Primordial black hole formation

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- (very...) Brief overview of cosmology
- Overview of formation mechanisms
- Collapse of large density perturbations
- Production of large density perturbations

(very...) Brief overview of cosmology

early Universe

today

log(time)

inflation?
(accelerated expansion)

more about this later

radiation domination

$$p = \frac{1}{3}\rho$$

$$\rho \propto a^{-3}$$

$$a \propto t^{1/2}$$

matter domination

$$p = 0$$

$$\rho \propto a^{-4}$$

$$a \propto t^{2/3}$$

late-time acceleration

not to scale!

(particle) horizon: maximum distance particles could have travelled in time t (size of the observable universe at that time).

closely related to Hubble radius:

$$R = cH^{-1}$$

horizon mass: mass within horizon

$$M_H \approx \frac{c^3 t}{G} \approx 10^{15}\,\mathrm{g}\left(\frac{t}{10^{-23}\,\mathrm{s}}\right)$$

at the Planck time: $t_{\rm Pl} \sim 10^{-43}\,\mathrm{s} \qquad M_{\rm H} \sim M_{\rm Pl} \sim 10^{-5}\,\mathrm{g}$

at nucleosynthesis: $t \sim 1\,\mathrm{s} \qquad M_{\rm H} \sim 10^5 M_\odot$

Overview of formation mechanisms

Need large density:

Schwarzschild radius:

$$R_{\rm s} = \frac{2GM}{c^2} = 2 \left(\frac{M}{M_\odot} \right) \, {\rm km}$$

corresponding density:

$$\rho_{\rm s} = 10^{18} \left(\frac{M}{M_\odot} \right)^{-2} \, {\rm g \, cm^{-3}}$$

Collapse of large density perturbations

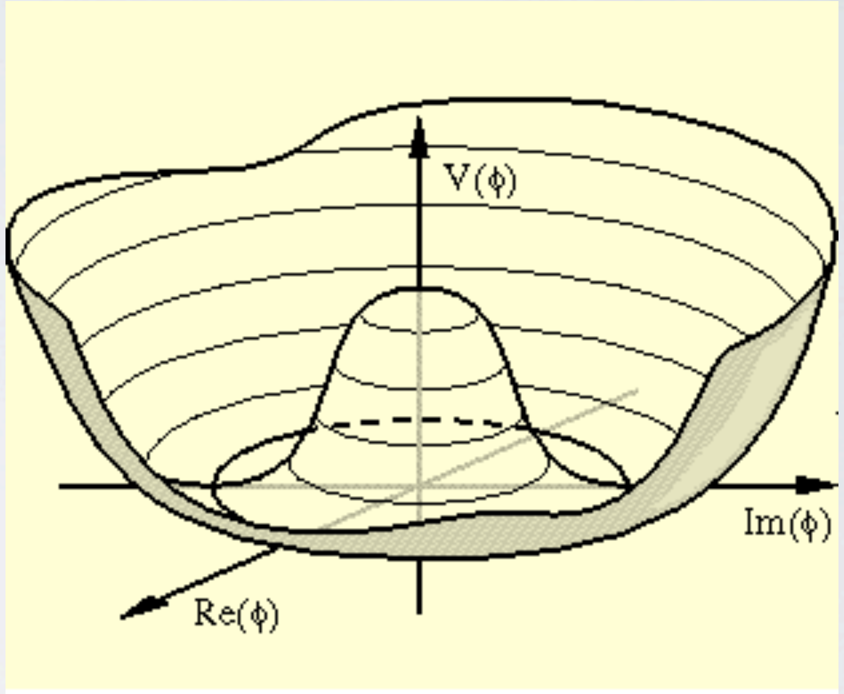
(lots) more about this later

Collapse of cosmic string loops

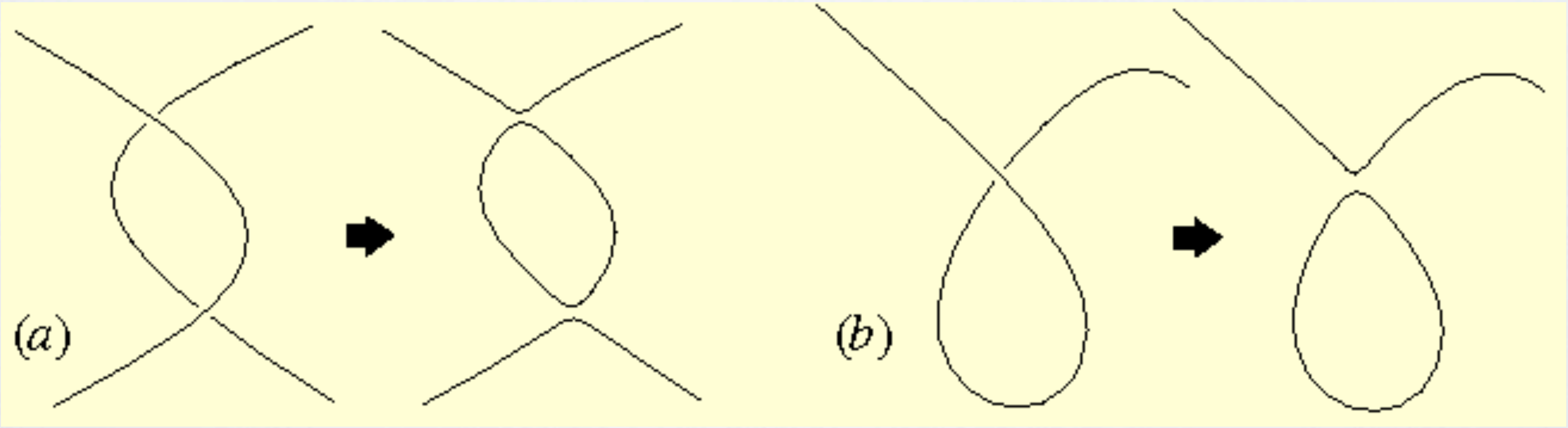
Hawking; Polnarev & Zemboricz;

1d topological defects formed during symmetry breaking phase transition

http://www.damtp.cam.ac.uk/research/gr/public/cs_top.html

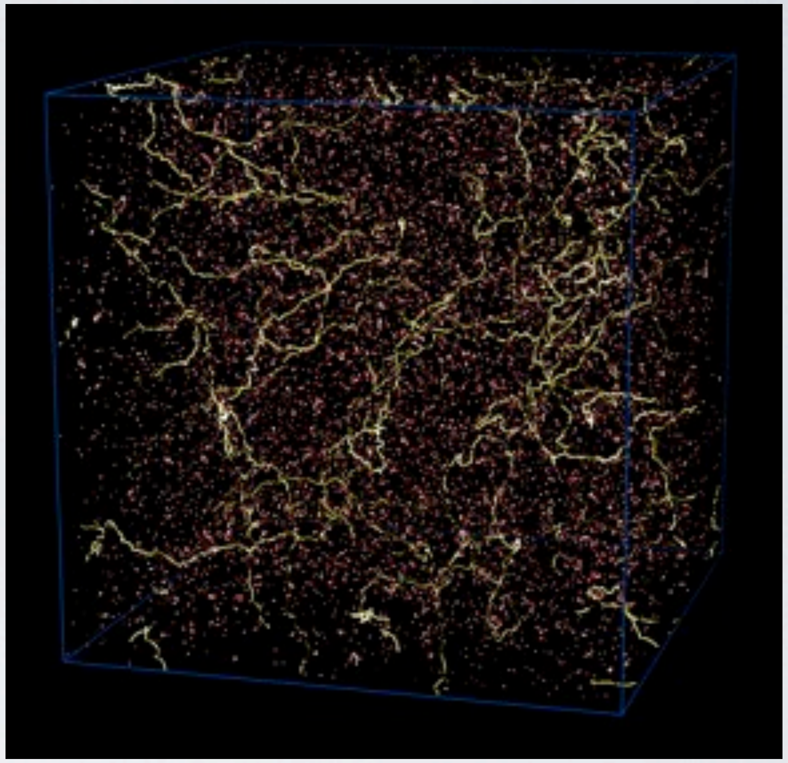


strings intercommute producing loops:



cosmic string network:

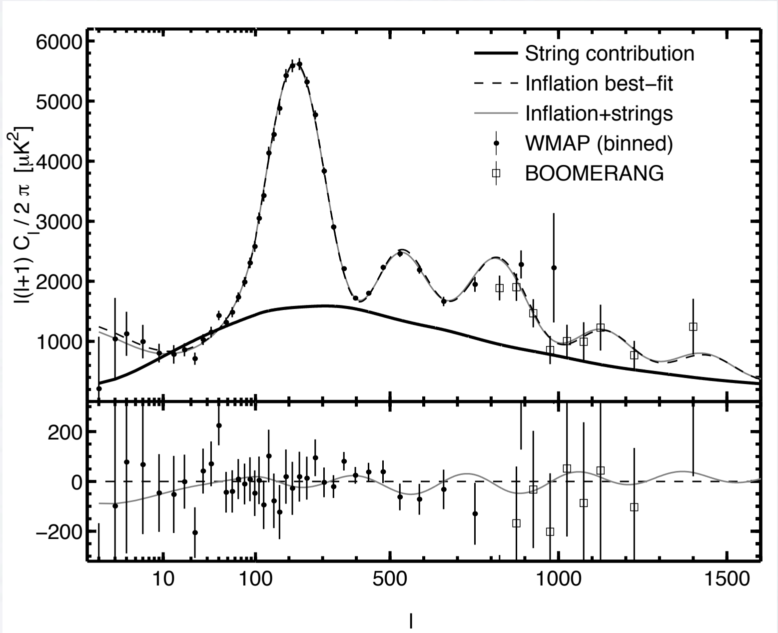
(some long strings and lots of loops)



Allen & Shellard

string tension, Gμ
constrained by CMB anisotropies:

$$G\mu \lesssim 10^{-6}$$



Bevis et al.

Small probability that loop will get into configuration where all dimensions lie within Schwarzschild radius (and hence collapse to from a PBH with mass of order the horizon mass at that time).

Probability depends on $G\mu$ and string correlation scale.

Probability is time independent, therefore PBHs have extended mass spectrum:

Carr

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto M^{-2.5}$$

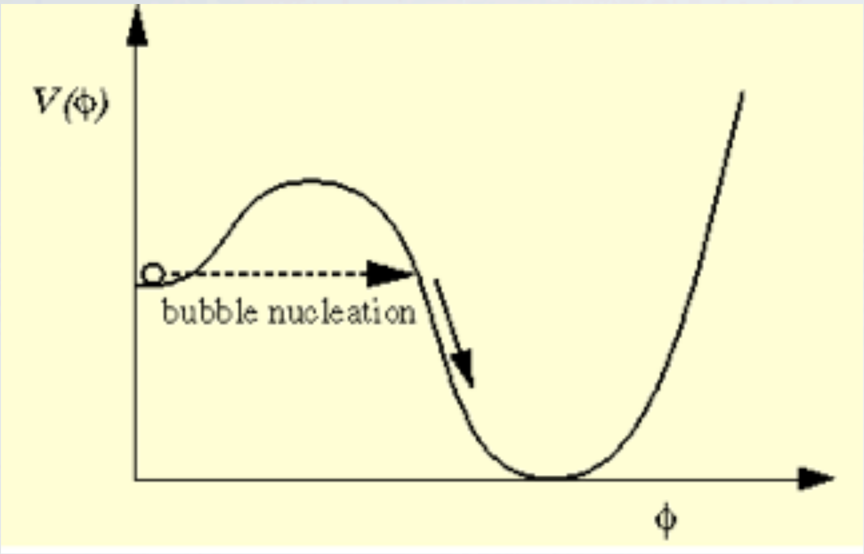
Constraints on PBH abundance (from gamma-rays & cosmic rays) place constraints on $G\mu$ which are similar to CMB constraints.

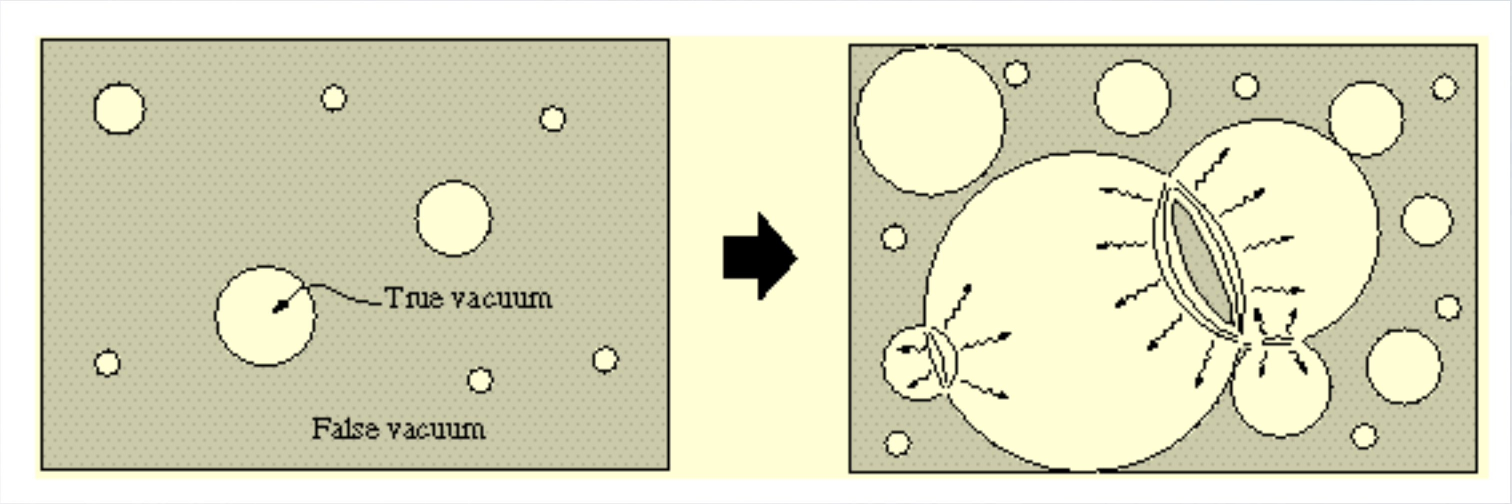
MacGibbon, Brandenberger & Wichoski

Bubble collisions

Hawking

1st order phase transitions occur via the nucleation of bubbles.





http://www.damtp.cam.ac.uk/research/gr/public/cs_phase.html

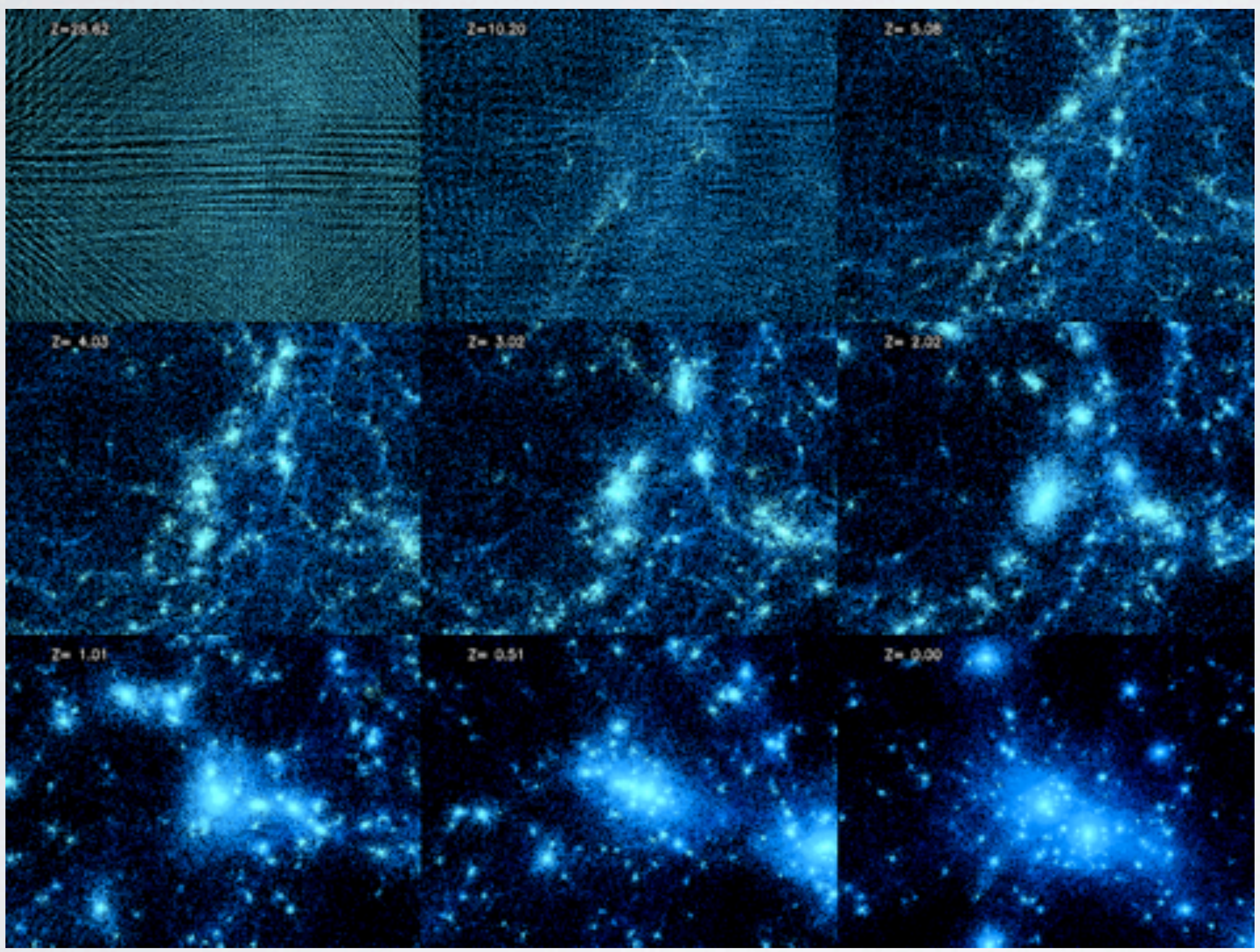
PBHs can form when bubbles collide (but bubble formation rate must be fine tuned).

PBH mass is of order horizon mass at phase transition:

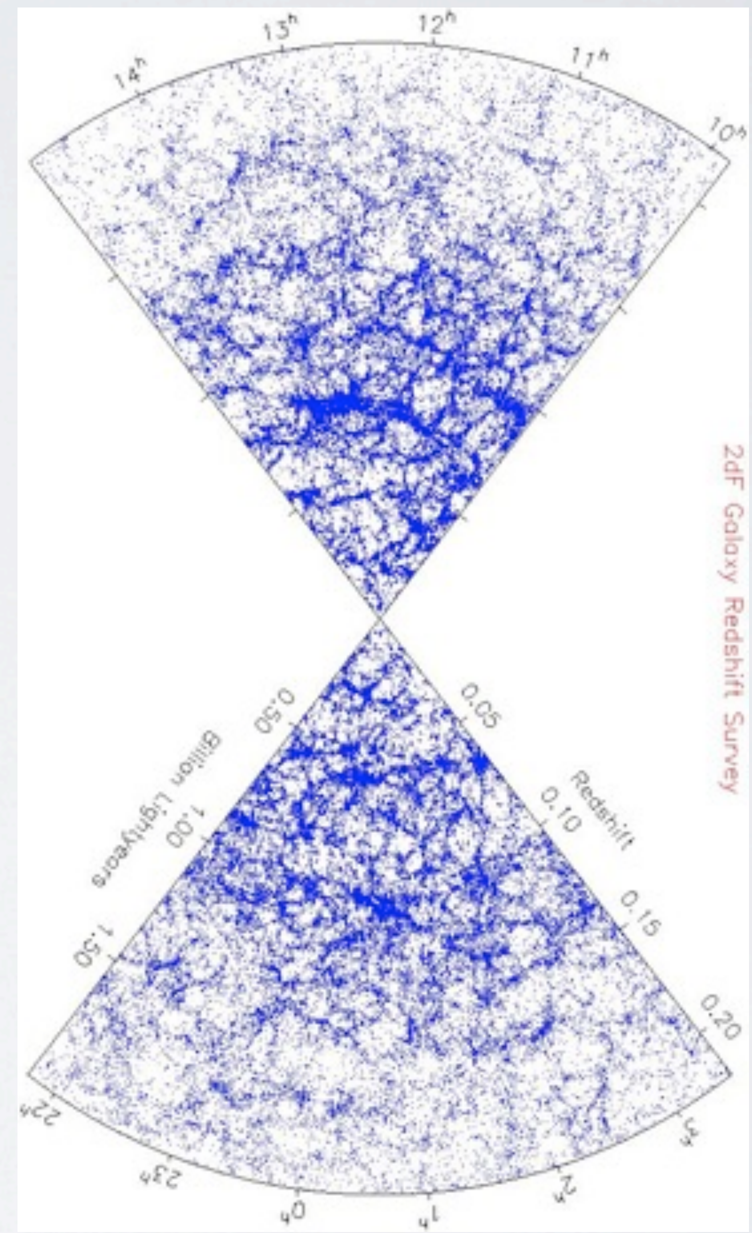
GUT scale:	$\sim 10^3$ g
electroweak scale:	$\sim 10^{28}$ g
QCD scale:	$\sim 10^{32}$ g

Collapse of large density perturbations

Observed structures in the Universe (galaxies, galaxy clusters) form via gravitational instability, from the growth and collapse of (initially small) density perturbations:



Kravtsov



2dF galaxy redshift survey

During radiation domination an initially large (at horizon entry) density perturbation can collapse to form a PBH with mass of order the horizon mass. Zeldovich & Novikov; Hawking; Carr & Hawking

For gravity to overcome pressure forces resisting collapse, size of region at maximum expansion must be larger than Jean's length.

Simple analysis:

density contrast: $$\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

equation of state: $$p = \gamma \rho$$

threshold for PBH formation: $$\delta \geq \delta_{\rm c} \approx \gamma = \frac{1}{3}$$

PBH mass: $$M \approx \gamma M_{\rm H}$$

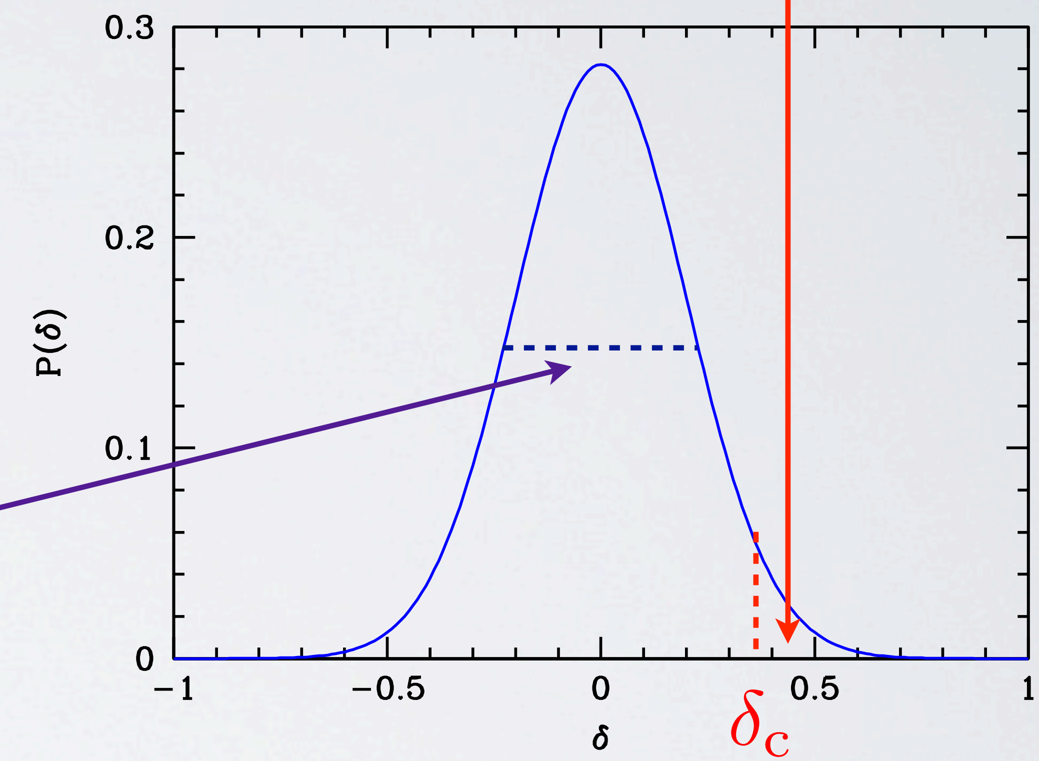
initial abundance of PBHs formed:

$$\beta(M) \sim \int_{\delta_c}^{\infty} P(\delta(M_{\rm H}))\,{\rm d}\delta(M_{\rm H})$$

PBH forming fluctuations

assuming a gaussian probability distribution:

$$\beta(M) \sim \sigma(M_{\rm H}) \exp\left[-\frac{1}{18\sigma^2(M_{\rm H})}\right]$$



σ(M) (mass variance)
typical size of fluctuations

Observational constraints on PBH abundance β (M) < O(10^{-20}) translate into σ (M_H) < O(0.01).

If σ (M_H) is independent of mass, PBHs have an extended mass function Carr.

Otherwise mass function is peaked where perturbations are largest.

Refinements:

i) accurate determination of the collapse threshold

Requires numerical simulations.

e.g. Nadezhin, Novikov & Polnarev; Bicknell & Henriksen; Shibata & Sasaki;

Musco et al. (with appropriate initial conditions) find $\delta_{\rm c} \approx 0.45$.

Hidalgo & Polnarev study dependence of threshold on shape of perturbation (but large perturbations are close to spherical Doroshkevich).

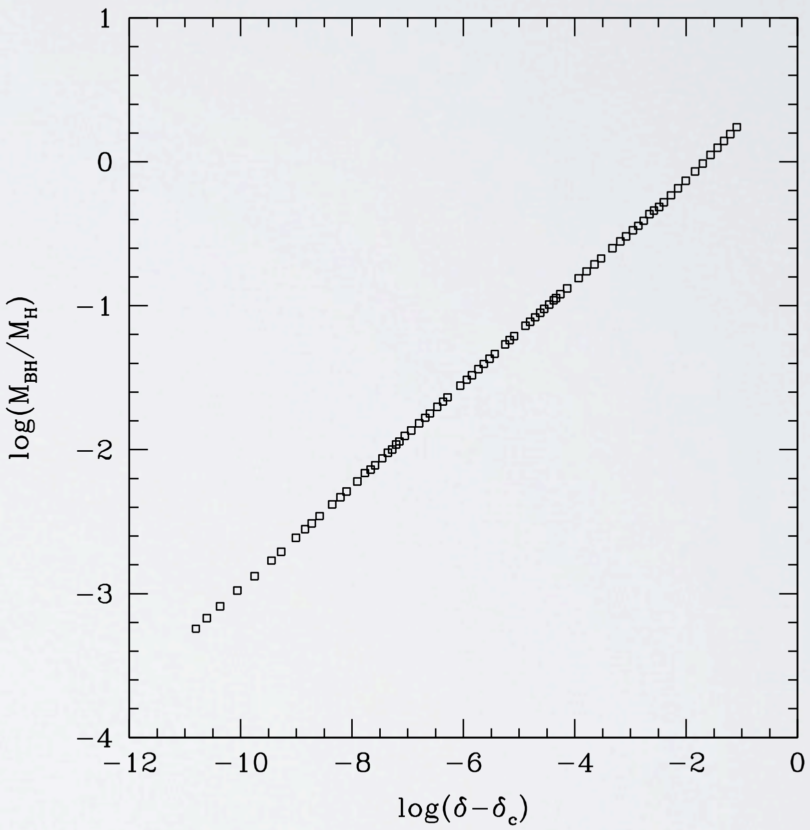
ii) critical phenomena

Choptuik; Evans & Coleman; Niemeyer & Jedamzik

BH mass depends on size of fluctuation it forms from:

$$M = kM_{\mathrm{H}}(\delta - \delta_{\mathrm{c}})^{\gamma}$$

Musco, Miller & Polnarev
using numerical simulations
(with appropriate initial conditions)
find k=4.02, γ =0.357



Mass function still peaked close to M_H.

Expect fluid description to break down so that $M \nrightarrow 0$ as $\delta \to \delta_{\mathrm{c}}$ Hawke & Stewart

iii) non-Gaussianity

Currently a hot-topic in cosmology.

How would non-gaussianity affect the abundance of PBHs?

Depends on exactly how density perturbations are generated (more later).

iv) upper limit on size of density perturbations forming PBHs

Originally thought that fluctuations with $\delta > 1$ would collapse to form separate closed Universe rather than a PBH. Carr & Hawking

Kopp, Hofmann & Weller, argue that an upper limit arises for other reasons, and argue curvature fluctuation should be used for PBH abundance calculations.

Tail of probability distribution is a rapidly falling function of δ so PBH abundance only weakly dependent on precise value (or existence) of upper limit.

Summary of collapse of large density perturbations:

In the radiation dominated early Universe, a PBHs can form from the collapse of a large ($\delta \geq \delta_c \approx 0.3$-$0.5$) density perturbations.

PBH mass is roughly equal to the horizon mass (but depends on ($\delta - \delta_c$)).

Initial PBH abundance is (roughly) given by

$$\beta(M) \sim \sigma(M_{\rm H}) \exp\left[- \frac{1}{18 \sigma^2(M_{\rm H})} \right]$$

An interesting abundance of PBHs (β (M) ~10^{-20}) requires σ (M_H) ~ 0.01.

Can often approximate PBH mass function with a delta-function (but for some applications, e.g. gamma-rays from evaporation, detailed mass function is important).

n.b. PBHs can also form from collapse of density perturbations during matter domination.

In this case regions must be sufficiently spherically symmetric Yu, Khlopov & Polnarev and $\beta(M) \approx 0.02 \sigma^{13/2}(M)$.

Production of large density perturbations

Inflation: A period of accelerated expansion ($\ddot{a} > 0$) in the early Universe.

Problems with the Big Bang:

Flatness: density evolves away from critical density (for which geometry is flat), to be so close to critical density today requires fine tuning of initial conditions.

Horizon: regions that have never been in causal contact have the same Cosmic Microwave Background temperature and anisotropy distribution.

Monopoles: formed when symmetry breaks, would dominate the density of the Universe today.

Inflation solves these problems by:

driving 'initial' density extremely close to critical density

allowing currently observable universe to originate from small region (originally in causal contact)

diluting monopoles

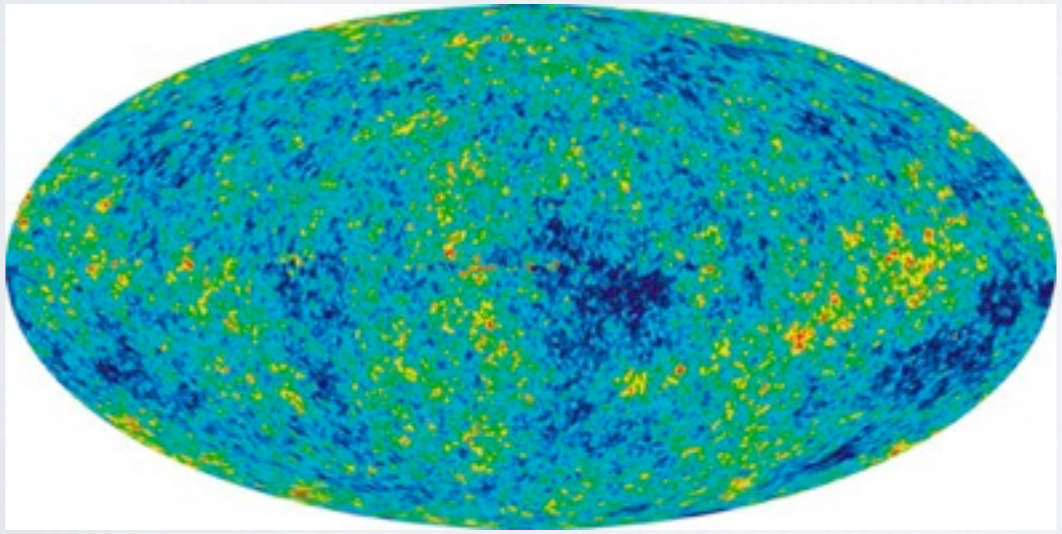
It can also generate density perturbations:

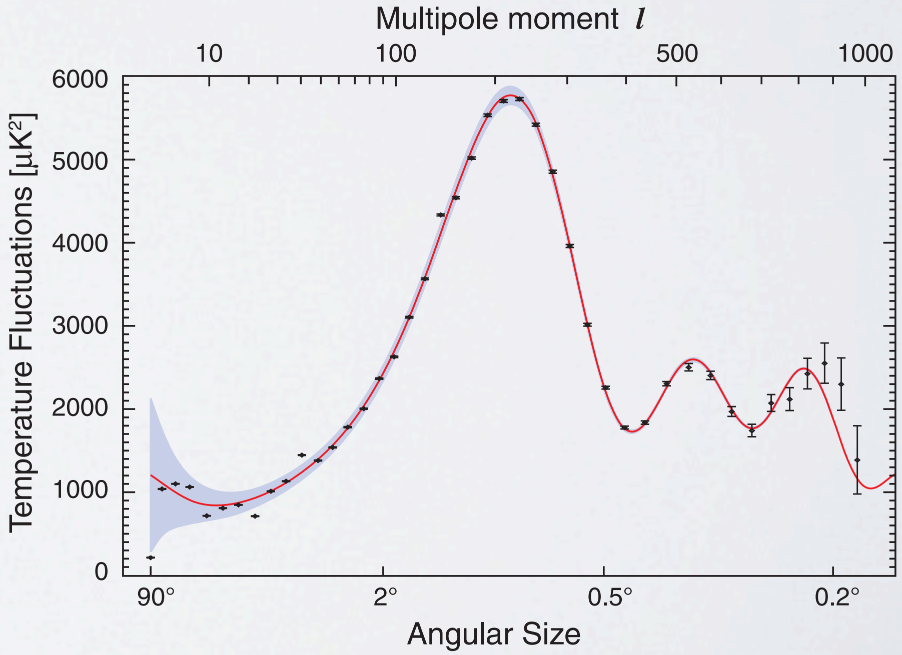
Quantum fluctuations

Curvature perturbations

Density perturbations

which are consistent with the temperature anisotropies in the cosmic microwave background radiation





WMAP [Komatsu et al.]

What drives inflation?

what do we need to get $\ddot{a} > 0$?

Fluid equation: $$\dot{\rho} + 3H(\rho + p) = 0$$

Combined with Friedmann equation: $$\frac{\ddot{a}}{a} = -4\pi G(\rho + 3p)$$

$$\ddot{a} > 0 \quad \Rightarrow \quad p < -\frac{1}{3}\rho$$

i.e. negative pressure!

Scalar field:

spin zero particle (unchanged under co-ordinate transformations)

required for spontaneous symmetry breaking

common in `beyond standard model' particle theories

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

if potential dominates: $$\rho \approx -p \approx V(\phi)$$

Scalar field dynamics-a quick overview

Friedman equation: $H^2 = \frac{8\pi G}{3}\left(V + \frac{1}{2}\dot{\phi}^2\right)$ Fluid equation: $\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi}$

[c.f. a ball rolling down a hill, with the expansion of the Universe acting as friction]

Slow roll approximation

Slow roll parameters:

$$\epsilon = \frac{m_{\rm Pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2$$

slope of potential

$$\eta = \frac{m_{\rm Pl}^2}{8\pi}\frac{V''}{V}$$

curvature of potential

If $\varepsilon, |\eta| \ll 1$ $H^2 \approx \frac{8\pi}{3m_{\rm Pl}^2}V$ $3H\dot{\phi} \approx -\frac{\mathrm{d}V}{\mathrm{d}\phi}$

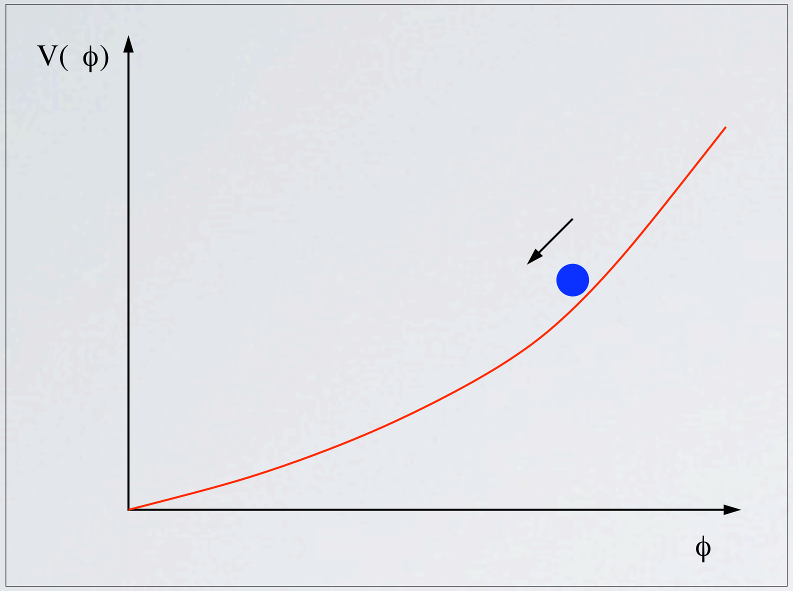
$\varepsilon < 1 \longrightarrow \ddot{a} > 0$

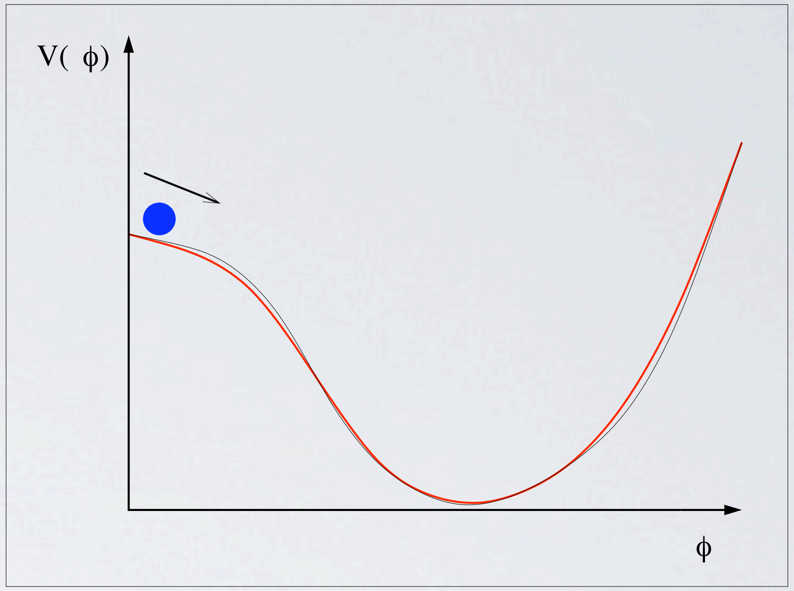
Inflation ends when potential becomes too steep: $\varepsilon \approx 1$

Field oscillates around minimum of potential.

Inflaton field decays creating radiation dominated Universe (reheating).

Classes of inflation models



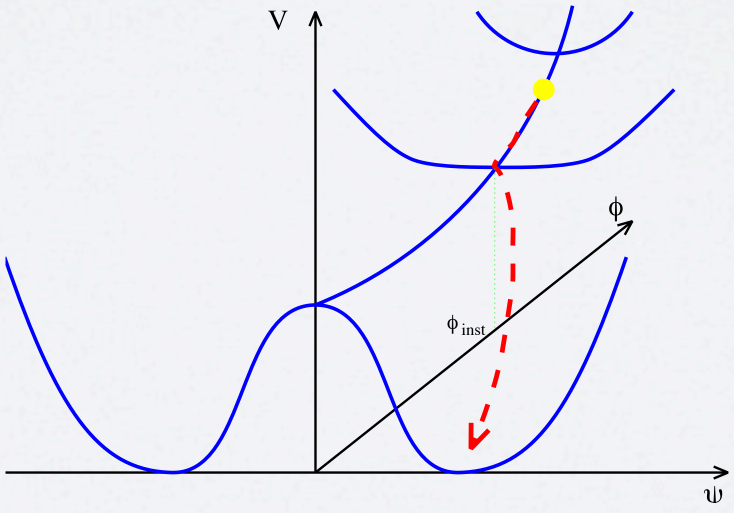


Large field

$\frac{1}{2}m^2\phi^2$ $\lambda\phi^4$

Small field

$\lambda(\phi^2 - M^2)$



Liddle

Hybrid

Waterfall transition of auxiliary field triggers end of inflation.

Density perturbations: theory v. observations

amplitude of fluctuations usually parameterised as: $\sigma^2(M_{\rm H}) \propto k^{n_s-1}$

k=comoving wavenumber $\quad$ n_s=spectral index

and $M_{\rm H} \propto k^{-2}$ during radiation domination

Theory (slow roll inflation):

$$\sigma^2(M_{\rm H}) \propto \frac{V^3}{(V')^2} \qquad\qquad n_{\rm s} = 1 - 6\epsilon + 2\eta + ...$$

Observations (CMB + large scale structure):

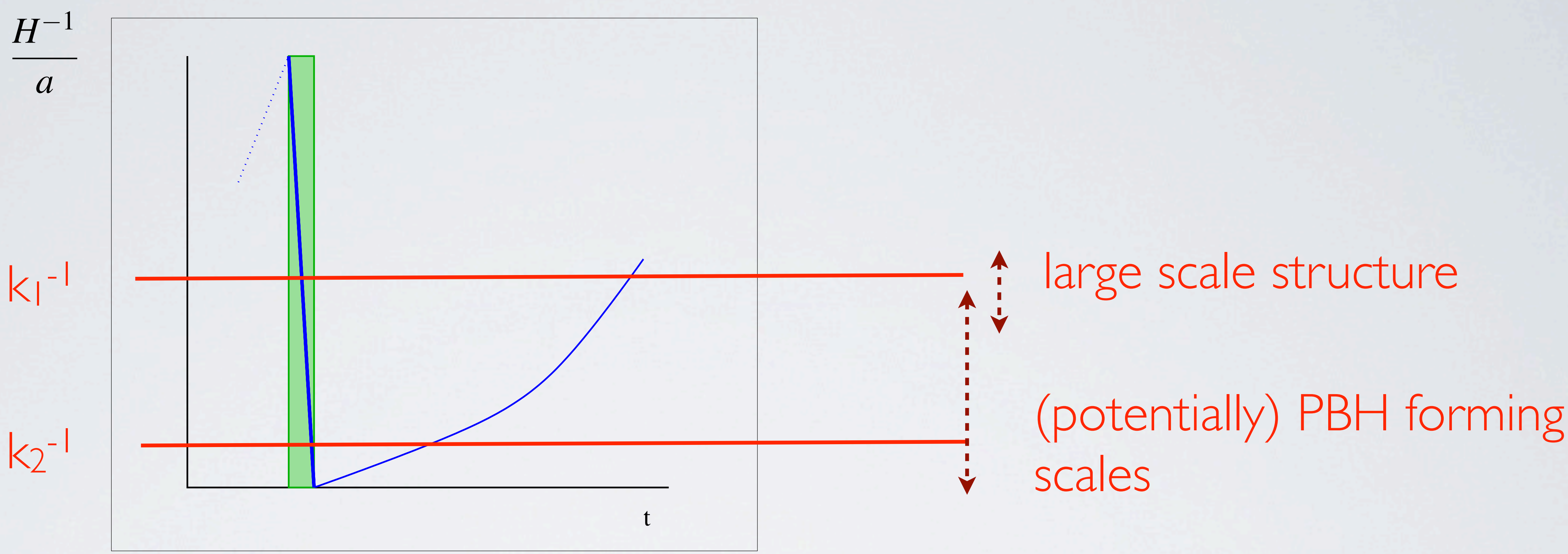
$$\sigma(M_{\rm H,CMB}) \sim 10^{-5} \qquad\qquad n_{\rm s} = 0.963^{+0.014}_{-0.015}$$

If we extrapolate the power law power spectrum down to small scales, the abundance of PBHs produced is negligible, however **this is not a valid thing to do.**

(power law form is essentially first term of Taylor expansion which is only valid over a, relatively, narrow range of scales).

$$\ddot{a} > 0 \quad \Rightarrow \quad \frac{\mathrm{d}(H^{-1}/a)}{\mathrm{d}t} < 0$$

i.e. comoving Hubble radius decreases during inflation



A scales exits the horizon during inflation when k = a H.

Re-enters when k = a H again (and perturbations collapse if sufficiently large)

CMB & LSS probe scales: $k \sim 1 - 10^{-3}\,\mathrm{Mpc}^{-1}$

PBHs can form on scales: $k \sim 10^{-2} - 10^{23}\,\mathrm{Mpc}^{-1}$

Lower limit on mass of PBHs which can form set by reheat temperature at the end of inflation: $M \sim M_{\rm H} = 10^{18}\,\mathrm{g}\left(\frac{10^7\,\mathrm{GeV}}{T}\right)^2$

How to generate large density perturbations on small scales?

see Kiefer talk tomorrow

i) stochastic generation of inflation models

Peiris & Easther stochastically generate inflation models using slow roll 'flow equations'.

In models where inflation can continue indefinitely (and is ended via an auxiliary mechanism) fluctuations on small scales can be large enough to form PBHs.

see also Josan & Green

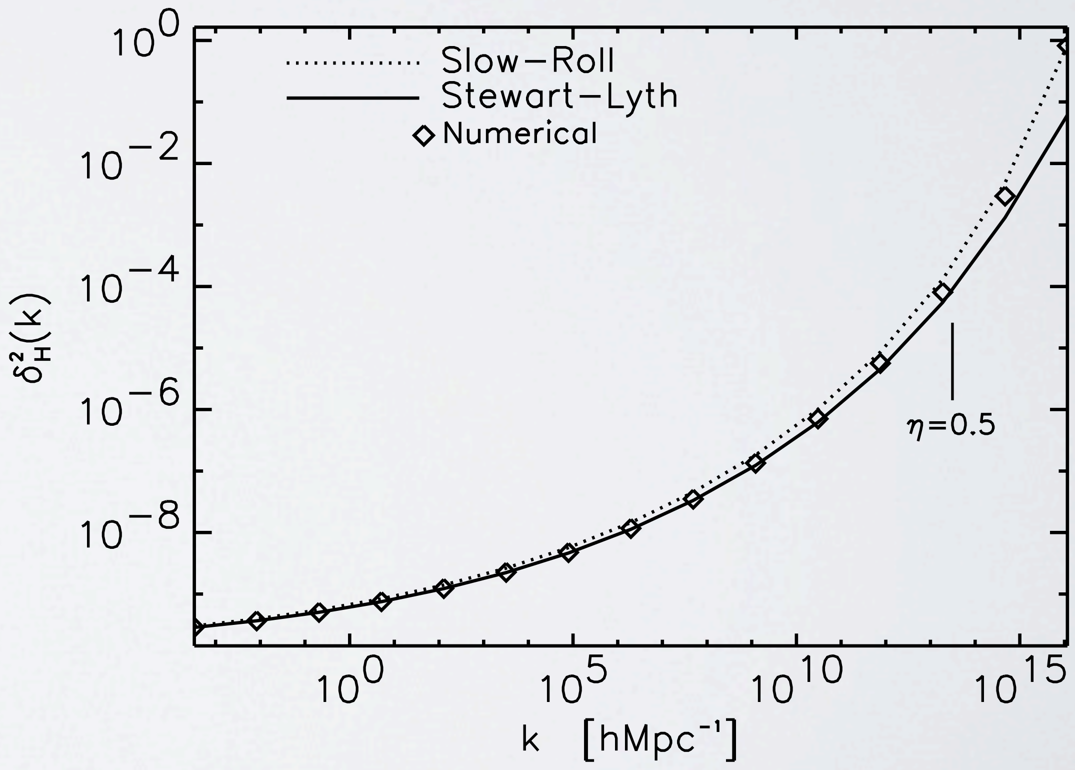
ii) running mass inflation model:

Stewart

Mass of inflation field varies:
(motivated by field theory)

$$V(\phi) = V_0 \pm m^2(\phi)\phi^2$$

Scale dependence of size of fluctuations can be large enough that PBHs are formed. (e.g. Leach, Grivell & Liddle; Kohri, Lyth & Melchiorri; Alabidi & Kohri; Drees & Erfani...)



Leach, Grivell & Liddle

iii) a feature in the inflaton potential/power spectrum

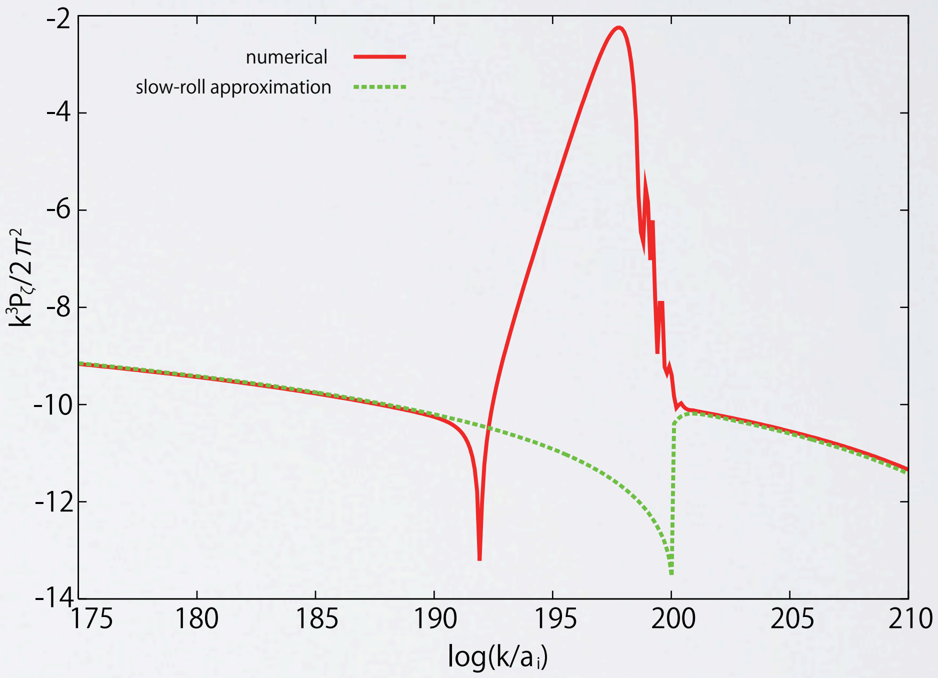
e.g. Ivanov, Naselsky & Novikov; Bullock & Primack; Ivanov; Bringmann, Kiefer & Polarski; Blais, Bringmann, Kiefer & Polarski;

$$\sigma^2(M_{\rm H}) \propto \frac{V^3}{(V')^2}$$

n.b. $\sigma(M) \nrightarrow \infty$ as $V' \to 0$ as assumptions in calculation break down. Kinney

iv) double inflation

e.g. Saito, Yokoyama & Nagata can get peak in amplitude of perturbations on scales which leave the horizon close to the end of the first period of inflation.



n.b. In general when large perturbations are produced standard calculation of amplitude breaks down (inflation is not slow roll and/or ends soon afterwards) and a numerical calculation is required.

Non-gaussianity

If the density perturbation probability distribution is non-gaussian this will in principle affect the PBH abundance. Bullock & Primack; Ivanov

Hidalgo change can be non-negligible, but not huge.

Saito, Yokoyama & Nagata for chaotic new inflation model (which produces an interesting abundance of PBHs), change is negligible.

Summary

- PBHs can form in the early Universe via various mechanisms (collapse of large density perturbations or cosmic string loops, bubble collisions).

- To form an interesting density of PBHs from collapse of density perturbations, need density perturbations on small scales to be substantially larger than on cosmological scales.

- Various inflation models can produce large density perturbations on small scales. Either on very smallest scales (determined by reheat temperature) or at particular scale (feature in potential, or break between two periods of inflation).