PERSPECTIVES IN TESTING POST-NEWTONIAN GRAVITY IN THE GRAVITATIONAL FIELD OF GC BLACK HOLE

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OUTLINE

1. ORBITAL MOTIONS AROUND SGR A*: THE SCENARIO
2. USING THE RADIAL VELOCITY
3. NON-KEPLERIAN SECULAR EFFECTS ON THE RADIAL VELOCITY
   - General relativistic effects on v_{ρ}
   - Classical non-Keplerian effects on v_{ρ}
4. NUMERICAL EVALUATIONS
5. CONCLUSIONS
6. ACKNOWLEDGEMENTS

THE GALACTIC BLACK HOLE AND ITS NEIGHBORING STARS

- The Galactic Center (GC) hosts a supermassive black hole (SBH) [Genzel et al. 1996, Schödel et al. 2002, Ghez et al. 2008] with a mass of the order of $M_\bullet = 4 \times 10^6 M_\odot$ [Ghez et al. 2008, Gillessen et al. 2009a, Gillessen et al. 2009b] and, thus, a Schwarzschild radius of $r_g = 0.084$ au
- In its immediate vicinity a number of rapidly orbiting main sequence stars have been detected and tracked in the *infrared* since 1992 [Eckart & Genzel, 1996, Ghez et al., 1998]
- The star S2 has eccentricity $e = 0.8831$, semi-major axis $a = 1031.69$ au and orbital period $P_b = 15.9$ yr [Gillessen et al. 2009a]
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IMPORTANCE OF GENERAL RELATIVISTIC EFFECTS IN DIFFERENT ASTRONOMICAL AND ASTROPHYSICAL SCENARIOS

If we take the *smallness* of the ratio $\mathfrak{r}$ of the test particle's average distance $\langle r \rangle = a(1 + e^2/2)$ from the central body to its Schwarzschild radius r_g as an index of the importance of the Einstein's General Theory of Relativity (GTR) in several astronomical and astrophysical systems, we have

- $\mathfrak{r}_{\rm LAGEOS} = 1.4 \times 10^9$
- $\mathfrak{r}_{\rm Mercury} = 2 \times 10^7$
- $\mathfrak{r}_{\rm WASP-19b} = 8.74 \times 10^5$
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THE AVAILABLE DATA

The *directly observable* quantities of the motions of the S stars around the SBH in the GC are

- astrometric measurements of their positions in the sky in terms of right ascension α and declination δ
- their radial velocities v_ρ. They cover a time span of $\Delta t = 7$ yr

From a practical point of view, the radial velocity data are *easier* to handle with respect to the astrometric observations. Indeed, the inclusion of new data into pre-existent records needs *no* special care because the radial velocities refer to the Local Standard of Rest (LSR). Instead, for the astrometric data it turns out that only an *approximate* realization of a *common* relative reference frame is possible. It implies that the exact definition of the coordinates is a matter of each data analysis in such a way that simply merging two different sets of astrometric positions would yield incorrect results

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SECULAR DYNAMICAL EFFECTS ON OBSERVABLES

- Let us assume that we have an *explicit, analytical* expression for a given observable Y as a function of all or some Keplerian orbital elements, i.e. $Y = Y(f, \{\kappa\})$, where f is the true anomaly, and κ denotes the ensemble of the Keplerian orbital elements explicitly entering Y apart from the mean anomaly $\mathcal{M}$
- Then, we straightforwardly compute its *secular* variation as

$$\left\langle \frac{dY}{dt} \right\rangle = \left(\frac{1}{P_{\rm b}}\right) \int_0^{2\pi} \left[\frac{\partial Y}{\partial f}\frac{df}{d\mathcal{M}}\frac{d\mathcal{M}}{dt} + \sum_{\kappa}\frac{\partial Y}{\partial \kappa}\frac{d\kappa}{dt}\right]\left(\frac{dt}{df}\right) df. \quad (1)$$

  In it, $d\mathcal{M}/dt$ and $d\kappa/dt$ are the instantaneous variations of the Keplerian orbital elements computed with the Gauss variation equations and evaluated onto the unperturbed Keplerian ellipse, while $df/d\mathcal{M}$ and dt/df are the usual Keplerian expressions for such derivatives
- We apply eq. (1) to the radial velocity v_{ρ} [Iorio 2011]

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- We apply eq. (1) to the radial velocity v_{ρ} [Iorio 2011]

The analytical expression of the radial velocity v_ρ is

$$v_\rho = \frac{na \sin i}{\sqrt{1-e^2}} \left[e \cos \omega + \cos(\omega + f) \right], \qquad (2)$$

where

- $n \doteq \sqrt{\frac{GM}{a^3}} = \frac{2\pi}{P_{\rm b}}$ is the Keplerian mean motion
- a is the semi-major axis
- e is the eccentricity
- i is the inclination to the plane of the sky
- ω is the argument of *perinigricon*[1]

In order to obtain $\langle \dot{v}_\rho \rangle$ due to a given dynamical perturbing force $\boldsymbol{F}$, eq. (2) has to be inserted into eq. (1) along with the analytical expressions of the variations of the Keplerian orbital elements induced by $\boldsymbol{F}$

[1] From *niger* ('black'). ω may be dubbed also *peribothron*, from βόθρος ('pit'), referring to the BH's gravitational well. Also *peripekton* may be used, from πηκτός ('frozen'): BHs were, indeed, also known as *frozen stars*

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SCHWARZSCHILD-TYPE ‘GRAVITOELECTRIC’ EFFECTS

- The secular variation $\langle \dot{v}_\rho \rangle$ of the star’s radial velocity caused by the general relativistic Schwarzschild-type ‘gravitoelectric’ *static* component of the BH’s gravitational field is [Iorio 2011]

$$\left\langle \dot{v}_\rho^{(\mathrm{GE})} \right\rangle = \left(n^2 \mathcal{R}_g \right) \frac{15e(1+e^2)\sin i \sin\omega}{8\left(1-e^2\right)^{5/2}}, \tag{3}$$

with

$$\mathcal{R}_g \doteq \frac{GM}{c^2}$$

- Note that eq. (3), which is an *exact* result in e, vanishes for *circular* orbits, i.e. for $e = 0$

Schwarzschild-type 'gravitoelectric' effects

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KERR-TYPE 'GRAVITOMAGNETIC' EFFECTS (I)

- The secular variation $\langle \dot{v}_\rho \rangle$ of the star's radial velocity caused by the general relativistic Kerr-type 'gravitomagnetic' *stationary* component of the BH's gravitational field is [Iorio 2011]

$$\left\langle \dot{v}_\rho^{(\mathrm{GM})} \right\rangle = \left(\frac{nGL}{c^2 a^2} \right) \frac{e}{4(1-e^2)^2} \left[\mathcal{V}_c \cos\omega + \mathcal{V}_s \sin\omega \right], \tag{4}$$

- with

$$\begin{aligned} \mathcal{V}_c &\doteq 11 \cot i \sin I \sin\Psi \sin\Omega, \\ \mathcal{V}_s &\doteq \frac{\csc i}{4} \{ \cos\Omega \sin 2I (\sin\Psi - \sin 3\Psi) - \\ &- \sin^2\Psi \cos\Psi [\cos 2I (3 + \cos 2\Omega) + \\ &+ 2\sin^2\Omega \Big] + 104 \sin^2 i \cos\Psi \Big\}. \end{aligned} \tag{5}$$

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KERR-TYPE 'GRAVITOMAGNETIC' EFFECTS (II)

- The parameters entering eq. (4) are
- $L = \chi \frac{M^2 G}{c}$ is the magnitude of the BH's angular momentum expressed in terms of the maximum value admissible for it to have a horizon in the Kerr metric $L_{\rm max} = \frac{M^2 G}{c}$. In the case of Sgr A* it is [Genzel et al.2003, Kato et al. 2010]

$$\chi = 0.44 - 0.52, \ L \simeq 7 \times 10^{54} \ \mathrm{kg \ m^2 \ s^{-2}} \tag{6}$$

- I is the angle between $\boldsymbol{L}$ and the line-of-sight's unit vector $\hat{\rho}$
- Ψ is the angle between the star's orbital angular momentum ℓ and the BH's angular momentum $\boldsymbol{L}$
- Ω is the longitude of the ascending node defined from $\sin\Psi \sin I \cos\Omega = \left(\hat{\boldsymbol{L}} \times \hat{\ell}\right) \cdot \left(\hat{\boldsymbol{L}} \times \hat{\rho}\right)$
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- According to the 'no-hair' or uniqueness theorems of GTR [Chrusciel 1994, Heusler 1998], an electrically neutral BH is *completely* characterized by its mass M and angular momentum L only. As a consequence, *all* the multipole moments of its external spacetime are functions of M and L
- In particular, the quadrupole mass moment is

$$Q_2 = -\frac{L^2 G}{c^2 M} = -\chi^2 \frac{G^3 M^3}{c^4} \quad (7)$$

- Since in the case of Sgr A* $\chi = 0.44$, it is

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OUTLINE

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2. USING THE RADIAL VELOCITY
3. NON-KEPLERIAN SECULAR EFFECTS ON THE RADIAL VELOCITY
   - General relativistic effects on v_ρ
   - Classical non-Keplerian effects on v_ρ
4. NUMERICAL EVALUATIONS
5. CONCLUSIONS
6. ACKNOWLEDGEMENTS

THE DIFFUSE INNER DARK MATTER (I)

- In addition to the dynamical effects *directly* related to the SBH *itself*, also the impact of a diffuse cluster of non-luminous ordinary matter around the BH due to massive remnants of various kinds [Morris 1993] should be taken into account. We adopt a Plummer density profile for it [Mouawad et al. 2005, Gillessen et al. 2009b]

$$\varrho_{\rm dm}(r) = \frac{3\mu M}{4\pi d_{\rm c}^3}\left(1 + \frac{r^2}{d_{\rm c}^2}\right)^{-5/2}.$$

- In it, the core radius is $d_{\rm c} = 15\ {\rm mpc}$, [Gillessen et al. 2009b]
- The mass parameter μ is the ratio of the total extended mass $M_{\overline{r}}$ at a given distance $\overline{r}$ to the central point mass. Fits involving S2 able to probe the mass enclosed between its aponigricon and perinigricon yield $\mu \leq 0.04 - 0.05$. [Mouawad et al. 2005, Gillessen et al. 2009b]

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By using the known orbital parameters of S2 [Gillessen et al. 2009b], the known mass $M_\bullet$ of the SBH in the GC [Gillessen et al. 2009b], the values for its angular momentum and quadrupole mass moment of eq. (6) and eq. (8), and those for the diffuse dark matter halo, it turns out that the nominal amplitudes of the long-term variations of $\dot{v}_\rho$ are of the order of

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ACCURACY IN MEASURING THE RADIAL VELOCITY

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- The *total* accuracy reachable in $\langle \dot{v}_\rho \rangle$ is actually impacted by the uncertainty in LSR itself. Indeed, as explained by [Ghez et al. 2008], to obtain v_ρ with respect to the LSR, each observed radial velocity has to be corrected for the Earth's rotation, its motion around the Sun, and the Sun's peculiar motion with respect to the LSR. Also the rotation speed Θ_0 of LSR does matter; recent evaluations by [Reid et al. 2007] yield an uncertainty of the order of 16 km s^{-1} corresponding to a future uncertainty of 2×10^{-5} m s^{-2} over $\Delta t = 20$ yr.
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Concluding remarks

- The cumulative, long term time variations of the radial velocity of S2 orbiting the SBH in the GC caused by several Newtonian and Einsteinian dynamical effects are 8×10^{-5} m s^{-2} (Schwarzschild), 4×10^{-6} m s^{-2} (dark matter), 1×10^{-8} m s^{-2} (Kerr), 1×10^{-10} m s^{-2} (quadrupole), respectively.
- By assuming a present-day uncertainty of about 15 km s^{-1} in the radial velocity measurements, its time changes may be detected in the future at a $\approx 10^{-5}$ m s^{-2} level over an observational time span of 20 yr; *at present*, radial velocity data cover just 7 yr. Even if such evaluations will turn out to be not too optimistic, a detection of the Kerr and the quadrupole-induced cumulative changes of the radial velocity seems to be unfeasible.

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ACKNOWLEDGEMENTS

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