Quantum Gravity

General overview and recent developments

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Contents

Why quantum gravity?

- Unification of all interactions
- Singularity theorems
  - Black holes
  - ‘Big Bang’
- Problem of time
- Absence of viable alternatives

The problem of time

- Absolute time in quantum theory:

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$$

- Dynamical time in general relativity:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

QUANTUM GRAVITY?

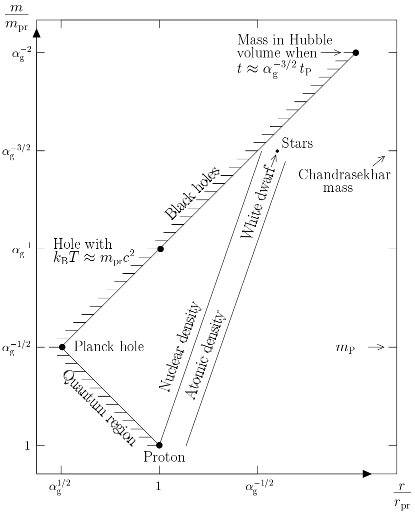
Planck units

$$
\begin{aligned}
l_{\mathrm{P}} &= \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33}\ \mathrm{cm} \\
t_{\mathrm{P}} &= \frac{l_{\mathrm{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44}\ \mathrm{s} \\
m_{\mathrm{P}} &= \frac{\hbar}{l_{\mathrm{P}} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-5}\ \mathrm{g} \approx 1.22 \times 10^{19}\ \mathrm{GeV}/c^2
\end{aligned}
$$

Max Planck (1899):

Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

Structures in the Universe



$$\alpha_g = \frac{G m_{pr}^2}{\hbar c} = \left(\frac{m_{pr}}{m_P}\right)^2 \approx 5.91 \times 10^{-39}$$

Steps towards quantum gravity

- Interaction of micro- and macroscopic systems with an external gravitational field
- Quantum field theory on curved backgrounds (or in flat background, but in non-inertial systems)
- Full quantum gravity

Black-hole radiation

Black holes radiate with a temperature proportional to $\hbar$:

$$T_{\mathrm{BH}} = \frac{\hbar\kappa}{2\pi k_{\mathrm{B}} c}$$

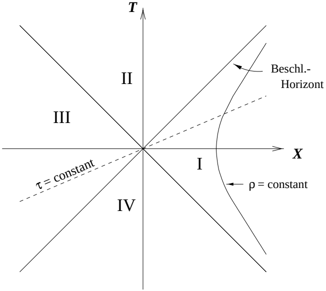
Schwarzschild case:

$$\begin{aligned} T_{\mathrm{BH}} &= \frac{\hbar c^3}{8\pi k_{\mathrm{B}} G M} \\ &\approx 6.17 \times 10^{-8} \left(\frac{M_\odot}{M}\right) \mathrm{K} \end{aligned}$$

Black holes also have an entropy:

$$S_{\mathrm{BH}} = k_{\mathrm{B}} \frac{A}{4 l_{\mathrm{P}}^2} \overset{\text{Schwarzschild}}{\approx} 1.07 \times 10^{77} k_{\mathrm{B}} \left(\frac{M}{M_\odot}\right)^2$$

Analogous effect in flat spacetime



Accelerated observer in the Minkowski vacuum experiences thermal radiation with temperature

$$T_{\mathrm{DU}} = \frac{\hbar a}{2\pi k_{\mathrm{B}} c} \approx 4.05 \times 10^{-23}\, a \left[\frac{\mathrm{cm}}{\mathrm{s}^2}\right]\ \mathrm{K}\ .$$

(Davies–Unruh temperature)

Main Approaches to Quantum Gravity

No question about quantum gravity is more difficult than the question, "What is the question?" (John Wheeler 1984)

- Quantum general relativity
  - Covariant approaches (perturbation theory, path integrals, ...)
  - Canonical approaches (geometrodynamics, connection dynamics, loop dynamics, ...)
- String theory
- Other approaches
  (Quantization of topology, causal sets ...)

Covariant quantum gravity

Perturbation theory:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{\frac{32\pi G}{c^4}} f_{\mu\nu}$$

- $\bar{g}_{\mu\nu}$: classical background
- Perturbation theory with respect to $f_{\mu\nu}$ (Feynman rules)
- "Particle" of quantum gravity: graviton (massless spin-2 particle)

Perturbative non-renormalizability

Effective field theory

Concrete predictions possible at low energies
(even in non-renormalizable theory)

Example:

Quantum gravitational correction to the Newtonian potential

$$V(r) = -\frac{Gm_1m_2}{r}\left(1 + \underbrace{3\frac{G(m_1+m_2)}{rc^2}}_{\mathrm{GR-correction}} + \underbrace{\frac{41}{10\pi}\frac{G\hbar}{r^2c^3}}_{\mathrm{QG-correction}}\right)$$

(Bjerrum–Bohr *et al*. 2003)

Analogy: Chiral perturbation theory (small pion mass)

Asymptotic Safety

Weinberg (1979): A theory is called asymptotically safe if all essential coupling parameters g_i of the theory approach for $k \to \infty$ a non-trivial fix point

Preliminary results:

- Effective gravitational constant vanishes for $k \to \infty$
- Effective gravitational constant increases with distance (simulation of Dark Matter?)
- Small positive cosmological constant as an infrared effect (Dark Energy?)
- Spacetime appears two-dimensional on smallest scales

(M. Reuter and collaborators from 1998 on)

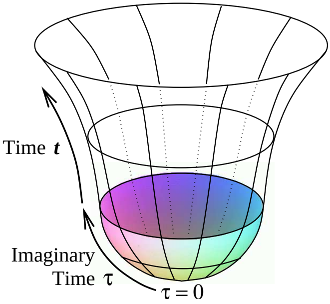
Path integrals

$$\boxed{Z[g] = \int \mathcal{D}g_{\mu\nu}(x)\ \mathrm{e}^{\mathrm{i}S[g_{\mu\nu}(x)]/\hbar}}$$

In addition: sum over all topologies?

- Euclidean path integrals
(e.g. for Hartle–Hawking proposal
or Regge calculus)
- Lorentzian path integrals
(e.g. for dynamical triangulation)

Euclidean path integrals

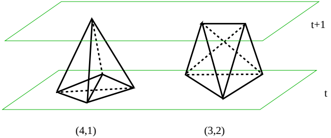


S. W. Hawking, Vatican conference 1982:

There ought to be something very special about the boundary conditions of the universe and what can be more special than the condition that there is no boundary.

Dynamical triangulation

- makes use of *Lorentzian* path integrals
- edge lengths of simplices remain fixed; sum is performed over all possible combinations with equilateral simplices
- Monte-Carlo simulations



Preliminary results:

- Hausdorff dimension $H = 3.10 \pm 0.15$
- Spacetime two-dimensional on smallest scales (cf. asymptotic-safety approach)
- positive cosmological constant needed
- continuum limit?

(Ambjørn, Loll, Jurkiewicz from 1998 on)

Canonical quantum gravity

Central equations are constraints:

$$\boxed{\hat{H}\Psi = 0}$$

Distinction according to choice of variables:

- Geometrodynamics –
  variables are three-metric and extrinsic curvature
- Connection dynamics –
  variables are connection (A_a^i) and non-abelian electric field (E_i^a)
- Loop dynamics –
  variables are holonomy connected with A_a^i and the flux of E_i^a

Quantum geometrodynamics

Formal quantization of the classical constraints yields (for pure gravity):

$$\hat{H}\Psi \equiv \left(-16\pi G\hbar^2 G_{abcd}\frac{\delta^2}{\delta h_{ab}\delta h_{cd}} - (16\pi G)^{-1}\sqrt{h}\big({}^{(3)}R - 2\Lambda\big)\right)\Psi = 0$$

Wheeler–DeWitt equation

$$\hat{D}^a\Psi \equiv -2\nabla_b\frac{\hbar}{\mathrm{i}}\frac{\delta\Psi}{\delta h_{ab}} = 0$$

quantum diffeomorphism (momentum) constraint

(guarantees the coordinate independence of Ψ)

Time, and therefore spacetime, have disappeared from the formalism!

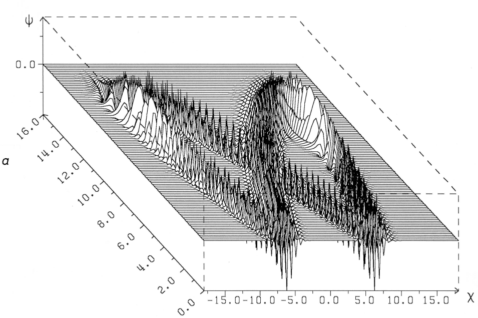
Problem of time and related problems

- External time t has vanished from the formalism
- This holds also for loop quantum gravity and probably for string theory
- Wheeler–DeWitt equation has the structure of a wave equation any may therefore allow the introduction of an “intrinsic time”
- Hilbert-space structure in quantum mechanics is connected with the probability interpretation, in particular with probability conservation *in time* t; what happens with this structure in a timeless situation?
- What is an observable in quantum gravity?

Example

Indefinite Oscillator

$$\hat{H}\psi(a,\chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2 \right)\psi = 0$$



(C. K. 1990)

Semiclassical approximation

Expansion of the Wheeler–De-Witt equation into powers of the Planck mass:

$$\Psi \approx \exp(\mathrm{i}S_0[h]/G\hbar)\,\psi[h,\phi]$$

(h: three-metric, ϕ: non-gravitational degrees of freedom)

- S_0 obeys the Hamilton–Jacobi equation of gravity
- ψ obeys approximately a (functional) Schrödinger equation:

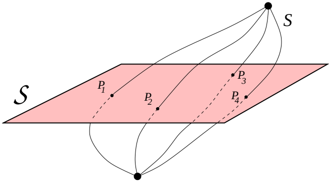
$$\mathrm{i}\hbar\,\underbrace{\nabla S_0\,\nabla\psi}_{\frac{\partial\psi}{\partial t}} \approx H_\mathrm{m}\,\psi$$

(H_m: Hamiltonian for non-gravitational fields ϕ)

Temps t retrouvé!

- Next order: Quantum gravitational corrections to the Schrödinger equation (C. K. and T. P. Singh 1991)

Loop quantum gravity



Quantization of area:

$$\hat{A}(\mathcal{S})\Psi_S[A] = 8\pi\beta l_{\mathrm{P}}^2 \sum_{P\in S\cap\mathcal{S}} \sqrt{j_P(j_P+1)}\Psi_S[A]$$

(β: free parameter in this approach)

String theory

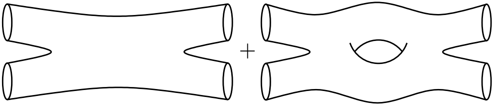
Important properties:

- Inclusion of gravity unavoidable
- Gauge invariance, supersymmetry, higher dimensions
- Unification of all interactions
- Perturbation theory proabably finite at all orders, but sum diverges
- Only three fundamental constants: $\hbar, c, l_{\mathrm{s}}$
- Branes as central objects
- Dualities connect different string theories

Space and time in string theory

$$\boxed{Z = \int \mathcal{D}X\mathcal{D}h \; \mathrm{e}^{-S/\hbar}}$$

(X: Embedding; h: Metric on worldsheet)



$+ \ldots$

Absence of quantum anomalies $\longrightarrow$

- Background fields obey Einstein equations up to $\mathcal{O}(l_{\mathrm{s}}^2)$; can be derived from an effective action
- Constraint on the number of spacetime dimensions: 10 resp. 11

Generalized uncertainty relation:

$$\Delta x \geq \frac{\hbar}{\Delta p} + \frac{l_{\mathrm{s}}^2}{\hbar} \Delta p$$

Problems

- Too many “string vacua” (problem of landscape)
- No background independence?
- Standard model of particle physics?
- What is the role of the 11th dimension? What is M-theory?
- Experiments?

Black holes

Microscopic explanation of entropy?

$$\boxed{S_{\rm BH} = k_{\rm B}\frac{A}{4l_{\rm P}^2}}$$

- Loop quantum gravity: microscopic degrees of freedom are the spin networks; $S_{\rm BH}$ only follows for a specific choice of β: $\beta = 0.237532\ldots$
- String theory: microscopic degrees of freedom are the "D-branes"; $S_{\rm BH}$ only follows for special (extremal or near-extremal) black holes
- Quantum geometrodynamics: entanglement entropy between the hole and other degrees of freedom?

Problem of information loss

- Final phase of evaporation?
- Fate of information is a consequence of the fundamental theory (unitary or non-unitary)
- Problem does not arise in the semiclassical approximation (thermal character of Hawking radiation follows from decoherence)
- Empirical problems:
  - Are there primordial black holes?
  - Can black holes be generated in accelerators?

Primordial black holes

Primordial Black Holes could form from density fluctuations in the early Universe (with masses from 1 g on); black holes with an initial mass of $M_0 \approx 5 \times 10^{14}\ \mathrm{Gramm}$ would evaporate "today" $\longrightarrow$ typical spectrum of Gamma rays

Fermi Gamma-ray Space Telescope; Launch: June 2008

Generation of mini black holes at the LHC?

CMS detector

Only possible if space has more than three dimensions

My own research on quantum black holes

- Primordial black holes from density fluctuations in inflationary models
- Quasi-normal modes and the Hawking temperature
- Decoherence of quantum black holes and its relevance for the problem of information loss
- Hawking temperature from solutions to the Wheeler–DeWitt equation (for the LTB model) as well as quantum gravitational corrections
- Area law for the entropy from solutions to the Wheeler–DeWitt equation (for the 2+1-dimensional LTB model)
- Origin of corrections to the area law
- Model for black-hole evaporation

(to be discussed elsewhere)

Observations and experiments

Up to now only expectations!

- Evaporation of black holes (but need primordial black holes or big extra dimensions)
- Origin of masses and coupling constants (Λ!)
- Quantum gravitational corrections observable in the anisotropy spectrum of the cosmic background radiation?
- Time-dependent coupling constants, violation of the equivalence principle, ...
- Signature of a discrete structure of spacetime (γ-ray bursts?)
- Signature of extra dimensions (LHC)? Supersymmetry?

Einstein (according to Heisenberg) : Erst die Theorie entscheidet darüber, was man beobachten kann.

More details in

C. K., *Quantum Gravity*, second edition (Oxford 2007).