# Orbital interaction between stars and SMBH surrounded by accretion disc

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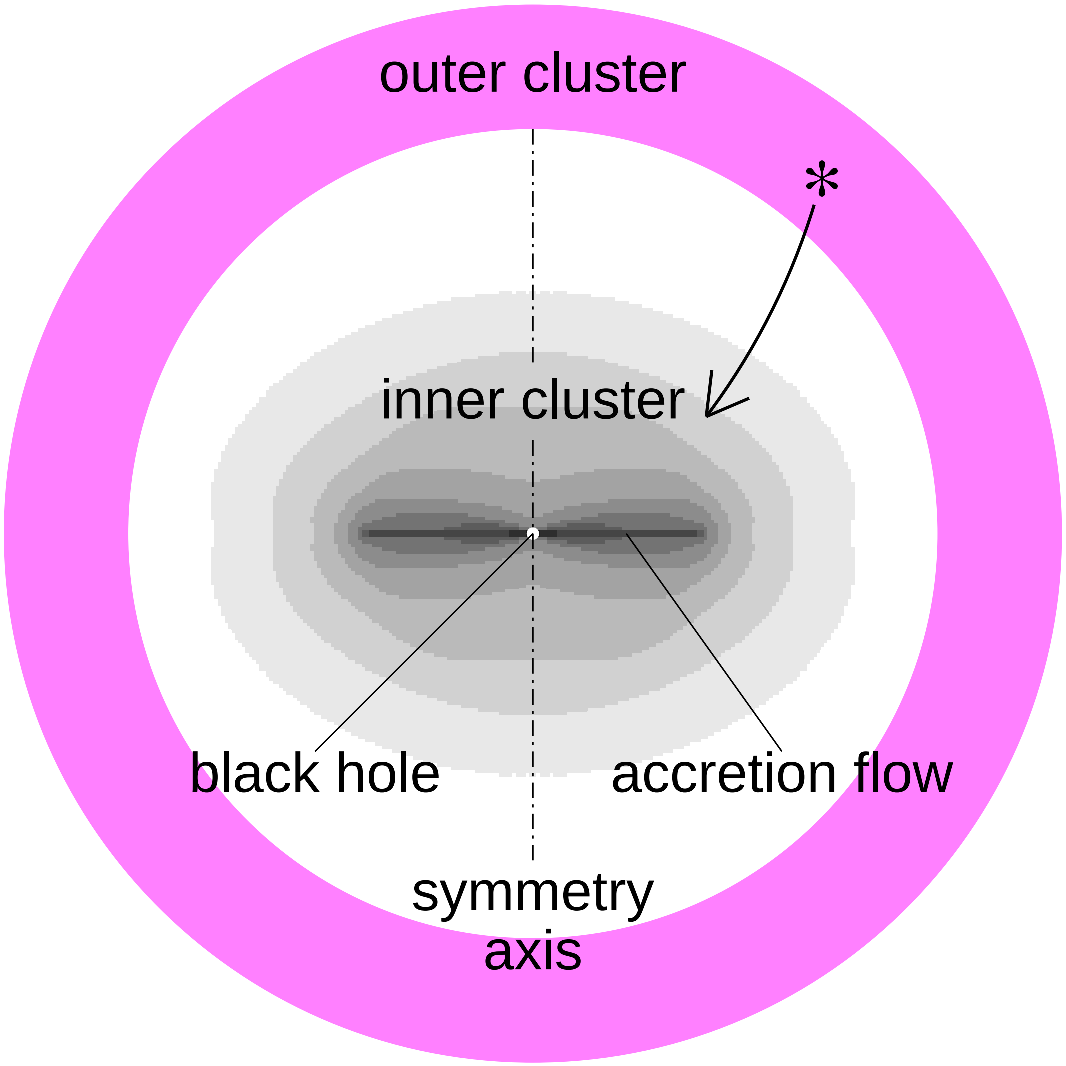
# *Model*




- Black hole
  $M_{\rm BH} \approx 10^3 - 10^8 M_\odot$
- Accretion flow
  $\Sigma_{\rm d} \propto r^s$
  $R_{\rm d} \approx 10^4 R_{\rm g} \approx 0.1\,{\rm pc}$
- ‘Outer’ cluster
  $n(r) = n_0 (r/r_{\rm h})^{-7/4}$
  $r_{\rm h} \approx 10\,{\rm pc}$
  $n_0 \approx 10^6 - 10^8 {\rm pc}^{-3}$
- ‘Inner’ cluster. . .

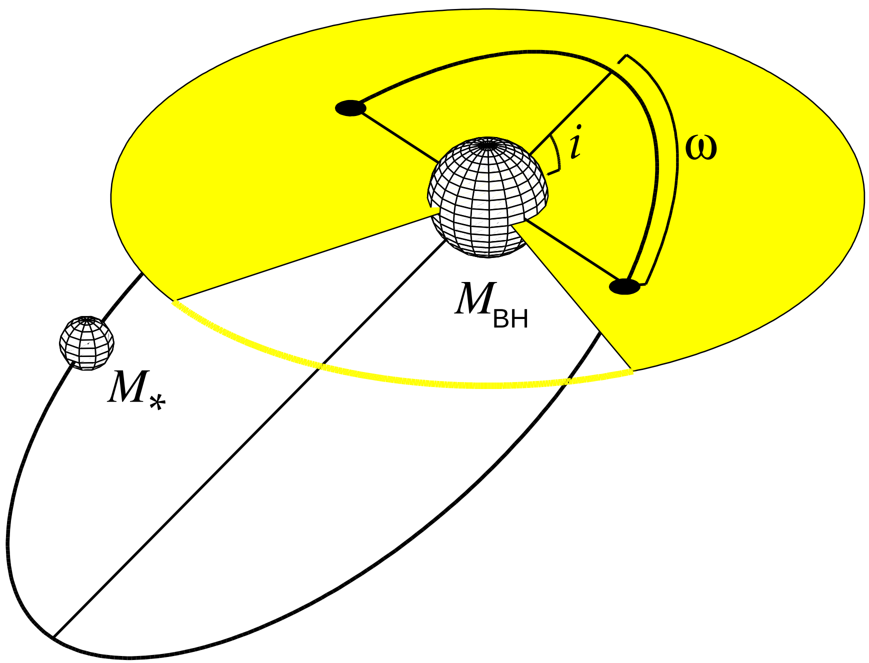
# Two-body Hamiltonian

Carthesian coordinates:

$$\mathcal{H} = \frac{1}{2}\left(v_1^2 + v_2^2 + v_3^2\right) - \frac{\mathcal{G}\,(m_0 + m_1)}{\sqrt{x_1^2 + x_2^2 + x_3^2}}$$

Delaunay variables:

$$\mathcal{H} = -\frac{\mathcal{G}^2(m_0 + m_1)^2}{2L^2}$$




$$L = \sqrt{\mathcal{G}(m_0 + m_1)a} \qquad l = M$$

$$G = L\sqrt{1 - e^2} \qquad g = \omega$$

$$H = G\cos i \qquad h = \Omega$$

# *Kozai–Lidov mechanism*

- evolution of a hiearchical triple system $M_1 > M_2 > M_3$

  Lidov 1961: Earth > Moon > satellite

  Kozai 1962: Sun > Jupiter > asteroid

- secular evolution of the orbital elements $e$, $i$ and $\omega$
- 'averaging' technique of the Hamiltonian perturbation theory allows to get rid of 'fast' variable (mean anomaly)
- integrals of motion: $a,\ C_1 \equiv \sqrt{1-e^2}\cos i$ and $\bar{V}_\mathrm{d}$
- motion of a star in the gravitational field of the central mass and an axisymmetric perturbation (ring, torus, disc. . . )

# *Kozai equations*

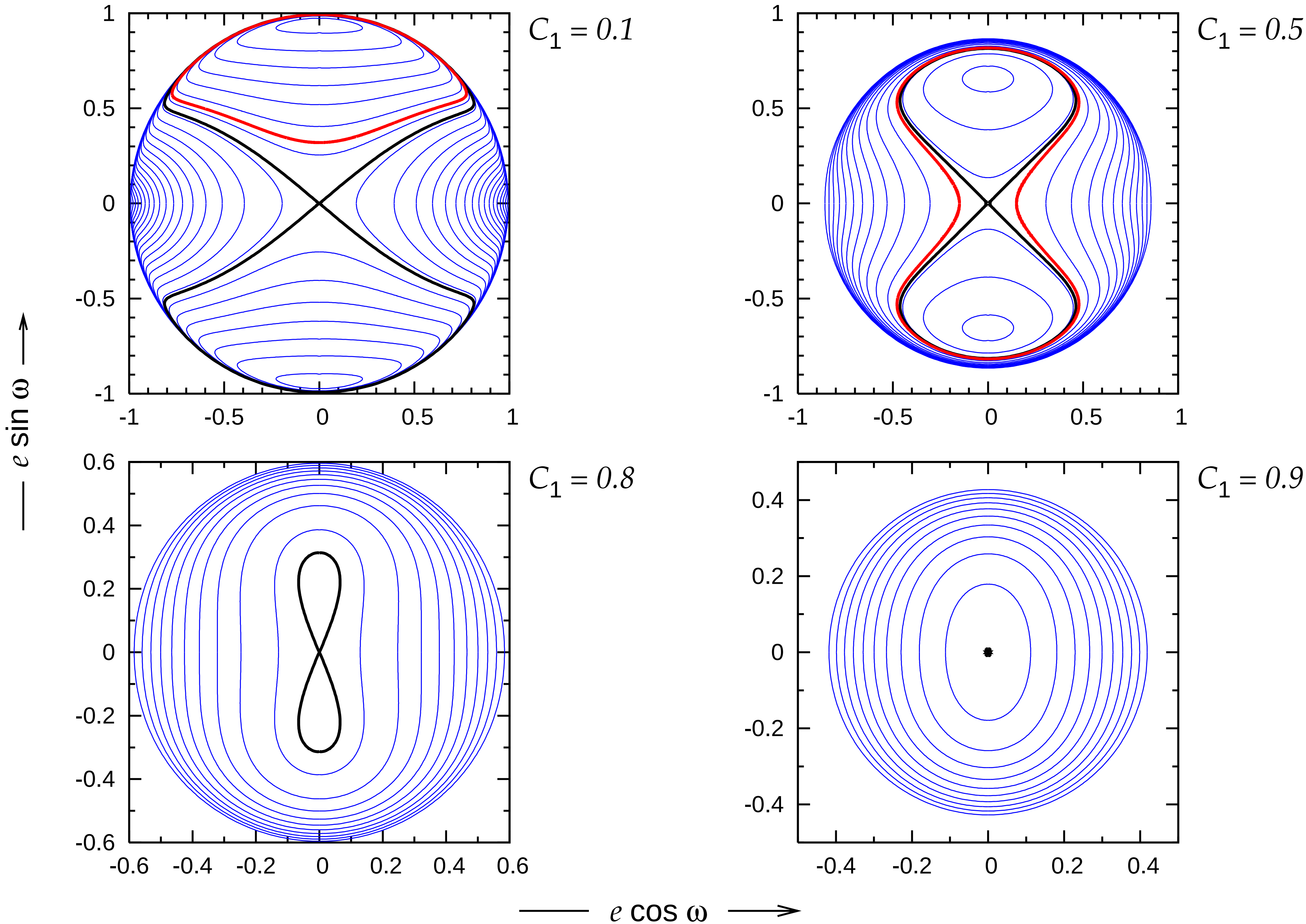
$$T_{\rm K}\sqrt{1-e^2}\,\frac{{\rm d}i}{{\rm d}t} = -5e^2\sin i\,\cos i\,\sin\omega\,\cos\omega$$

$$T_{\rm K}\sqrt{1-e^2}\,\frac{{\rm d}e}{{\rm d}t} = 5e(1-e^2)\sin^2 i\,\sin\omega\,\cos\omega$$

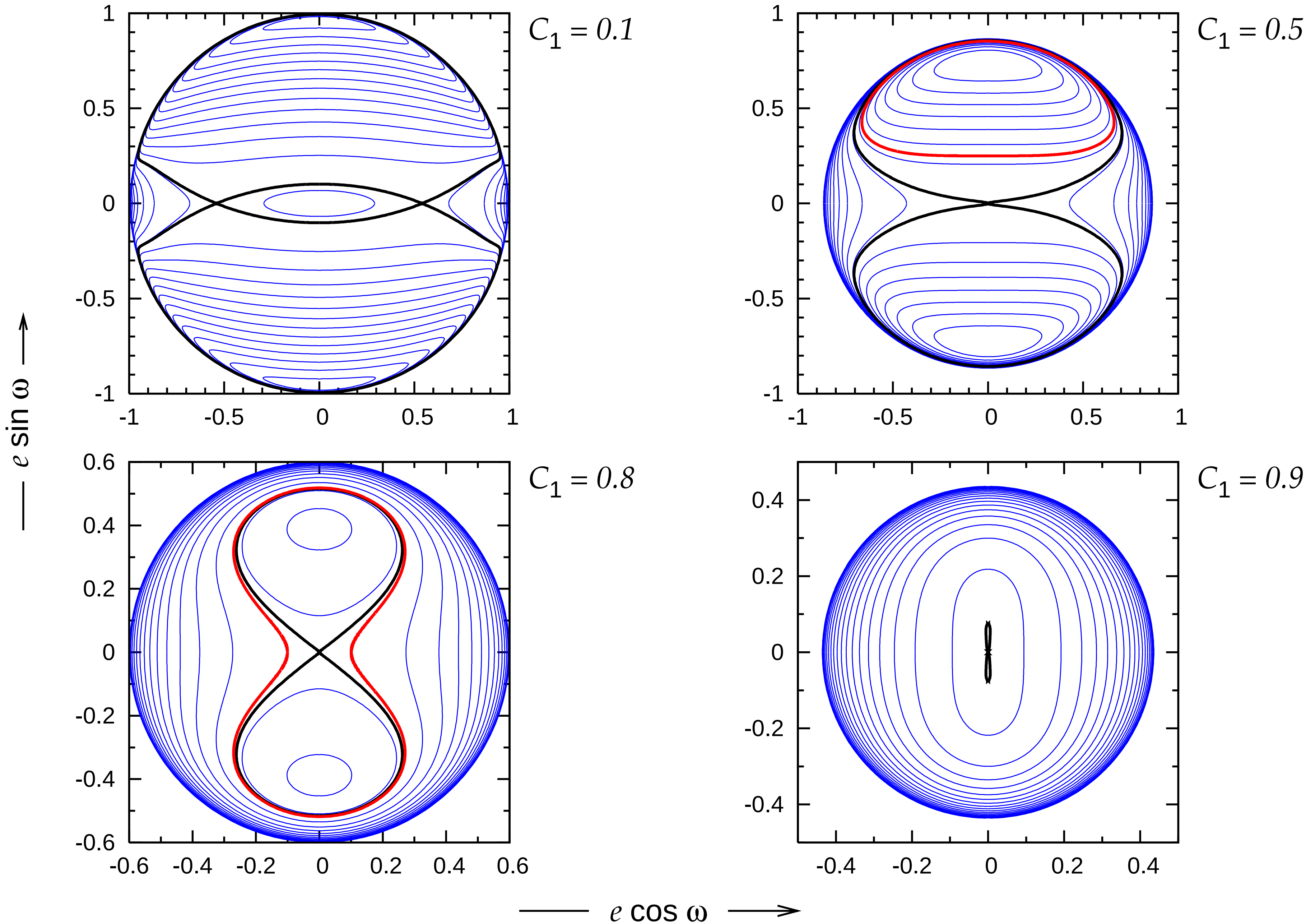
$$T_{\rm K}\sqrt{1-e^2}\,\frac{{\rm d}\omega}{{\rm d}t} = 2(1-e^2)+5(e^2-\sin^2 i)\sin^2\omega$$

$$T_{\rm K} \equiv \frac{4}{3}\frac{M_{\rm BH}}{M_{\rm d}}\left(\frac{R_{\rm d}}{a}\right)^3 P$$

# $\bar{V}_{\mathrm{d}}$ = const. *contours for a ring*



# $\overline{V}_{\mathrm{d}}$ = const. *contours for a disc*



# *Damping effect of the relativistic pericentre advance*

- characteristic timescales:

$$T_{\mathrm{K}} = \frac{4}{3}\frac{M_{\mathrm{BH}}}{M_{\mathrm{d}}}\left(\frac{R_{\mathrm{d}}}{a}\right)^3 P \qquad \text{vs.} \qquad T_{\mathrm{E}} = \frac{1}{3}\frac{a(1-e^2)}{R_{\mathrm{g}}}P$$

- Kozai oscillations are suppressed for

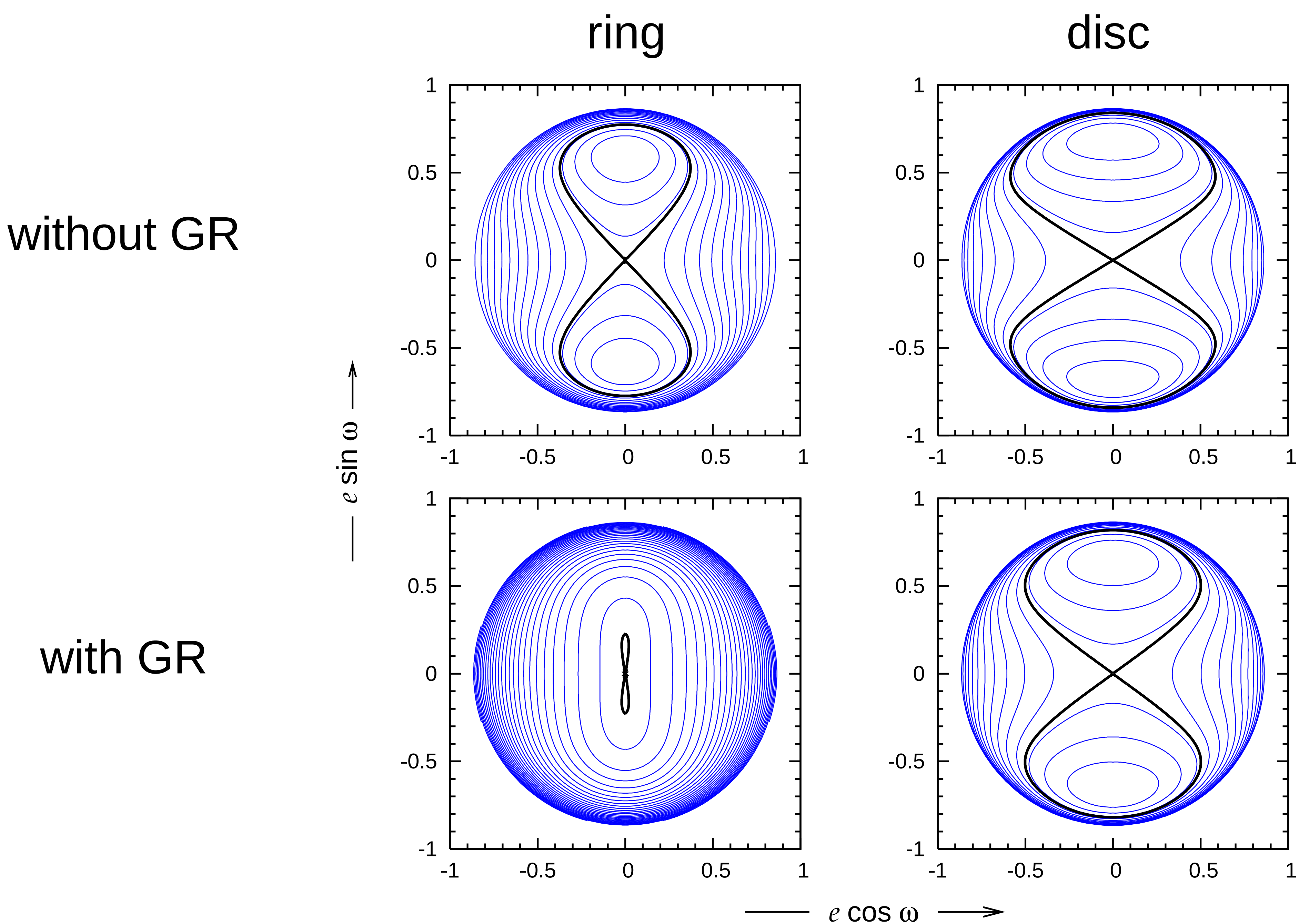
$$a < a_{\mathrm{min}} \approx \left(\frac{M_{\mathrm{BH}}}{M_{\mathrm{d}}}\right)^2 \left(\frac{R_{\mathrm{d}}}{R_{\mathrm{g}}}\right)^{6/7} \left(\frac{R_{\mathrm{min}}}{R_{\mathrm{g}}}\right)^{-1/7} R_{\mathrm{g}}$$

(Karas & Šubr, 2007, 2010)

- Paczyński-Wiita description of the central mass potential:

$$V_{\mathrm{PW}} = -\frac{GM_{\mathrm{BH}}}{r-2R_{\mathrm{g}}} = -\frac{GM_{\mathrm{BH}}}{r} - \frac{2GM_{\mathrm{BH}}R_{\mathrm{g}}}{r(r-2R_{\mathrm{g}})}$$

# *Damping effect of the relativistic pericentre advance*



# *Effect of the extended star cluster*

Stellar cusp in the sphere of influence of the central black hole

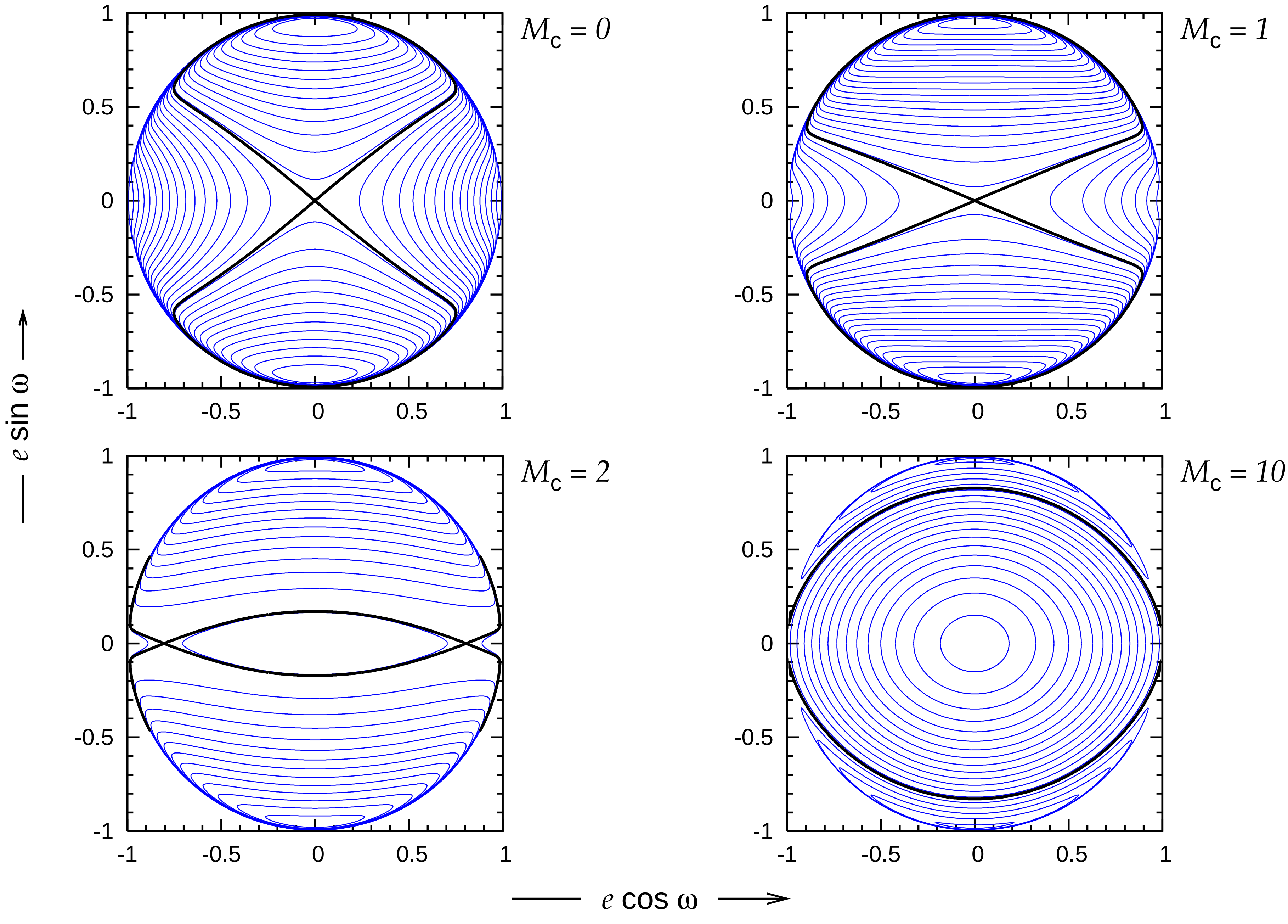
- Bahcall & Wolf (1976): $\rho(r) \propto r^{-7/4}$
- Galactic centre:

$$\rho(r) \approx 1.2 \times 10^6 \left(\frac{r}{0.4\mathrm{pc}}\right)^{-\alpha} M_\odot\,\mathrm{pc}^{-3}$$

$$\alpha = \begin{cases} 1.4 & r \lesssim 0.4\mathrm{pc} \\ 2.0 & r \gtrsim 0.4\mathrm{pc} \end{cases}$$

$$V_c(r) \propto \begin{cases} r^{2-\alpha} & \alpha < 2 \\ \ln(r) & \alpha = 2 \end{cases}$$

# Effect of the extended star cluster



# *Tidal disruptions*

- enhanced tidal disruptions due to the eccentricity oscillations
- supply of gas for accretion discs; food for black holes

$$R_{\rm t} = \left(\frac{M_{\rm BH}}{M_*}\right)^{1/3} R_* = 47\left(\frac{M_{\rm BH}}{10^6 M_\odot}\right)^{-2/3}\left(\frac{M_*}{M_\odot}\right)^{-1/3}\left(\frac{R_*}{R_\odot}\right) R_{\rm g}$$

- characteristic radius of the stellar cluster $\sim 10^6 R_{\rm g} \Longrightarrow$ extreme eccentricities needed
- $\mathcal{F}(R_{\rm min})$: fraction of stars from an enssemble with given (initial) distribution of orbital elements $D_{\rm f}(a, e, i, \omega)$ that pass the centre within $R_{\rm min}$ at some moment.

# Fractional probabilities — definitions

$$\mathcal{F}(R_{\rm min}) \equiv \int_{a_{\rm min}}^{a_{\rm max}} {\rm d}a \int_0^1 {\rm d}C_1 \int_0^{\sqrt{1-C_1^2}} {\rm d}e \int_0^{2\pi} {\rm d}\omega \; \Theta(e_{\rm max} - e_{\rm min}) D_{\rm f}(a, C_1, e, \omega)$$

$$\mathcal{F}_1(R_{\rm min}; a) \equiv \frac{1}{D_1} \int_0^1 {\rm d}C_1 \int_0^{\sqrt{1-C_1^2}} {\rm d}e \int_0^{2\pi} {\rm d}\omega \; \Theta(e_{\rm max} - e_{\rm min}) D_{\rm f}(a, C_1, e, \omega)$$

$$D_1(a) \equiv \int_0^1 {\rm d}C_1 \int_0^{\sqrt{1-C_1^2}} {\rm d}e \int_0^{2\pi} {\rm d}\omega \; D_{\rm f}(a, C_1, e, \omega)$$

$$e_{\rm min} \equiv 1 - R_{\rm min}/a$$

$$e_{\rm max} \equiv e_{\rm max}(a, C_1, e, \omega)$$

# *Analytical estimates*

$$D_{\rm f}(a,i,e,\omega) = D_0\, a^{1/4} e \cos i \quad \Longleftrightarrow \quad D_{\rm f}(a,C_1,e,\omega) = D_0'\, a^{1/4} \frac{e}{\sqrt{1-e^2}}$$

- central potential only:

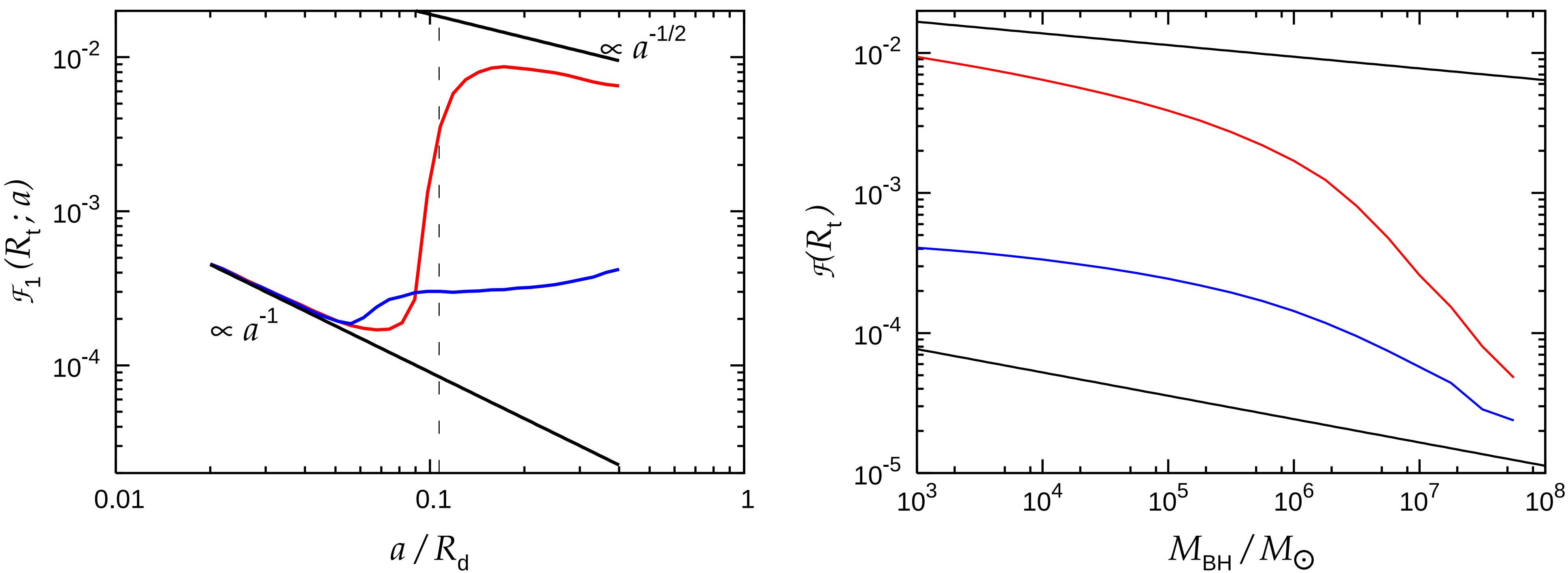
$$\mathcal{F}(R_{\rm min}) \approx 10 R_{\rm min}/a_{\rm max}$$
$$\mathcal{F}_1(R_{\rm min};a) = 2R_{\rm min}/a - R_{\rm min}^2/a^2$$

- Kozai without damping:

$$\mathcal{F}(R_{\rm min}) \approx \frac{10}{3}\sqrt{2R_{\rm min}/a_{\rm max}}$$
$$\mathcal{F}_1(R_{\rm min};a) \approx 2\sqrt{2R_{\rm min}/a}$$

# Single-parameter model — numerical results

- mass-sigma relation: $\sigma \approx 20\,(M_{\rm BH}/10^4 M_\odot)^{1/4}\,{\rm km\,s^{-1}}$
- characteristic radius of the star cluster: $R_{\rm h} = GM_{\rm BH}/\sigma^2$
- $R_{\rm d} = R_{\rm h}\,,\ 0.04R_{\rm h} \le a \le 0.4R_{\rm h}$
- $M_{\rm d} = 0.01M_{\rm BH}\,,\ M_{\rm c} = M_{\rm BH}\,,\ M_* = M_\odot\,,\ R_* = R_\odot$



# In the Galactic centre

molecular torus (CND; ring):

$$M_{\rm d} = 0.1 M_{\rm BH}$$
$$R_{\rm d} = 1.6\,{\rm pc}$$

star cluster:

$$\rho(r) \propto r^{-1.75}$$
$$M_{\rm c}(1.6{\rm pc}) = M_{\rm BH}$$

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$$\mathcal{F}(R_{\rm t}) \approx 3 \times 10^{-4}$$
$$N \approx 100$$

(clockwise) stellar disc:

$$M_{\rm d} = 0.01 M_{\rm BH}$$
$$R_{\rm in} = 0.03\,{\rm pc}$$
$$R_{\rm out} = 0.3\,{\rm pc}$$
$$\Sigma(r) \propto r^{-2}$$

star cluster:

$$\rho(r) \propto r^{-1.4}$$
$$M_{\rm c}(0.4{\rm pc}) = 0.2 M_{\rm BH}$$

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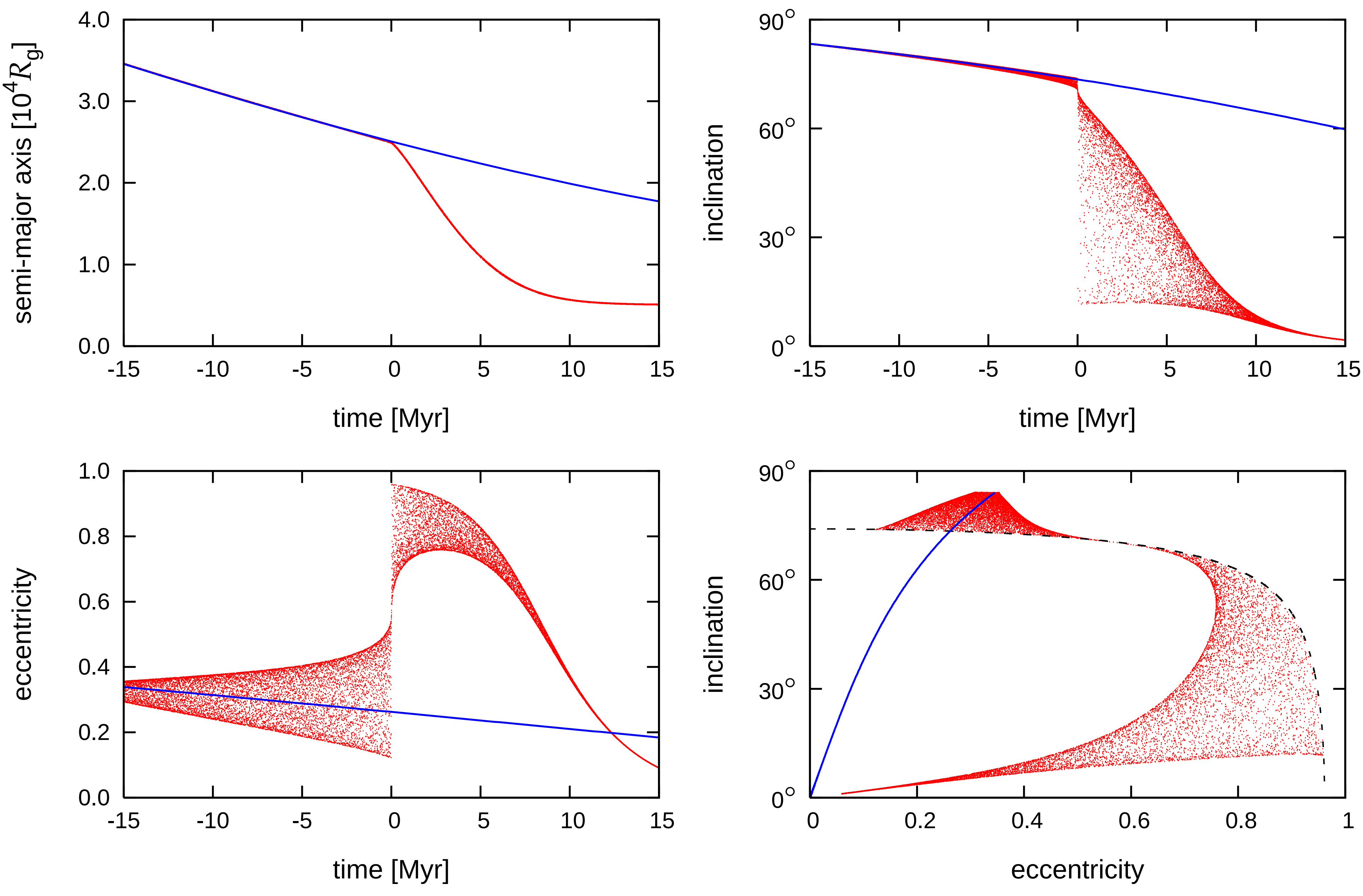
$$\mathcal{F}(R_{\rm t}) \approx 2 \times 10^{-3}$$
$$N \approx 100$$

# Example of temporal evolution for S2



# *References*


- Kozai oscillations acting together with dissipative drag of a gaseous disc — transporting S-stars toward the centre (Šubr & Karas, 2005, A&A 433, 405)
- Stellar tidal disruptions due to the eccentricity oscillations (Karas & Šubr, 2007, A&A 470, 11)
- Gravitating disks around black holes (Karas & Šubr, 2010, IAU Symp. 267)

# Three-body problem

$$\mathcal{H} = \mathcal{H}_{\mathrm{K}}(p^1, q^1) + \mathcal{H}_{\mathrm{K}}(p^2, q^2) + \frac{\mathcal{G} m_2 (m_0 + m_1)}{|r_2|} - \frac{\mathcal{G} m_0 m_2}{|r_2|} - \frac{\mathcal{G} m_1 m_2}{|r_{12}|}$$

$$\bar{\mathcal{H}} = -\frac{\mu a_1^2}{8 a_2^3 (1 - e_2^2)^{3/2}} \left( (2 + 3e_1^2)(3\cos^2 I - 1) + 15 e_1^2 \sin^2 I \cos 2\omega \right)$$