On the Production of Primordial Black Holes in Inflationary Cosmology

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Primordial black holes and inflation

Inflation: hypothetical phase in the early universe ($\sim 10^{-34}$ to 10^{-32} s) with $\ddot{a}(t) > 0$

- Primordial black holes (PBHs) produced *during* inflation will be diluted away
- Inflation produces a spectrum of fluctuations that can be relevant for PBH formation
- Of relevance is the size of the fluctuations when they re-enter the Hubble scale (when their wavelength becomes $< H^{-1} \equiv a/\dot{a}$)

B. Carr, A. Green, A. Liddle, J. Lidsey, ...

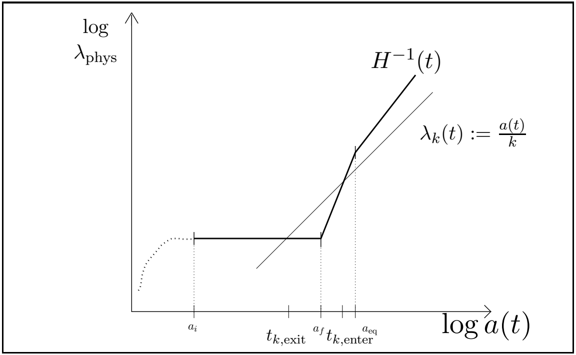

Figure: Time development of a physical scale $\lambda(t)$ and the Hubble scale $H^{-1}(t)$. During an inflationary phase, $H^{-1}(t)$ remains approximately constant. After the end of inflation, $H^{-1}(t)$ increases faster than any scale. Therefore λ_k enters the Hubble scale again at $t_{k,\text{enter}}$ in the radiation- (or matter-) dominated phase.

- Assumption of scale-free power spectrum for the primordial curvature perturbation $\mathcal{R}_k$:

$$\Delta^2_{\mathcal{R}}(k) = \frac{k^3\langle|\mathcal{R}_k|^2\rangle}{2\pi^2} \equiv \frac{k^3}{2\pi^2}P(k) \equiv \Delta^2_{\mathcal{R}}(k_0)\left(\frac{k}{k_0}\right)^{n_s-1},$$

$k_0 = 0.002$ Mpc^{-1}

- WMAP-7 data (2011):

$$n_s = 0.968 \pm 0.012 \quad (68\%\ \text{C.L.})$$

- A significant number of PBHs can only be produced for a ‘blue spectrum’ ($n > 1$), but this is observationally excluded
- Discuss in the following a model with *broken scale invariance* (BSI)[1]

[1] D. Blais, T. Bringmann, C. Kiefer, D. Polarski, PRD **67** (2003) 024024.

Assumption: primordial fluctuations obey Gaussian statistics

- Probability density

$$p(\delta) = \frac{1}{\sqrt{2\pi}\,\sigma(R)}\, e^{-\frac{\delta^2}{2\sigma^2(R)}},$$

- Dispersion (mass variance)

$$\sigma^2(R) = \frac{1}{2\pi^2}\int_0^\infty dk\; k^2\; W_{\rm TH}^2(kR)\; P(k),$$

$P(k)$: power spectrum; $\sigma^2(R) \equiv \langle(\delta M/M)_R^2\rangle$

- Probability that that a PBH is formed with mass $M_{\rm PBH} \geq M_H(t_k)$:

$$\beta(M_H) = \frac{1}{\sqrt{2\pi}\,\sigma_H(t_k)}\int_{\delta_{\rm min}}^{\delta_{\rm max}} e^{-\frac{\delta^2}{2\sigma_H^2(t_k)}}\,{\rm d}\delta \approx \frac{\sigma_H(t_k)}{\sqrt{2\pi}\,\delta_{\rm min}} e^{-\frac{\delta_{\rm min}^2}{2\sigma_H^2(t_k)}}$$

The BSI model

Inflationary model with a jump in the first derivative of the inflaton potential $V(\phi)$ at some scale k_s and corresponding value ϕ_s of the inflaton `(A. Starobinsky (1992))`. In the radiation-dominated phase one has

$$k^3\Phi^2(k,t_k) = \frac{2H_s^6}{A_-^2}\left[1 - 3(p-1)\frac{1}{y}\left(\left(1-\frac{1}{y^2}\right)\sin 2y + \frac{2}{y}\cos 2y\right)\right.$$
$$\left. + \frac{9}{2}(p-1)^2\frac{1}{y^2}\left(1+\frac{1}{y^2}\right)\left(1+\frac{1}{y^2}+\left(1-\frac{1}{y^2}\right)\cos 2y - \frac{2}{y}\sin 2y\right)\right]$$

with

$$y := \frac{k}{k_s}, \qquad p := \frac{A_-}{A_+}, \qquad H_s^2 = \frac{8\pi G V(\phi_s)}{3} \ ,$$

where Φ is the (peculiar) gravitational potential, and $A_-,\ A_+$ are the inflaton potential derivatives on both sides of the jump.

`cf.  Ivanov, Naselsky, Novikov 1994`

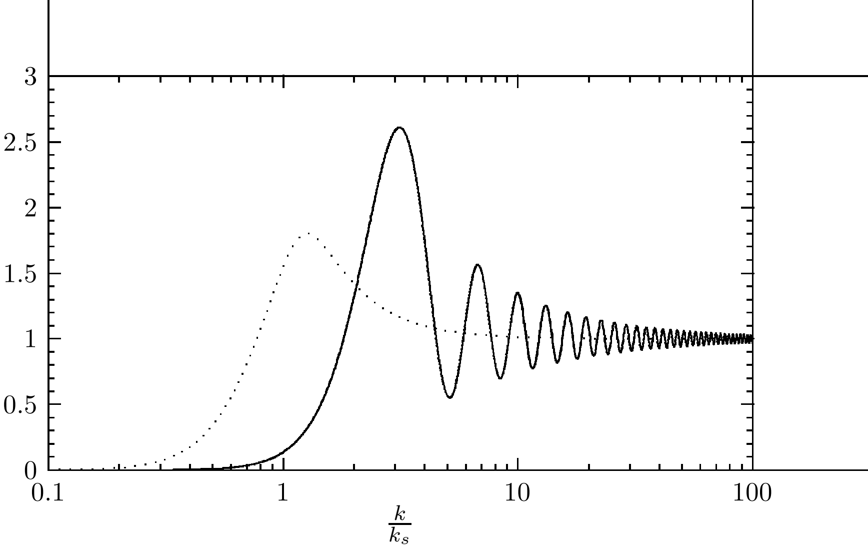

Figure: Mass variance $\sigma_H^2(t_k)$ (dotted line) and primordial power spectrum $k^3\Phi^2(k,t_k) \propto \delta_H^2(t_k)$ (solid line) for the Starobinsky-type BSI spectrum are displayed for $p = 7.58 \times 10^{-4}$. This particular value of the parameter p corresponds to p_γ, the absolute minimal value allowed by the constraint set by the contribution of evaporated PBHs to the diffuse γ-ray background.

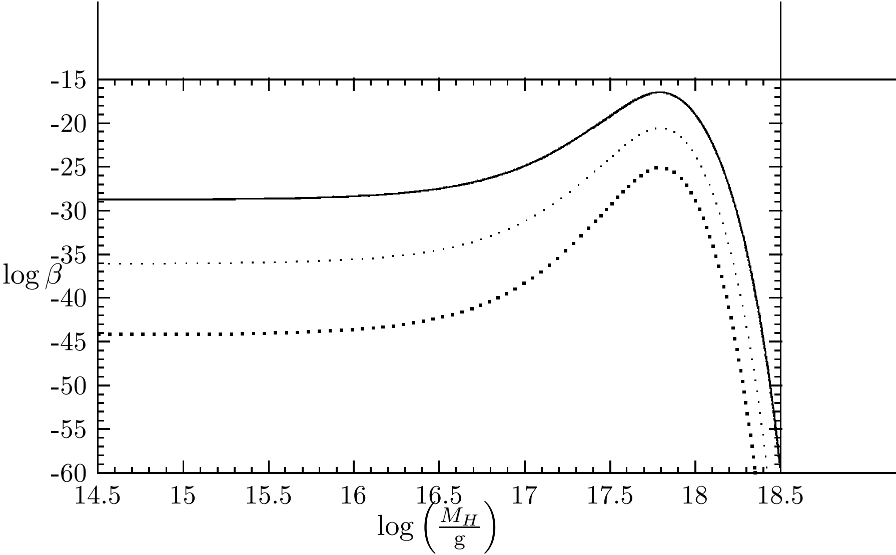

Figure: $\beta(M_H)$ is shown for the BSI-spectrum containing a jump in the inflaton potential derivative; $M_H(t_{k_s}) = 10^{18}$g, and from bottom to top $p = 10^{-3},\ 9\times10^{-4},\ 8\times10^{-4}$. As can be seen, $\beta(M_H)$ acquires a well localized bump in the vicinity of $M_s \equiv M_H(t_{k_s})$.

Observational constraints

We demand

$$\Omega_{\mathrm{PBH},0} \approx \Omega_{\mathrm{PBH},0}(M_{\mathrm{peak}}) \leq 0.3$$

This leads to

$$\beta(M_H) \leq 0.48 \times 10^{-17} \left(\frac{M_H}{10^{15}\mathrm{g}} \right)^{\frac{1}{2}} \; h^2$$

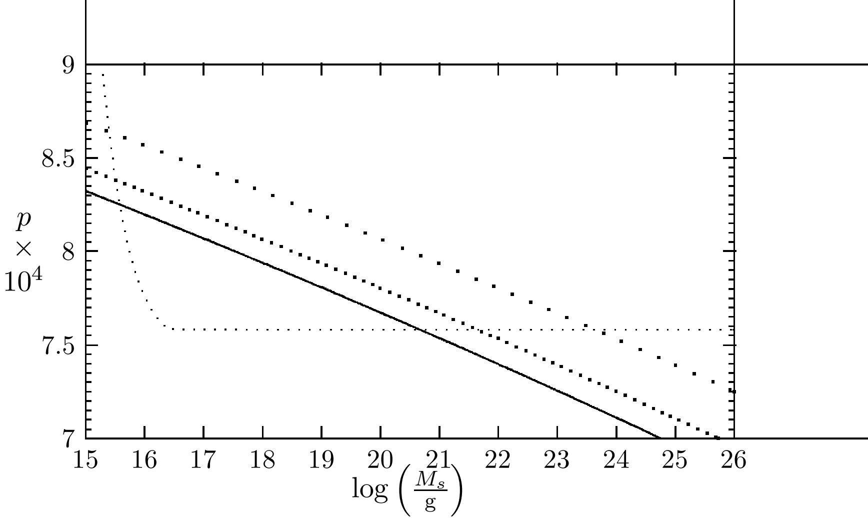

Figure: The allowed region in parameter space $(p, M_s \equiv M_H(t_{k_s}))$ is shown. The solid line represents those points for which $\Omega_{\mathrm{PBH},0}(M_{\mathrm{peak}}) = 0.3$. Below the solid line we have $\Omega_{\mathrm{PBH},0}(M_{\mathrm{peak}}) > 0.3$. The two lines parallel to the solid line represent, from bottom to top, those points for which $\Omega_{\mathrm{PBH},0}(M_{\mathrm{peak}}) = 0.1$ and 0.01, respectively. Below the dotted line, the γ-ray background constraint is violated.

Contribution to Dark Matter?

In the context of this model, a significant part of the cold dark matter could exist in the form of primordial black holes with mass M in the range $5 \times 10^{15}\ \mathrm{g} \lesssim M \lesssim 10^{21}\ \mathrm{g}$.

Conclusion

- Significant production of PBHs possible in inflationary cosmology with broken scale invariance;
- this could yield a significant part of cold dark matter
- Influence of non-Gaussian fluctuations?
  (cf. Bullock and Primack 1997)
- How was inflation realized (if it was)?
- Formation of quantum PBHs?