Modelling the evolution of small black holes

Elizabeth Winstanley

Astro-Particle Theory and Cosmology Group
School of Mathematics and Statistics
University of Sheffield
United Kingdom

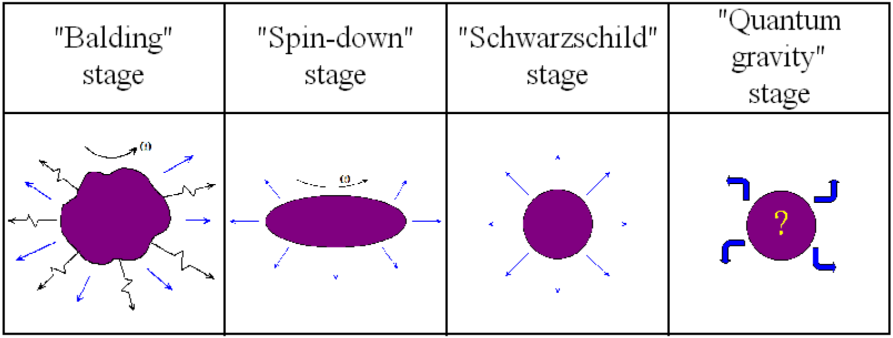
Thanks to STFC UK for financial support

Outline

Stages in the evolution of small black holes

Black holes formed will be rapidly rotating, highly asymmetric, and have gauge field hair

Four stages of subsequent evolution:

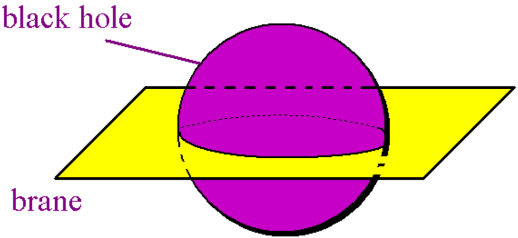


[Giddings and Thomas, hep-ph/0106219]

Modelling small black holes at the end of the balding stage

Small black holes in ADD

- Metric of higher-dimensional black holes in general relativity is known [Myers and Perry, *Annals Phys.* **172**, 304 (1986)]
- Take a 'slice' through a higher-dimensional black hole to give a brane black hole



Modelling small black holes in ADD

Myers-Perry higher-dimensional black hole

$$
\begin{aligned}
ds^2 &= \left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^2 + \frac{2a\mu \sin^2\theta}{\Sigma r^{n-1}} dt\, d\varphi - \frac{\Sigma}{\Delta_n} dr^2 - \Sigma\, d\theta^2 \\
&\quad - \left(r^2 + a^2 + \frac{a^2\mu \sin^2\theta}{\Sigma r^{n-1}}\right) \sin^2\theta\, d\varphi^2 - r^2 \cos^2\theta\, d\Omega_n^2
\end{aligned}
$$

where

$$
\Delta_n = r^2 + a^2 - \frac{\mu}{r^{n-1}}, \qquad \Sigma = r^2 + a^2 \cos^2\theta
$$

Black hole mass M and angular momentum J:

$$
M = \frac{(n+2)\, A_{n+2}\mu}{16\pi G_{4+n}}, \qquad J = \frac{2aM}{n+2}
$$

Modelling small black holes in ADD

Slice of Myers-Perry black hole

$$ds^2 = \left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^2 + \frac{2a\mu \sin^2\theta}{\Sigma r^{n-1}} dt\, d\varphi - \frac{\Sigma}{\Delta_n} dr^2 - \Sigma\, d\theta^2 - \left(r^2 + a^2 + \frac{a^2 \mu \sin^2\theta}{\Sigma r^{n-1}}\right) \sin^2\theta\, d\varphi^2$$

where

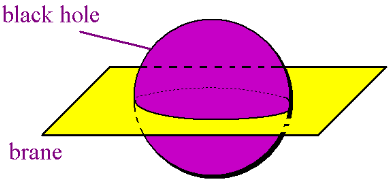
$$\Delta_n = r^2 + a^2 - \frac{\mu}{r^{n-1}}, \qquad \Sigma = r^2 + a^2 \cos^2\theta$$

and n is the number of extra dimensions.

Usual Kerr black hole

Set $n = 0$ in the above metric

Hawking radiation on the brane and in the bulk



Hawking temperature

$$T_H = \frac{(n+1)r_h^2 + (n-1)a^2}{4\pi(r_h^2 + a^2)r_h}$$

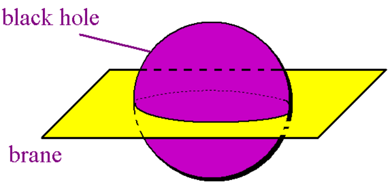
Particles on the brane

- Standard model particles: fermions, gauge bosons, Higgs
- Also gravitons and scalars
- Live on the brane "slice" of the black hole geometry

Particles in the bulk

- Gravitons and scalars
- Will be invisible
- Live on the higher-dimensional black hole geometry

Hawking radiation on the brane and in the bulk



Hawking temperature

$$T_H = \frac{(n+1)r_h^2 + (n-1)a^2}{4\pi(r_h^2 + a^2)r_h}$$

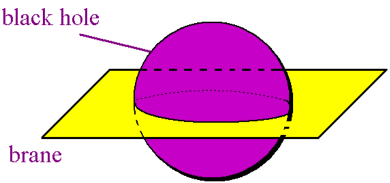
Particles on the brane

- Standard model particles: fermions, gauge bosons, Higgs
- Also gravitons and scalars
- Live on the brane "slice" of the black hole geometry

Particles in the bulk

- Gravitons and scalars
- Will be invisible
- Live on the higher-dimensional black hole geometry

Hawking radiation on the brane and in the bulk



Hawking temperature

$$T_H = \frac{(n+1)r_h^2 + (n-1)a^2}{4\pi(r_h^2 + a^2)r_h}$$

Particles on the brane

- Standard model particles: fermions, gauge bosons, Higgs
- Also gravitons and scalars
- Live on the brane "slice" of the black hole geometry

Particles in the bulk

- Gravitons and scalars
- Will be invisible
- Live on the higher-dimensional black hole geometry

Quantum fields on black hole space-times

Quantum field theory in curved space-time

- Black hole geometry is fixed and classical
- Quantum fields (scalars, fermions, gauge bosons, gravitons) propagate on this background

Quantum field modes

- "Master" equation for fields of spin 0, $\frac{1}{2}$, 1 and 2 on Kerr [Teukolsky, *Phys. Rev. Lett.* **29** 1114 (1972); *Astrophys. J.* **185** 635 (1973)]
- Expand field Ψ in terms of modes of frequency ω:

$$\Psi = \sum_{\omega \ell m} R_{s\omega\ell m}(r) S_{s\omega\ell m}(\theta) e^{-i\omega t} e^{im\varphi}$$

Quantum fields on black hole space-times

Quantum field theory in curved space-time

- Black hole geometry is fixed and classical
- Quantum fields (scalars, fermions, gauge bosons, gravitons) propagate on this background

Quantum field modes

- "Master" equation for fields of spin 0, $\frac{1}{2}$, 1 and 2 on Kerr [Teukolsky, *Phys. Rev. Lett.* **29** 1114 (1972); *Astrophys. J.* **185** 635 (1973)]
- Expand field Ψ in terms of modes of frequency ω:

$$\Psi = \sum_{\omega\ell m} R_{s\omega\ell m}(r) S_{s\omega\ell m}(\theta) e^{-i\omega t} e^{im\varphi}$$

Computing Hawking radiation

Differential emission rates, integrated over all angles:

$$\frac{d^2}{dt\,d\omega}\begin{pmatrix} N \\ E \\ J \end{pmatrix} = \frac{1}{4\pi}\sum_{\text{modes}} \frac{|\mathcal{A}_{s\omega\ell m}|^2}{e^{\tilde{\omega}/T_H} \mp 1}\begin{pmatrix} 1 \\ \omega \\ m \end{pmatrix}$$

where $\tilde{\omega} = \omega - m\Omega_H$

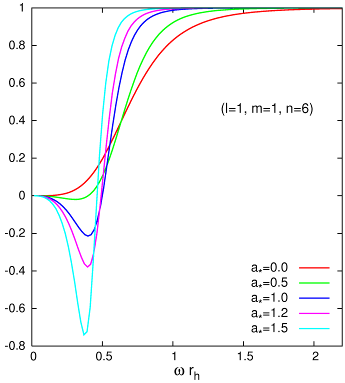
Grey-body factor $|\mathcal{A}_{s\omega\ell m}|^2$

- Emitted radiation is not precisely thermal
- Interaction of emitted quanta with gravitational potential around the black hole
- For an incoming wave from infinity incident on the black hole:

$$|\mathcal{A}_{s\omega\ell m}|^2 = 1 - |\mathcal{R}_{s\omega\ell m}|^2 = \frac{\mathcal{F}_{\text{horizon}}}{\mathcal{F}_{\text{infinity}}}$$

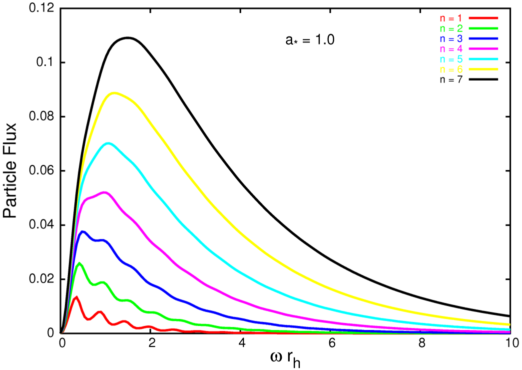
Grey-body factors and emission spectra

Grey-body factors for gauge boson emission and $n = 6$



[Figure taken from Casals et al, `hep-th/0511163`]

Fermion emission spectra for a rotating black hole, integrated over all angles



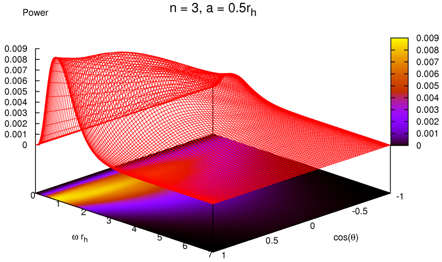
[Figure taken from Casals et al, `hep-th/0608193`]

Angular distribution of energy flux

Differential energy emission rate:

$$\frac{d^3E}{dt\,d\omega\,d(\cos\theta)} = \frac{1}{4\pi}\sum_{\text{modes}} \frac{\omega\,|\mathcal{A}_{s\omega\ell m}|^2}{e^{\tilde{\omega}/T_H} \mp 1}\left[S_{|s|\omega\ell m}(\theta)^2 + S_{-|s|\omega\ell m}(\theta)^2\right]$$

Energy emission for positive helicity fermions and gauge bosons for $n = 3$ and $a_* = 0.5$

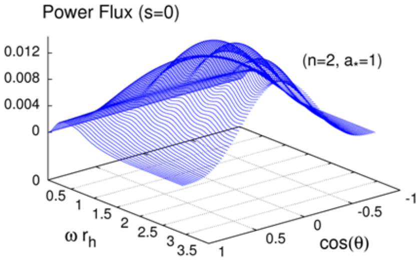


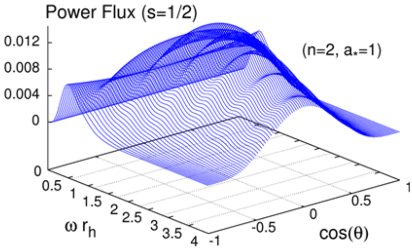
[Figures taken from Casals et al `arXiv:0907.1511 [hep-th]`]

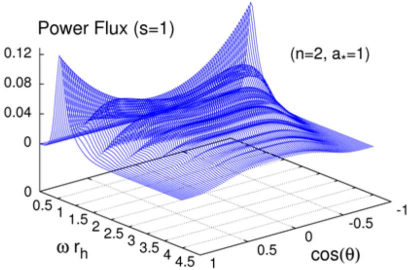
Angular distribution of energy flux

Six-dimensional black hole

n=2







What we know about the Hawking radiation phases

"Spin-down" phase

- Brane emission - scalars, fermions, gauge bosons done
- Bulk emission - scalars done
- Graviton emission - partial results only

"Schwarzschild" phase

- Brane emission - scalars, fermions, gauge bosons done
- Bulk emission - scalars done
- Graviton emission - bulk and brane done

"Black holes radiate mainly on the brane"

[Emparan, Horowitz and Myers, hep-th/0003118]

Ratio of bulk/brane emission for massless scalars, $n = 2$

$a_* = 0.0$	$a_* = 0.2$	$a_* = 0.4$	$a_* = 0.6$	$a_* = 0.8$	$a_* = 1.0$
19.9%	18.6%	15.3%	11.7%	9.0%	7.1%

[Casals et al, arXiv:0801.4910 [hep-th]]

What we know about the Hawking radiation phases

"Spin-down" phase

- Brane emission - scalars, fermions, gauge bosons done
- Bulk emission - scalars done
- Graviton emission - partial results only

"Schwarzschild" phase

- Brane emission - scalars, fermions, gauge bosons done
- Bulk emission - scalars done
- Graviton emission - bulk and brane done

"Black holes radiate mainly on the brane"

[Emparan, Horowitz and Myers, hep-th/0003118]

Ratio of bulk/brane emission for massless scalars, $n = 2$

$a_* = 0.0$	$a_* = 0.2$	$a_* = 0.4$	$a_* = 0.6$	$a_* = 0.8$	$a_* = 1.0$
19.9%	18.6%	15.3%	11.7%	9.0%	7.1%

[Casals et al, arXiv:0801.4910 [hep-th]]

What we know about the Hawking radiation phases

"Spin-down" phase

- Brane emission - scalars, fermions, gauge bosons done
- Bulk emission - scalars done
- Graviton emission - partial results only

"Schwarzschild" phase

- Brane emission - scalars, fermions, gauge bosons done
- Bulk emission - scalars done
- Graviton emission - bulk and brane done

"Black holes radiate mainly on the brane"

[Emparan, Horowitz and Myers, hep-th/0003118]

Ratio of bulk/brane emission for massless scalars, $n = 2$

$a_* = 0.0$	$a_* = 0.2$	$a_* = 0.4$	$a_* = 0.6$	$a_* = 0.8$	$a_* = 1.0$
19.9%	18.6%	15.3%	11.7%	9.0%	7.1%

[Casals et al, arXiv:0801.4910 [hep-th]]

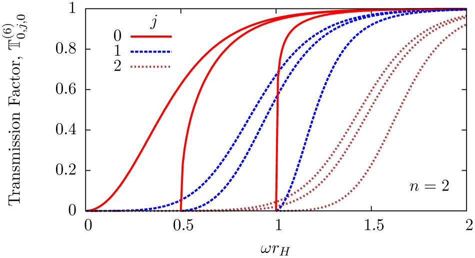
More complicated effects in Hawking radiation

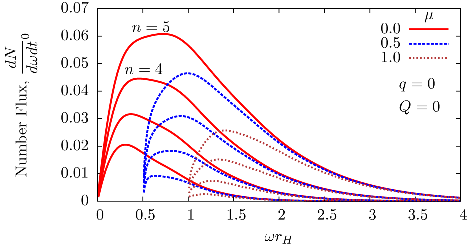
Massive particles

- Sharp cut-off in grey-body factor at particle mass
- Reduction in number of particles emitted

[Rogatko and Szyplowska, `arXiv:0904.4544 [hep-th]`]
[Kanti and Pappas, `arXiv:1003.5125 [hep-th]`]

[Figures taken from Sampaio, `arXiv:0911.0688 [hep-th]`]





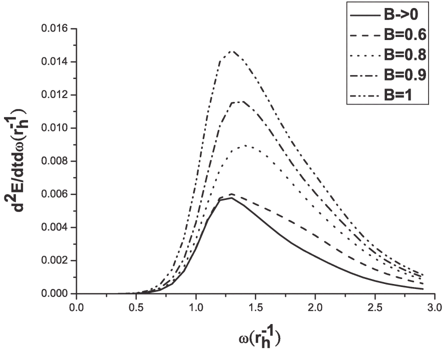
More complicated effects in Hawking radiation

Brane tension

Exact codimension-2 solutions for a black hole with a tense brane
[Kaloper and Kiley, hep-th/0601110]
[Kiley, arXiv:0708.1016 [hep-th]]

Bulk emission suppressed by brane tension

[Figure taken from Dai et al, hep-th/0611184]
[Kobayashi et al, arXiv:0711.1395 [hep-th]]
[Rogatko and Szyplowska, arXiv:0905.4342 [hep-th]]

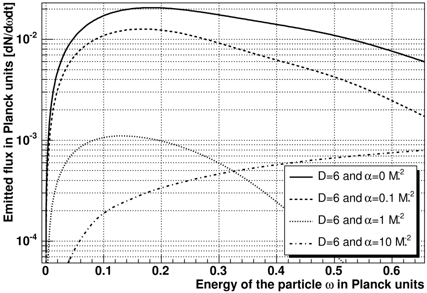


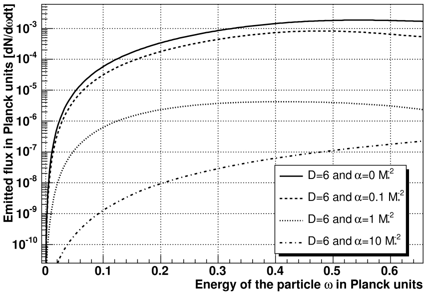
More complicated effects in Hawking radiation

Gauss-Bonnet gravity

Exact metric for spherically symmetric black hole with Gauss-Bonnet corrections [Boulware and Deser, *Phys. Rev. Lett.* **55**, 2656 (1985)]

Suppression of emission of both brane (left) and bulk (right) particles





[Figures taken from Grain et al, `hep-th/0509128`]

[Konoplya and Zhidenko, `arXiv:1004.3772 [hep-th]`]

Balding phase

Shedding of mass and angular momentum through gravitational radiation modeled as part of formation process

Electromagnetic effects

- Classical Maxwell field on the brane only - modifies the “slice” of the Myers-Perry black hole
- Loss of black hole charge is not rapid in TeV gravity models

[Sampaio, arXiv:0907.5107 [hep-th]] ;

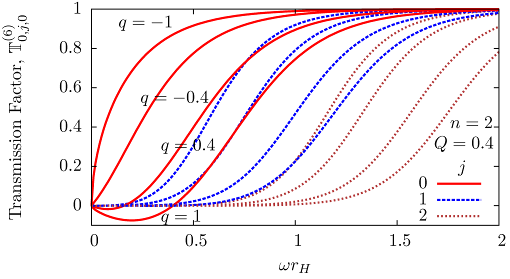
QCD effects

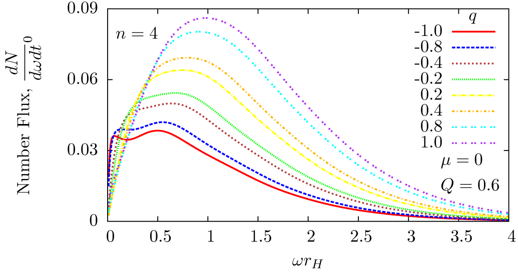
Likely to be significant, but little work on this

[Calmet et al, arXiv:0806.4605 [hep-ph]]

[Gingrich, arXiv:0912.0826 [hep-ph]]

Hawking radiation of charged particles





Modification of grey-body factor and emission spectrum for charged particles on the brane by a charged black hole

[Figures taken from Sampaio, `arXiv:0911.0688 [hep-th]`]

Quantum gravity effects in small black hole evolution

Some possible end-points of black hole evaporation

- Emits final burst all at once
- Remnant
- String ball
- ???????

- Quantum gravity scattering processes are much more likely than semi-classical black hole formation
  [Meade and Randall, `arXiv:0708.3017 [hep-ph]`]
- Quantum black holes
  [Calmet et al, `arXiv:1005.1805 [hep-ph]`]
  [Gingrich, `arXiv:0912.0826 [hep-ph]`]

Open issues in modelling the evolution of small black holes

- Complete computation of graviton radiation
  - Requires full gravitational perturbation equations for rotating higher-dimensional black holes
  - Recent work only for tensor-type gravitational perturbations with $n \geq 3$
    [Doukas et al, arXiv:0906.1515 [hep-th]]
    [Kanti et al, arXiv:0906.3845 [hep-th]]
- Realistic evolution will be a stochastic process
  - Individual quanta emitted rather than a continuum
  - Black hole will recoil, possibly even come off the brane
  - Black hole may not have time to approach thermal equilibrium between emissions
- Quantum gravity effects important in last stage of the evolution

Open issues in modelling the evolution of small black holes

- Complete computation of graviton radiation
  - Requires full gravitational perturbation equations for rotating higher-dimensional black holes
  - Recent work only for tensor-type gravitational perturbations with $n \geq 3$
    [Doukas et al, arXiv:0906.1515 [hep-th]]
    [Kanti et al, arXiv:0906.3845 [hep-th]]

- Realistic evolution will be a stochastic process
  - Individual quanta emitted rather than a continuum
  - Black hole will recoil, possibly even come off the brane
  - Black hole may not have time to approach thermal equilibrium between emissions

- Quantum gravity effects important in last stage of the evolution

Open issues in modelling the evolution of small black holes

- Complete computation of graviton radiation
  - Requires full gravitational perturbation equations for rotating higher-dimensional black holes
  - Recent work only for tensor-type gravitational perturbations with $n \geq 3$
    [Doukas et al, arXiv:0906.1515 [hep-th]]
    [Kanti et al, arXiv:0906.3845 [hep-th]]
- Realistic evolution will be a stochastic process
  - Individual quanta emitted rather than a continuum
  - Black hole will recoil, possibly even come off the brane
  - Black hole may not have time to approach thermal equilibrium between emissions
- Quantum gravity effects important in last stage of the evolution

Conclusions

Four stages in the evolution of small black holes

- Balding phase
- Spin-down phase
- Schwarzschild phase
- Quantum gravity phase

- Modelling of balding phase is very complicated due to lack of symmetry and matter coupling to the black hole
- Detailed analysis of semi-classical Hawking radiation apart from graviton modes for rotating black hole
- End-point of black hole evolution not fully understood
- Need to understand small black holes as quantum rather than semi-classical objects