} \times B_p$ is a poloidal vector, normal to $B_p$. Its magnitude is $E = \frac{ωΩ}{c} B_p$. 
So far, we’ve used Maxwell’s eqs, Ohm’s law and the continuity.

The energy and momentum equations remain. The latter:

\[ \gamma \rho_0 (\mathbf{V} \cdot \nabla) (\xi \gamma \mathbf{V}) = -\nabla P + \frac{J \times B}{c} - \gamma^2 \rho_0 \xi \frac{G M}{r^2} \hat{r} \]

or,

\[ \gamma \rho_0 (\mathbf{V} \cdot \nabla) (\xi \gamma \mathbf{V}) = -\nabla P + \frac{\nabla \cdot E}{4\pi} \times E + \frac{\nabla \times B}{4\pi} \times B - \gamma^2 \rho_0 \xi \frac{G M}{r^2} \hat{r} \]

Due to axisymmetry, the toroidal component can be integrated to give the total angular momentum-to-mass flux ratio:

\[ \xi \gamma \varpi V_\phi \left\{ -\frac{\varpi B_\phi}{\Psi_A} \right\} = L(A) \]

\[ \frac{-\varpi B_p B_\phi / 4\pi}{\rho V_p} \]
For polytropic flows:

- the energy eq gives \( \frac{P}{\rho_0^{\Gamma_{\text{eff}}}} = Q(A) \)
  (the effective entropy is a constant of motion).

- the momentum along \( V_p \) gives \( (h \xi \gamma - 1)c^2 - h\varpi\Omega B_\phi/\Psi_A = \mathcal{E}(A) \)
  
  - In relativistic MHD,
    \[
    h \xi \gamma - h\varpi\Omega B_\phi/\Psi_A c^2 = 1 + \mathcal{E}/c^2 \equiv \mu(A) = \text{maximum } \gamma
    \]

  - In nonrelativistic, expansion wrt \((\ldots)/c^2\) gives
    \[
    \frac{V^2}{2} + \frac{\Gamma_{\text{eff}} P}{\Gamma_{\text{eff}} - 1 \rho} - \frac{GM}{r} - \frac{\varpi\Omega B_\phi}{\Psi_A} \left( \frac{c}{4\pi} \right) \frac{E |B_\phi|}{\rho V_p} = \mathcal{E}(A)
    \]
Info from integrals (nonrelativistic case)

- near the source,
  \( V_\phi = V_p B_\phi / B_p + \varpi \Omega \to V_\phi \approx \varpi \Omega \)
  for disk-driven flows \( \Omega \approx \Omega_K \)
solid-body rotation (rotating wires up to Alfvén surface – \(|B_\phi|/B_p\) increases)
- at large distances
  \( \varpi V_\phi - \frac{\varpi B_\phi}{\Psi_A} = L(A) \to V_\phi \sim L/\varpi \)

- \( A = \text{const} \to B_p \propto 1/\varpi^2 \)
  \( \rho V_p \propto B_p \to \rho V_p \propto 1/\varpi^2 \)
- near the source, for Poynting-dominated flows \( \mathcal{E} \approx -\varpi \Omega B_{\phi i}/\Psi_A \to B_{\phi i} \approx \frac{-\Psi_A \mathcal{E}}{\varpi_i \Omega} \).

- Alfvén surface: combination of integrals \( B_\phi = -\frac{L \Psi_A 1 - \varpi^2 \Omega/L}{\varpi 1 - M^2} \to \varpi_A = \sqrt{L/\Omega} \)
  \( (M = V_p \sqrt{4\pi \rho / B_p} = \text{Alfvén Mach}) \)
  \( L = \Omega \varpi_A^2, \varpi_A = \text{lever arm for } L\text{-extraction} \)
- maximum asymptotic velocity \( V_\infty = \sqrt{2\mathcal{E}} \approx \sqrt{2L \Omega} = \varpi_A \Omega \sqrt{2} = \frac{\varpi_A}{\varpi_i} \sqrt{2} V_{\phi i} \)
- \( L \approx \varpi_\infty V_{\phi \infty}, \text{ so } V_\infty \approx \sqrt{2 \varpi_\infty V_{\phi \infty} \Omega} \)
  (connection between \( V_\infty, V_{\phi \infty}, \varpi_\infty - \varpi_i \))
The partial integration of the system, the relations between the integrals, and the definition of \( M = V_p/(B_p/\sqrt{4\pi \rho}) \), yield

\[
B = \frac{\nabla A \times \hat{\phi}}{\omega} - \frac{L \Psi_A}{\omega} \frac{1 - \omega^2\Omega/L}{1 - M^2} \phi,
\]

\[
V = \frac{M^2 \nabla A \times \hat{\phi}}{\Psi_A \omega} + \frac{L}{\omega} \frac{\omega^2\Omega/L - M^2}{1 - M^2} \phi,
\]

\[
\rho = \frac{\Psi_A^2}{4\pi M^2}
\]

Thus, we have only two unknowns, the \( A \) and \( M \), given by the two poloidal components of the momentum equation

\( M \) comes from momentum along the flow and \( A \) from the transfield

these two eqs are coupled!
Poloidal components of the momentum eq

\[ \gamma \rho_0 (V \cdot \nabla) (\xi \gamma V) = -\nabla P + \frac{(\nabla \cdot E) E + (\nabla \times B) \times B}{4\pi} \Leftrightarrow 0 = f_G + f_T + f_C + f_I + f_P + f_E + f_B \]

- \( f_G = -\gamma \rho_0 \xi (V \cdot \nabla) V \) : “temperature” force
- \( f_T = -\gamma^2 \rho_0 (V \cdot \nabla \xi) V \) : centrifugal force
- \( f_C = \omega \gamma^2 \rho_0 \xi V_\phi^2 / \omega \) : inertial force
- \( f_I = -\gamma^2 \rho_0 \xi (V \cdot \nabla) V - f_C \)
- \( f_P = -\nabla P \) : pressure force
- \( f_E = (\nabla \cdot E) E / 4\pi \) : electric force
- \( f_B = (\nabla \times B) \times B / 4\pi \) : magnetic force
Acceleration mechanisms

- **thermal** (due to $\nabla P$) $\rightarrow$ velocities up to $C_s$
- **magnetocentrifugal** (beads on wire)
  - initial half-opening angle $\vartheta > 30^o$ (only for cold flows)
  - velocities up to $\lesssim \varpi_i \Omega$
  - in reality due to magnetic pressure: the constancy of the integral $L$ gives
    \[
    f_C\parallel = -\rho V\phi \frac{\partial V\phi}{\partial \ell} + \frac{V\phi}{V_p}(B_p/|B\phi|)f_B\parallel
    \]
- **relativistic thermal** (thermal fireball) gives $\gamma \sim \xi_i$, where $\xi = \frac{\text{enthalpy}}{\text{mass} \times c^2}$.
- **magnetic** due to $f_B\parallel \propto$ gradient of $\varpi B\phi$ $\rightarrow$ velocities up to complete matter domination (not always the case)

All acceleration mechanisms can be seen in

\[
\frac{V^2}{2} + \frac{\Gamma_{\text{eff}}}{\Gamma_{\text{eff}} - 1} \frac{P}{\rho} - \frac{G\mathcal{M}}{r} - \frac{\Omega}{\Psi_A} \varpi B\phi = \mathcal{E}, \text{ or, } \xi \gamma - \frac{\Omega}{\Psi_A c^2} \varpi B\phi = \mu
\]

So $V, \gamma \uparrow$ when $P/\rho_0, \xi \downarrow$ (thermal, relativistic thermal), or, $\varpi |B\phi| \downarrow$ (magnetocentrifugal, magnetic).
On the magnetic acceleration

\[(\nabla \times B) \times B = (\nabla \times B_p) \times B_p + (\nabla \times B_p) \times B_\phi + (\nabla \times B_\phi) \times B_p + (\nabla \times B_\phi) \times B_\phi\]

\[-\frac{B_\phi^2}{\omega} \nabla \phi - \frac{B_\phi^2}{2} = -\frac{B_\phi}{\omega} \nabla (\omega B_\phi)\]

\[f_B \| = -\frac{B_\phi}{4\pi \omega} \nabla \ell (\omega B_\phi)\]

\[\omega B_\phi\] is related to the poloidal current

\[J_p = \frac{c}{4\pi} \nabla \times B_\phi = \frac{1}{2\pi \omega} \nabla I \times \hat{\phi},\] with

\[I = \int J_p \cdot dS = \frac{c}{2} \omega B_\phi\]
Currents or magnetic fields?

Although in MHD $B$ drive the currents and not the opposite (since $J = ne(V_+ - V_-)$ with $|V_+ - V_-| \ll V$), currents are useful in understanding acceleration/collimation.
The efficiency of the magnetic acceleration

The $\mathbf{J}_p \times \mathbf{B}_\phi$ force strongly depends on the angle between field-lines and current-lines (for zero angle no acceleration at all)

Are we free to choose these two lines? NO! All MHD quantities are related to each other and should be found by solving the full system of equations.

At classical fast surface $V_p \approx B / \sqrt{4\pi \rho}$,

$$\frac{\text{kinetic}}{\text{Poynting}} = \frac{V_p^2/2}{-\varpi \Omega B_\phi / \Psi_A} \approx \frac{1}{2} \left( 1 - \frac{V_\phi}{\varpi \Omega} \right) \frac{B^2}{B_\phi^2} \sim \frac{1}{2}$$

(using $\Psi_A = 4\pi \rho V_p / B_p$ and $V_\phi = \varpi \Omega + V_p B_\phi / B_p$)

For relativistic flows $\gamma_f \approx \mu^{1/3} (\ll \mu)$ and this ratio is $\mu^{-2/3} \ll 1$!
\[ V_p = \left( \frac{M^2}{\Psi_A} \right) B_p \quad \text{(from} \quad M^2 = 4\pi \rho V_p^2 / B_p^2 \quad \text{and} \quad \Psi_A = 4\pi \rho V_p / B_p) \]

At \( M \gg 1, \omega \gg \omega_A \),
\[ -\frac{\omega \Omega B_\phi}{\Psi_A} \approx \frac{\Omega^2 B_p \omega^2}{\Psi_A V_p} \]

(from \( B_\phi = -\frac{L \Psi_A}{\omega} \frac{1 - \omega^2 \Omega / L}{1 - M^2} \approx -\frac{L \Psi_A \omega^2 \Omega / L}{\omega} \frac{M^2}{M^2} \) at large distances)

The kinetic Poynting = 1/2 at fast gives

\[ V_f = \left( \frac{\Omega^2 B_p \omega^2}{\Psi_A} \right)^{1/3} \]

(The relativistic version is \( \gamma_f = \mu^{1/3} \).)
Acceleration continues after the fast (crucial especially for relativistic flows)

Defining the constant of motion

\[
\sigma_m = \frac{A\Omega^2}{\Psi A\mathcal{E} V_\infty} = \frac{A\Omega^2(1 + \mathcal{E}/c^2)}{\Psi A\mathcal{E}^{3/2}\sqrt{2 + \mathcal{E}/c^2}}
\]

we find (by combining the integral relations)

\[
\frac{\text{Poynting}}{\text{total energy flux}} = \sigma_m \left(1 - \frac{V_\phi}{\varpi \Omega}\right) \frac{V_\infty B_p \varpi^2}{V_p A} \propto \frac{B_p \varpi^2}{A}
\]

So, the transfield force-balance determines the acceleration through the "bunching function" \(B_p \varpi^2/A\).

Another ways to see it: Poynting \(\propto \varpi B_\phi\) and \(B_\phi/B_p \approx E/B_p \approx \varpi \Omega/c\), so Poynting \(\propto \varpi^2 B_p\)
The magnetic field minimizes its energy under the condition of keeping the magnetic flux constant.

So, $\omega B_\phi \downarrow$ for decreasing

$$\omega^2 B_p = \frac{\omega^2}{2\pi \omega \, dl_\perp} (B_p dS) \propto \frac{\omega}{dl_\perp}.$$ 

Expansion with increasing $dl_\perp/\omega$ leads to acceleration

The expansion ends in a more-or-less uniform distribution $\omega^2 B_p \approx A$ (in a quasi-monopolar shape).
Conclusions on the magnetic acceleration

- In nonrelativistic flows efficiency $\sim 1/3$ already at classical fast (increases further – the final value depends on the fieldline shape).

- In relativistic flows: If we start with a uniform distribution the magnetic energy is already minimum $\rightarrow$ no acceleration. Example: Michel’s (1969) solution which gives $\gamma_\infty \approx \mu^{1/3} \ll \mu$.

  Also Beskin+1998; Bogovalov 2001 who found quasi-monopolar solutions.

  For any other (more realistic) field distribution we have efficient acceleration!
The acceleration efficiency in relativistic flows

Applying
\[
\frac{\text{Poynting total energy flux}}{\text{total energy flux}} = \sigma_m \left(1 - \frac{V_\phi}{\varpi \Omega} \right) \frac{V_\infty B_p \varpi^2}{V_p A}
\]
at fast, we get \(\sigma_m \approx (A/B_p \varpi^2)_{\text{fast}}\)

Applying the same relation at infinity – where \(B_p \varpi^2 \approx A\) – yields
\[
(P\text{oynting/total energy flux})_\infty = \sigma_m, \text{ or, } \frac{\mathcal{E} - \gamma_\infty c^2}{\mathcal{E}} = \sigma_m \rightarrow
\]

\[
\gamma_\infty \approx \frac{\mathcal{E}}{c^2} (1 - \sigma_m) \approx \frac{\mathcal{E}}{c^2} \left(1 - \frac{A}{(\varpi^2 B_p)_f}\right)
\]

The more bunched the fieldlines near the fast surface the higher the acceleration efficiency.

\[
I = c \varpi B_\phi / 2 \approx -c \varpi E / 2 \approx -c \varpi (\varpi \Omega B_p / c) / 2 = (A \Omega / 2)(\varpi^2 B_p / A).
\]

So, \(\gamma_\infty \approx (\mathcal{E} / c^2) (1 - A \Omega / 2 I_f)\)

Since the flow is force-free up to the fast, \(I_i \approx I_f\) and we connect the acceleration efficiency with conditions at the base of the flow
\[
\gamma_\infty \approx (\mathcal{E} / c^2) (1 - A \Omega / 2 I_i)
\]
On the collimation

hoop-stress:

+ electric force

degree of collimation? Role of environment?
The $\mathbf{J}_p \times \mathbf{B}_\phi$ force contributes to the collimation (hoop-stress paradigm). In nonrelativistic flows works fine. In relativistic flows the electric force plays an opposite role (a manifestation of the high inertia of the flow).

- collimation by an external wind
- self-collimation mainly works at small distances where the velocities are mildly relativistic
- surrounding medium plays a role
some key steps on MHD modeling

• Michel 1969: assuming monopole flow (crucial) → inefficient acceleration with $\gamma_\infty \approx \mu^{1/3} \ll \mu$

• Li, Chiueh & Begelman 1992; Contopoulos 1994: cold self-similar model → $\gamma_\infty \approx \mu/2$ (50% efficiency)

• Vlahakis & Königl 2003, 2004: generalization of the self-similar model (including thermal and radiation effects) → $\gamma_\infty \approx \mu/2$ (50% efficiency)

• Vlahakis 2004: complete asymptotic transfield force-balance connect the flow-shape (collimation) with acceleration explain why Michel’s model is inefficient

• Beskin & Nokhrina 2006: parabolic jet with $\gamma_\infty \approx \mu/2$
some key steps (cont’d)

• Komissarov, Barkov, Vlahakis & Königl 2007 and Komissarov, Vlahakis, Königl & Barkov 2009: possible for the first time to simulate high γ MHD flows and follow the acceleration up to the end
  + analytical scalings
  + role of causality, role of external pressure

• Tchekhovskoy, McKinney & Narayan, 2009: simulations of nearly monopolar flow (more detailed than in Komissarov+2009)
  Even for nearly monopolar flow the acceleration is efficient near the rotation axis
some key steps (cont’d)

• Lyubarsky 2009: generalization of the analytical results of Vlahakis 2004 and Komissarov+2009

• recent simulations by McKinney, Tchekhovskoy, Blandford, and Tchekhovskoy, McKinney, Narayan

• Contopoulos, The Force-free Magnetosphere of a Rotating Black Hole
“Standard” model for magnetic acceleration

component of the momentum equation

\[ \gamma \rho_0 (\mathbf{V} \cdot \nabla) (\gamma \xi \mathbf{V}) = -\nabla p + J^0 \mathbf{E} + \mathbf{J} \times \mathbf{B} \]

along the flow (wind equation): \( \gamma \approx \mu - \mathcal{F} \)

where \( \mathcal{F} \propto \varpi^2 B_p \)

since \( B_p \delta S = \text{const} \),
\[ \mathcal{F} \propto \varpi^2 / \delta S \propto \varpi / \delta \ell_\perp \]

acceleration requires the separation between streamlines to increase faster than the cylindrical radius

the collimation-acceleration paradigm:
\( \mathcal{F} \downarrow \) through stronger collimation of the inner streamlines relative to the outer ones (differential collimation)
transfield component of the momentum equation

\[ \frac{\gamma^2 \varpi}{R} \approx \left( \frac{2I}{\Omega B_p \varpi^2} \right)^2 \varpi \nabla_\perp \ln \left| \frac{I}{\gamma} \right| - \gamma^2 \frac{\varpi_{1c}^2}{\varpi^2} \nabla_\perp \varpi, \text{ with } \nabla_\perp \sim \frac{1}{\varpi}, \]

\[ \varpi_{1c} = \frac{c}{\Omega}, \]

simplifies to

\[ \frac{\gamma^2 \varpi}{R} \approx \left( \frac{1}{EM} \right)_{\text{inertia}} - \left( \frac{\gamma^2 \varpi_{1c}^2}{\varpi^2} \right)_{\text{centrifugal}} \]

- if centrifugal negligible then \( \gamma \approx \frac{z}{\varpi} \) (since \( R^{-1} \approx -\frac{d^2 \varpi}{dz^2} \approx \frac{\varpi}{z^2} \) power-law acceleration regime (for parabolic shapes \( z \propto \varpi^a \), \( \gamma \) is a power of \( \varpi \))

- if inertia negligible then \( \gamma \approx \varpi/\varpi_{1c} \) linear acceleration regime

- if electromagnetic negligible then \text{ballistic regime}
role of external pressure

\[ p_{\text{ext}} = B_{\text{co}}^2 / 8\pi \simeq (B^\phi)^2 / 8\pi \gamma^2 \propto 1/\varpi^2 \gamma^2 \]

Assuming \( p_{\text{ext}} \propto z^{-\alpha_p} \) we find \( \gamma^2 \propto z^{\alpha_p} / \varpi^2 \).

Combining with the transfield \( \frac{\gamma^2 \varpi}{R} \approx 1 - \gamma^2 \frac{\varpi_\perp^2}{\varpi^2} \) we find the funnel shape (we find the exponent \( a \) in \( z \propto \varpi^a \)).

- if the pressure drops slower than \( z^{-2} \) then
  - shape more collimated than \( z \propto \varpi^2 \)
  - linear acceleration \( \gamma \propto \varpi \)
- if the pressure drops as \( z^{-2} \) then
  - parabolic shape \( z \propto \varpi^a \) with \( 1 < a \leq 2 \)
  - first \( \gamma \propto \varpi \) and then power-law acceleration
    \[ \gamma \sim z/\varpi \propto \varpi^{a-1} \]
- if pressure drops faster than \( z^{-2} \) then
  - conical shape
  - linear acceleration \( \gamma \propto \varpi \) (small efficiency)
Black Holes at all scales

18 September 2013, Ioannina
Asymptotic flow-shape

From
\[ \frac{\text{Poynting total energy flux}}{A} = \sigma_m \left( 1 - \frac{V_\phi}{\varpi \Omega} \right) \frac{V_\infty B_p \varpi^2}{A} \]
we find
\[ \sigma_m \left( 1 - \frac{V_\phi}{\varpi \Omega} \right) \frac{B_p \varpi^2}{A} = \frac{V_p}{V_\infty} \frac{\text{Poynting total energy flux}}{A} < 1. \]

For \( V_\phi \ll \varpi \Omega \), this gives \( B_p \varpi^2 / A < 1 / \sigma_m \) (same as in Heyvaerts & Norman asymptotic analysis).

Assume two lines that cross a given cylinder \( \varpi = \text{const} \), one reference line \( A_i \) and another \( A \) that crosses the cylinder at higher \( z \) (so \( A < A_i \)).

The \( \hat{\varpi} \) component of \( B_p = \nabla A \times \hat{\phi} / \varpi \) is \( B_\varpi = -(1/\varpi) \partial A / \partial z \), or,
\[ \partial z / \partial A = -1 / (\varpi B_\varpi) \]
and can be integrated along the cylinder to give
\[ z(A, \varpi) = z(A_i, \varpi) + \int_{A_i}^{A} \frac{\sigma_m A}{A} dA. \]

But, \( B_\varpi < B_p < A / (\varpi^2 \sigma_m) \), and thus,
\[ z(A, \varpi) > z(A_i, \varpi) + \varpi \int_{A}^{A_i} \frac{\sigma_m}{A} dA \]
lines cannot bend towards equator, they are situated above some minimal cone.

Black Holes at all scales

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From
\[
\text{Poynting total energy flux} = \sigma_m \left( 1 - \frac{V_\phi}{\varpi \Omega} \right) \frac{V_\infty B_p \varpi^2}{V_p A}
\]
with Poynting \(= -\varpi \Omega B_\phi / \Psi_A = -2I\Omega/(c\Psi_A)\) and \(B_p/V_p = 4\pi \rho / \Psi_A\) we find
\[
\frac{I}{\gamma} = -2\pi c \frac{\Omega}{\Psi_A} \rho_0 \varpi^2
\]

• if \(\rho_0 \varpi^2 \to \infty\) \(\to f(A)\) (conical lines \(z/\varpi \to \infty\) const) then
\(I_\infty(A)/\gamma_\infty(A) = \text{const}\)
this constant is independent of \(A\), from
\[
\frac{\gamma^2 \varpi}{R} \approx \left[ \left( \frac{2I}{\Omega B_p \varpi^2} \right)^2 \varpi \nabla \ln \left| \frac{I}{\gamma} \right| - \gamma^2 \frac{\varpi^2}{\varpi^2} \varpi \hat{\omega} \right] \cdot \frac{\nabla A}{|\nabla A|} \text{ with } R/\varpi \to \infty \to \infty
\]

• if \(\rho_0 \varpi^2 \to 0\) (parabolic lines \(z/\varpi \to \infty\) \(\to \infty\)) then \(I_\infty/\gamma_\infty = 0\)
(100% acceleration efficiency)
\( I_\infty(A)/\gamma_\infty(A) = \text{const: solvability condition at infinity} \)

(Heyvaerts & Norman 1989; Chiueh, Li, & Begelman 1999; see also Vlahakis 2004 for generalized analysis)

- nonrelativistic case: \( I_\infty(A) = \text{const} \)
  - no current \( J_p \) flows between lines
  - If the flow carries some finite Poynitng flux at infinity, the corresponding \( J_p \) flows inside a **cylindrical core** (this is the only way to have \( I \) smoothly varying from zero on the axis to \( I_\infty \) at the edge of the core).
  - Note that for cylindrical lines the previous analysis (based on \( \varpi \to \infty \)) doesn’t hold.

- relativistic case:
  - again the **cylindrical core** is the only way to have \( I_\infty(A = 0) = 0 \) and \( I_\infty(A)/\gamma_\infty(A) = \text{const} \neq 0 \) at larger \( A \)
Self-similarity

• simple 1-D models (e.g., Weber & Davis, or spherically symmetric, monopole-like) cannot describe jets

• giving the flow-shape and solve for velocity is incomplete (solving the transfield is crucial for the acceleration)

• self-similarity: 1-D from mathematical point of view (ODEs), 2-D from physical
  we need an algorithm to produce all lines from a reference one
  Example: \( A = r^x f(\theta) \):
  If a line \( A \) starts from \( r_0 \) at \( \theta = \pi/2 \), then \( A = r_0^x f(\pi/2) \) and so,
  \[ r = r_0 \left[ f(\pi/2)/f(\theta) \right]^{1/x}. \]
  If we know one line – we know \( f(\theta) \) – from the eq above we find all other lines.

choose a coordinate system on the poloidal plane
give the dependence on the one coordinate
solve for the dependence on the other
### $r$ self-similarity

This model corresponds to boundary conditions at a cone $\theta = \theta_i$:

\[ V_r = D_1 r^{-1/2}, \quad V_\theta = D_2 r^{-1/2}, \quad V_\phi = D_3 r^{-1/2}, \quad \rho = D_4 r^{-2x-3}, \quad P = D_5 r^{-2x-4}, \quad B_\phi = D_6 r^{x-2}, \quad B_r = D_7 r^{x-2}. \]

For points on the same cone $\theta = \text{const}$,

\[ \frac{\varpi_1}{\varpi_2} = \frac{r_1}{r_2} = \frac{\mathcal{F}_1(A_1)}{\mathcal{F}_1(A_2)}. \]

**ODEs**

\[
\begin{align*}
\psi &= \psi(G, M, \theta), \quad \text{(Bernoulli)} \\
\frac{dG}{d\theta} &= N_0(G, M, \psi, \theta), \quad \text{(definition of } \psi) \\
\frac{dM}{d\theta} &= N(G, M, \psi, \theta) \quad \frac{d\theta}{D(G, M, \psi, \theta)}, \quad \text{(transfield)}
\end{align*}
\]

\[ \mathcal{D} = 0 : \text{ singular points} \]

(Alfvén, modified slow - fast)

**caveats:** singular at $A = 0$, absence of scale (need to give $A_{\text{in}}, A_{\text{out}}$)
Vlahakis+2000

![Graph showing velocities in units $V_z(z=0)$ vs. $z/\omega(z=0)$.

- $V_z$: Mod. slow Alfven
- $V_\omega$: Mod. fast
- $V_\phi$:

The graph plots the velocities $V_z$, $V_\omega$, and $V_\phi$ as functions of $z/\omega(z=0)$. The x-axis represents $z/\omega(z=0)$, ranging from $10^{-3}$ to $10^4$, while the y-axis represents velocities in units $V_z(z=0)$, ranging from -200 to 1000. The graph highlights the behavior of each velocity component under varying ratios of $z/\omega(z=0)$.]
Black Holes at all scales

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energy in units $V^2 \alpha^{-1/2}$

$V_\phi^2/2$

$V_p^2/2$

$E$

poynting

enthalpy

mod. fast

mod. slow

Alfven

gravity

Black Holes at all scales

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GRB Jets (NV & Königl 2001, 2003a,b)

\[-EB_\phi/(4\pi\gamma_0c V_p)=\text{(Poynting flux)/(mass flux)})^2\]

- \(\varpi_1 < \varpi < \varpi_6\): Thermal acceleration - force free magnetic field
  \((\gamma \propto \varpi, \rho_0 \propto \varpi^{-3}, T \propto \varpi^{-1}, \varpi B_\phi = \text{const}, \text{parabolic shape of fieldlines: } z \propto \varpi^2)\)

- \(\varpi_6 < \varpi < \varpi_8\): Magnetic acceleration \((\gamma \propto \varpi, \rho_0 \propto \varpi^{-3})\)

- \(\varpi = \varpi_8\): cylindrical regime - equipartition \(\gamma_\infty \approx (\text{-}EB_\phi/4\pi\gamma_0c V_p)_\infty\)

\(\tau\cdot\varpi\) is used to represent the ratio of time to \(\varpi\).

\(\xi\) is the enthalpy relating to rest mass.

\(\varpi_2, \varpi_3, \varpi_4, \varpi_5, \varpi_6, \varpi_7, \varpi_8\) are specific values or regimes.

Black Holes at all scales

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\[ \frac{-EB_\phi}{4\pi\gamma_0 c V_p} = \frac{\text{Poynting flux}}{\text{mass flux}} c^2 \]

\[ \xi = \frac{\text{enthalpy}}{\text{rest mass}} c^2 \]

- Thermal acceleration \( (\gamma \propto \varpi^{0.44}, \rho_0 \propto \varpi^{-2.4}, T \propto \varpi^{-0.8}, B_\phi \propto \varpi^{-1}, z \propto \varpi^{1.5}) \)
- Magnetic acceleration \( (\gamma \propto \varpi^{0.44}, \rho_0 \propto \varpi^{-2.4}) \)
- Cylindrical regime - equipartition \( \gamma_\infty \approx \left( -\frac{EB_\phi}{4\pi\gamma_0 V_p} \right)_\infty \)

Black Holes at all scales 18 September 2013, Ioannina
* At $\varpi = 10^8 \text{cm}$ – where $\gamma = 10$ – the opening half-angle is already $\vartheta = 10^\circ$

* For $\varpi > 10^8 \text{cm}$, collimation continues slowly ($\mathcal{R} \sim \gamma^2 \varpi$)
They used prescribed fieldlines (with $\omega^2 B_p/A \propto \omega^{-q}$) and found efficient acceleration with $\gamma_\infty$ (their $u_{p,\infty}$) $\sim \mu$ (their $\sigma$).

Although the analysis is not complete (the transfield is not solved), the results show the relation between line-shape and efficiency.
By expanding the equations wrt $2/\mu$ (their $1/\sigma$) they found a parabolic solution. The acceleration in the superfast regime is efficient, reaching $\gamma_\infty \sim \mu$. The scaling $\gamma \propto \varpi$ is the same as in Vlahakis & Königl (2003a).
Simulations of relativistic jets
Komissarov, Barkov, Vlahakis, & Königl (2007)

Left panel shows density (colour) and magnetic field lines. Right panel shows the Lorentz factor (colour) and the current lines.
Note the difference in $\gamma(r)$ for constant $z$.

It depends on the current $I$, which is related to $\Omega$:

$$I \approx r^2 B_p \Omega / 2$$
γσ (solid line), μ (dashed line) and γ (dash-dotted line) along a magnetic field line as a function of cylindrical radius for models C1 (left panel), C2 (middle panel) and A2 (right panel).
The external pressure is given by
\[
P_{\text{ext}} = \frac{B^2 - E^2}{8\pi}
\]

The graph illustrates different power laws for the external pressure as a function of the radius:

- **Solid line:** $p_{\text{ext}} \propto R^{-3.5}$ for $z \propto r$
- **Dashed line:** $p_{\text{ext}} \propto R^{-2}$ for $z \propto r^{3/2}$
- **Dash-dotted line:** $p_{\text{ext}} \propto R^{-1.6}$ for $z \propto r^2$
- **Dotted line:** $p_{\text{ext}} \propto R^{-1.1}$ for $z \propto r^3$
left: density/field lines, right: Lorentz factor/current lines (wall shape $z \propto r^{1.5}$)

Differential rotation $\rightarrow$ slow envelope
Uniform rotation $\rightarrow \ \gamma$ increases with $r$
Caveat: $\gamma \vartheta \sim 1$ (for high $\gamma$)

During the afterglow $\gamma$ decreases
When $1/\gamma > \vartheta$ the $F(t)$ decreases faster
• with $\gamma \vartheta \sim 1$ very narrow jets ($\vartheta < 1^\circ$ for $\gamma > 100$) $\longrightarrow$ early breaks or no breaks at all

• this is a result of causality (across jet): outer lines need to know that there is space to expand

• Mach cone half-opening $\theta_m$ should be $> \vartheta$

With $\sin \theta_m = \frac{\gamma f C_f}{\gamma V_p} \approx \frac{\sigma^{1/2}}{\gamma}$ the requirement for causality yields $\gamma \vartheta < \sigma^{1/2}$.

For efficient acceleration ($\sigma \sim 1$ or smaller) we always get $\gamma \vartheta \sim 1$
Rarefaction acceleration
Rarefaction acceleration

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Rarefaction acceleration

$R_s / \gamma_i$

$R_s$
Rarefaction simple waves

At $t = 0$ two uniform states are in contact:

This Riemann problem allows self-similar solutions that depend only on $\xi = x/t$.

- when right=vacuum, simple rarefaction wave
At $t > 0$:

For the cold case the Riemann invariants imply

$$v_x = \frac{1}{\gamma_j} \frac{2\sigma_j^{1/2}}{1 + \sigma_j} \left[ 1 - \left( \frac{\rho}{\rho_j} \right)^{1/2} \right], \quad \gamma = \frac{\gamma_j (1 + \sigma_j)}{1 + \sigma_j \rho / \rho_j}, \quad \rho = \frac{4\rho_j}{\sigma_j} \sinh^2 \left[ \frac{1}{3} \text{arcsinh} \left( \sigma_j^{1/2} - \frac{\mu_j x}{2 \gamma j} \right) \right]$$

$$V_{%h e%a d} = \frac{-\sigma_j^{1/2}}{\gamma_j}, \quad V_{%t ai l} = \frac{1}{\gamma_j} \frac{2\sigma_j^{1/2}}{1 + \sigma_j}, \quad \Delta \theta = V_{%t ai l} < 1/\gamma_i$$
The colour image in the Minkowski diagrams represents the distribution of the Lorentz factor and the contours show the worldlines of fluid parcels initially located at $x_i = -1, -0.8, -0.6, -0.4, -0.2, -0.02, 0$. 
Simulation results
Komissarov, Vlahakis & Königl 2010

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Steady-state rarefaction wave
Sapountzis & Vlahakis (2013)

- “flow around a corner”
- planar geometry
- ignoring $B_p$ (nonzero $B_y$)
- similarity variable $x/z$ (angle $\theta$)
- generalization of the nonrelativistic, hydrodynamic rarefaction (e.g. Landau & Lifshitz)
- in addition, allow for inhomogeneity in the “left” state
\[
\theta_{\text{head}} = -\frac{\sigma_j^{1/2}}{\gamma_j}
\]
\[
\theta_{\text{tail}} = \frac{2\sigma_j^{1/2}}{\gamma_j(1+\sigma_j)}
\]
\[
\sigma = \left(\sigma_j \gamma_j \frac{x_i}{z}\right)^{2/3}
\]
\[
\sigma = 1\ \text{at}\ r = \sigma_j \gamma_j \frac{|x_i|}{R_*} \approx 7 \times 10^{11} \sigma_j \left(\frac{|x_i|}{R_*/\gamma_j}\right) \left(\frac{R_*}{10R_{\odot}}\right) \text{ cm}
\]

Time-dependent (left) and steady-state (right) rarefaction (similar; \(ct \rightarrow z\))

(distance unit = \(R_*/\gamma_j \sim 10^{10}\) cm)
Axisymmetric model
Solve steady-state axisymmetric MHD eqs using the method of characteristics (Sapountzis & Vlahakis in preparation)

(not in scale!)

typical value of $R_* = 10^{12}$ cm
Black Holes at all scales

100 200 300 400 500 600

1 1.2 1.4 1.6 1.8 2 2.2 2.4 2.6 2.8 3

$\gamma$

$z/R_*$
Jet kinematics

• due to precession? (e.g., Caproni & Abraham)

• instabilities? (e.g., Hardee, Meier)

bulk jet flow may play at least a partial role

to explore this possibility, we used the relativistic self-similar model (Vlahakis & Königl 2004)

since the model gives the velocity (3D) field, we can follow the motion of a part of the flow
For given $\theta_{\text{obs}}$ (angle between jet axis and line of sight) and ejection area on the disk ($r_o, \phi_o$), we project the trajectory on the plane of sky and compare with observations. Find the best-fit parameters $r_o, \theta_{\text{obs}}, \phi_o$. 
For $\theta_{\text{obs}} = 1^\circ$ and $\phi_o = 0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ$ (from top to bottom):
best-fit to Unwin+ results for C7 component in 3C 345:

\[ r_o \approx 2 \times 10^{16} \text{cm}, \ \phi_o=180^\circ \ \text{and} \ \theta_{\text{obs}}=9^\circ \]
apparent trajectory

Trajectory of C7

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Black Holes at all scales

viewing angle (degr)
5 x (Doppler factor)
10
20
30
40
50
60

Lorentz factor
apparent velocity / c
2
4
6
8
10

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$\gamma = 10$, $\theta_{obs} = 1/2\gamma$, jet half-opening=1 degree, pitch angle at a reference distance = 0.1 degrees

electron’s energy spectrum $\propto \gamma_e^{-2.4}$
\[ \gamma = 10, \quad \theta_{\text{obs}} = 1/2\gamma, \] 
jet half-opening=1 degree, pitch angle at a reference diastance = 0.05 degrees
electron's energy spectrum \( \propto \gamma_e^{-2.4} \)
Summary

★ $B + \text{rotation} = \text{energy extraction}$

With $P_{ext} \rightarrow \text{jet}$

★ The \textit{collimation-acceleration paradigm} provides a viable explanation of the dynamics of relativistic jets

★ bulk acceleration up to Lorentz factors $\gamma_\infty \gtrsim 0.5 \frac{E}{M c^2}$

caveat: in ultrarelativistic GRB jets $\vartheta \sim 1/\gamma$

★ \textit{Rarefaction acceleration}

• further increases $\gamma$
• makes GRB jets with $\gamma \vartheta \gg 1$
• interesting for AGN jets as well

the intrinsic rotation of jets could be related to the observed kinematics and to the rotation of EVPA (Marscher+2008)