Cosmological production of noncommutative black holes

Piero Nicolini

Frankfurt Institute for Advances Studies (FIAS)
Institute for Theoretical Physics, Goethe University Frankfurt
Frankfurt am Main, Germany

R. Mann and P. N., arXiv:1102.5096 [gr-qc]
Bologna, April 2011

deSitter space instability?

State of the art

- dS space is stable with respect classical perturbation
- dS space is **quantum mechanically unstable**
- In Euclidean quantum gravity

$$\Psi = \int D\left[g_{ab}\right] \; e^{-I_E[g]}. \tag{1}$$

- dominant contributions $\rightarrow$ saddle points of I_E (**instantons**)
- Two instantons: **background** I_{bg}, **object** nucleated I_{obj}

$$\Gamma = |\Psi|^2 \sim \frac{\exp\left(-2 I_{\mathrm{obj}}\right)}{\exp\left(-2 I_{\mathrm{bg}}\right)} \propto e^{-1/\Lambda G}. \tag{2}$$

deSitter space instability?

New issue

- Quantum gravity corrected black hole spacetimes
  - Noncommutative geometry
  - Generalized Uncertainty Principle
  - Loop Quantum Gravity
  - Asymptotically Safe Gravity
- Do they affect the quantum (in)stability of the deSitter spacetime?
- Possible detection since they have longer lives due to an incomplete evaporation (remnant formation).
- We start from **noncommutative black holes** (NCBHs)
- NCBHs are reliable in the semi-classical regime in which the instanton formalism works.

The NC inspired Schwarzschild-deSitter spacetime

The energy-momentum tensor **delocalization**

$$T_0^0 = -\rho_\ell(r) = -\frac{M}{(4\pi\ell^2)^{3/2}} \exp\left(-\frac{r^2}{4\ell^2}\right). \qquad (3)$$

$$T^{\mu\nu}\,;\,\nu = 0, \quad g_{00} = -g_{rr}^{-1} \Rightarrow \qquad (4)$$
$$T^\mu{}_\nu = \mathrm{Diag}\left(-\rho_\ell\left(r\right),\ p_r\left(r\right),\ p_\perp\left(r\right),\ p_\perp\left(r\right)\right).$$

Einstein/fluid equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad (5)$$

$$ds^2 = -V(r)\,dt^2 + V(r)^{-1}\,dr^2 + r^2\,d\Omega^2. \qquad (6)$$

The NC inspired Schwarzschild-deSitter spacetime

The solution

$$V(r) = 1 - \frac{4MG\gamma(3/2; r^2/4\ell^2)}{r\sqrt{\pi}} - \frac{\Lambda r^2}{3} \qquad (7)$$

where $\gamma\left(3/2\,, r^2/4\ell^2\right) \equiv \int_0^{r^2/4\ell^2} dt\, t^{1/2} e^{-t}$.

$$r \gg \ell \Rightarrow V(r) \approx 1 - \frac{2MG}{r} - \frac{\Lambda r^2}{3}. \qquad (8)$$

$$r \ll \ell \Rightarrow V(r) \approx 1 - \frac{\Lambda_{\rm eff}}{3} r^2, \quad \Lambda_{\rm eff} = \Lambda + \frac{1}{\sqrt{\pi}} \frac{MG}{\ell^3}. \qquad (9)$$

The NC inspired Schwarzschild-deSitter spacetime

The horizon equation

$$V(r_H) = 0. \tag{10}$$

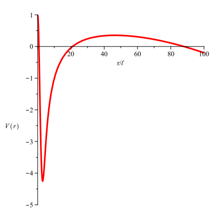
There is a mass $M_0 = M_0(\Lambda)$ such that

a) for $M > M_0$ there are **three horizons**, an inner and an outer black hole horizon r_+ and a cosmological horizon r_c.

b) for $M = M_N > M_0 \Rightarrow r_N \equiv r_+ = r_c$ (**Nariai-like solution**).

c) for $M = M_0 \Rightarrow r_0 \equiv r_+ = r_-$ (**single degenerate horizon**) and there is also a **cosmological horizon** r_c.

d) $M < M_0$ there is just **one (cosmological) horizon**, yielding a soliton.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r/ℓ
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−1
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−3
−4
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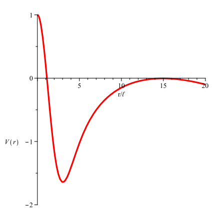
20
40
60
80
100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r/ℓ
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−1
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15
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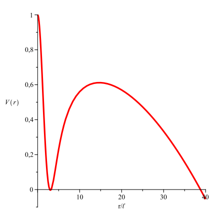
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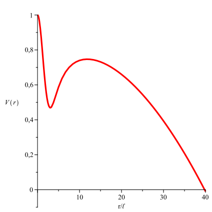
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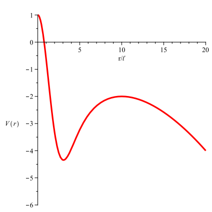
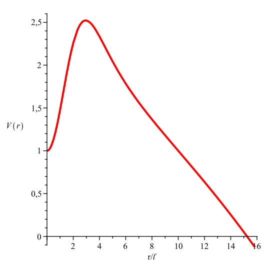

V(r)
r/ℓ
1
0
−1
−2
−3
−4
−5
−6
5
10
15
20

Negative mass solution

$$r \to -r \quad \text{or} \quad M \to -|M| \tag{11}$$

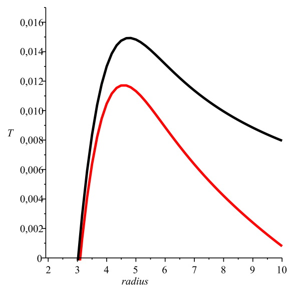
$$\Rightarrow V_{-}(r) = 1 + \frac{4|M|G\gamma(3/2; r^2/4\ell^2)}{r\sqrt{\pi}} - \frac{\Lambda r^2}{3}. \tag{12}$$



Thermodynamics

$$T(r_+) = \frac{1}{4\pi r_+}\left(1 - \frac{r_+^3}{4\ell^3}\frac{e^{-r_+^2/4\ell^2}}{\gamma(3/2;r_+^2/4\ell^2)}\right)\left(1 - \frac{\Lambda}{3}r_+^2\right) - \frac{\Lambda}{6\pi}\ r_+$$

$$T_c \approx \frac{3}{4\pi}\sqrt{\frac{\Lambda}{3}} > T_{dS}.$$



Gravitational Instantons

By analytically continuing $t \to i\tau$ one gets the Euclidean line element

$$ds_E^2 = V(r)\, d\tau^2 + V(r)^{-1}\, dr^2 + r^2\, d\Omega^2. \qquad (13)$$

$$\begin{aligned} I_{\mathrm{obj}} &= -\int_{\mathbb{M}_+} d^4x\sqrt{g}\left[\frac{\Lambda}{8\pi G} - \frac{\mathrm{T}}{2} + L_m\right] \\ &+ \ (\text{gravitational boundary terms}) \end{aligned} \qquad (14)$$

where $\mathrm{T} = T^{\mu}_{\mu}$ and $L_m = p_r + \frac{r^2}{4\ell^2}\frac{M}{\left(4\pi\ell^2\right)^{3/2}}\, e^{-\frac{r^2}{4\ell^2}}$.

$$I_{\mathrm{bg}} = -\int_{\mathbb{M}_+} d^4x\sqrt{g}\,\frac{\Lambda}{8\pi G} = -\frac{3}{2}\frac{\pi}{\Lambda G} \quad \Rightarrow \quad P_{\mathrm{bg}} = e^{\frac{3\pi}{\Lambda G}}. \qquad (15)$$

Cold Instanton

- $M > 0$, $V(r) \geq 0$, $r_+ = r_- = r_0 \Rightarrow r_0 \leq r \leq r_c$, $\tau = 2\pi/\kappa_c$
- $I_{\rm cold} \approx -\frac{\pi}{\Lambda G}\left(1 - 4M_0 G\sqrt{\frac{\Lambda}{3}}\left[\sqrt{\pi} - 0.67\right]\right)$
- $\Gamma_{\rm cold} \approx e^{-\frac{\pi}{\Lambda G}\left(1+8M_0 G\sqrt{\frac{\Lambda}{3}}\left[\sqrt{\pi}-0.67\right]\right)}$
- NB: for $\Lambda G \sim 1$ there is no $I_{\rm cold}$, but I_1.

Lukewarm Instanton

- $M > 0$, $V(r) \geq 0 \Rightarrow r_+ \leq r \leq r_c$, $\kappa_+ = \kappa_c$.
- $|I_{\rm cold}| \leq |I_{\rm lw}|$
- $\Gamma_{\rm lw} \approx e^{-\frac{\pi}{\Lambda G}}$.

Nariai Instanton

- $M > 0$, $V(r) \geq 0$, $r_+ = r_c \Rightarrow$

$$ds_E^2 = \frac{1}{A}\left(d\chi^2 + \sin^2\chi d\psi^2\right) + \frac{1}{B}\left(d\vartheta^2 + \sin^2\vartheta d\phi^2\right) \quad (16)$$

no pair production.

Single Horizon Instanton

- $0 \leq r \leq r_1$, $\tau = 0, \beta_1/2$ *with* $\beta_1 = 2\pi/\kappa_1$

$$I_1 = \frac{\beta_1 M}{\sqrt{\pi}}\left[2\gamma(3/2; r_1^2/4\ell^2) - \frac{1}{4}\frac{r_1^3}{\ell^3}e^{-r_1^2/4\ell^2}\right] - \frac{\beta_1 \Lambda}{12G}r_1^3.$$

NB: potential instability for negative mass solutions.

Summary

- Regular NC-Schwarzschild-deSitter solutions
- one, two, three or no horizon
- solutions for positive and negative mass parameter
- stable thermodynamics $T \to 0$, $T_c > T_{dS}$
- deSitter space is at the present time stable in agreement with experience.
- the production is relevant only for $\Lambda G \sim 1$
- Planck size black holes would not have been produced
- potential instability toward production of $M < 0$ solitons.

Work in progress: too much!