BONN meeting, 24 June 2010 : WG 1 1

"FORMATION OF SMALL BLACK HOLES IN HIGH-ENERGY COLLISIONS"

EINLEITUNG

VIELE QUELLEN → RHEIN

1. R. PENROSE (1973/4) TRAPPED SURFACES IN SPEED-OF-LIGHT COLLISIONS
2. P.D.D. (1975-77) STRONG-FIELD GRAV^L. RAD^N. FROM $\gamma \to \infty$ COLLISIONS OR ENCOUNTERS
3. PHILIP PAYNE (Ph.D. 1978-83) " " AT $\gamma = \infty$: MORE ACCURATE INFORMATION. (PUB. 1992)
4. (1993/4 ONWARDS) IDEA TAKEN UP BY G. VENEZIANO AND OTHERS
5. (1997 →) P.D.D. FINDS THAT, IN 4-DIM^L. S/GRAVITY OR GAUGE-INVARIANT S/G (SU(2), SU(3), ...) WITH SUITABLE BOUNDARY DATA ('FIELD DATA', NOT 'PARTICLE-STATE DATA'), QUANTUM AMPLITUDES ARE SEMI-CLASSICAL (IN A PRECISE SENSE)!
6. A. FARLEY (Ph.D. 1997-2002) HENCE CALCULATES QUANTUM <u>AMPLITUDES</u> (NOT JUST <u>PROBABILITIES</u>) FOR QUANTUM PROCESSES INVOLVING BLACK-HOLE FORMATION AND EVAPORATION (PUB. 2004-10)

ASSUMING (say) GAUGE-INVARIANT s/g

7 (~2000 ONWARD) POSSIBILITY THAT THERE MIGHT BE SOLITONIC SOLUTIONS OF THE CLASSICAL FIELD EQUATIONS.

IF THIS WORKED, THEN THE SEMI-CLASSICAL NATURE OF THE QUANTUM AMPLITUDE WOULD IMPLY THAT QUANTUM PROCESSES WOULD BE DESCRIBED via CLASSICAL CALCULATIONS (!!).

8 (200½ GIDDINGS + EARDLEY, et al. →) (ESSENTIALLY) REPEAT R. PENROSE'S CALCULATIONS OF APPARENT HORIZONS IN $\gamma = \infty$ COLLISION, but INCLUDING THE CASE OF DIMENSION $D > 4$.

9 ... ~~REFINEMENTS~~ ELABORATIONS OF " " ", WITH A POSSIBLE VIEW TOWARDS TESTING MODELS AT THE L.H.C.

... WIE EINE SYMPHONIE VON GUSTAV MAHLER. VIELE MOTIVE, JEDESMAL SEHR VERÄNDERT...

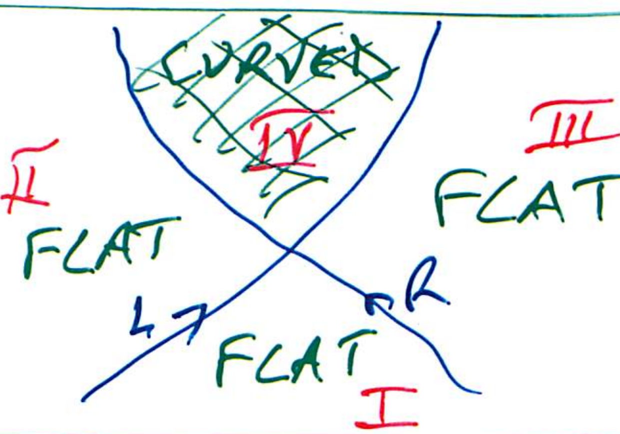
R. PENROSE (via P. PAYNE):

Speed-of-light collisions — the head-on (axisymmetric) case.
(1973/6?)

AICHELBURG–SEXL metric (1971):

$$ds^2 = du\,dv + dx^2 + dy^2 - 4\mu \log(x^2+y^2)\,\delta(u)\,du^2$$

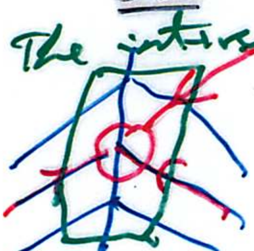
COLLISION (schematic):



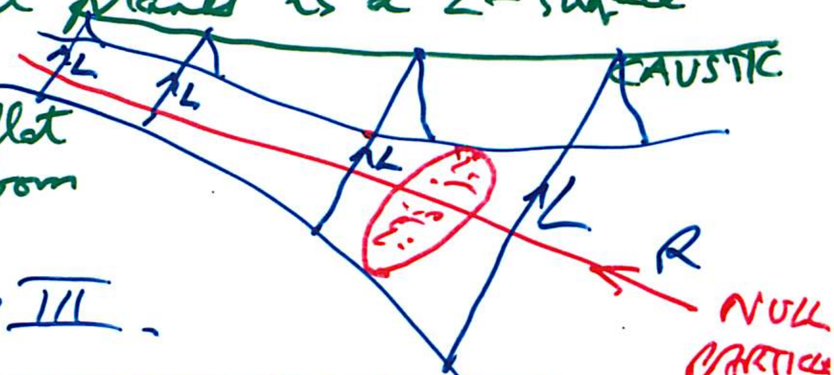
R.P. looked for closed trapped surfaces.
If COSMIC CENSORSHIP holds, then $\exists$ an event horizon, outside the trapped surface. But the area of a B.H. can only increase with time; hence, find a LOWER BOUND on the final B.H. area, and, further, on the FINAL MASS of B.H.
$\Longleftrightarrow$ an UPPER BOUND on ΔE radiated.

Case IMPACT PARAMETER $b=0$

The intersection of the null planes is a 2-surface



Viewed in flat coordinates from region III



And symmetrically II $\leftrightarrow$ III.

CHOOSE RADIUS OF 'RED CIRCLE' (really, CLOSED 2-SURFACE) s.t. the CONVERGENCE of the OUTGOING NULL GEODESICS is ZERO. (MARGINALLY TRAPPED SURFACE)

RADIUS $= 4\mu$, whence AREA OF APPARENT HORIZON IS $32\pi\mu^2$.

Hence, AREA OF FINAL BLACK HOLE $\geq 32\pi\mu^2$.

$\therefore$ MASS OF FINAL B.H. $\geq \left(\frac{1}{\sqrt{2}}\right)(2\mu)$ – EFFICIENCY OF GRAV^{c}. WAVE PRODUCTION $\leq \left(1-\frac{1}{\sqrt{2}}\right)$.

GRAV^L. RAD^N. FROM HIGH-SPEED BLACK-HOLE ENCOUNTERS

Take γ very large, but $\gamma < \infty$.

(PDD 1975–78, following a suggestion from R. PENROSE to G.E. CURTIS)

Advantage: all singularities due to previous $\gamma = \infty$ are SMOOTHED OUT. CONCEPTUALLY SIMPLER.

Technique: Perturbation theory for different regions of the SPACE-TIME in powers of γ^{-1}. Match the different regions together using MATCHED ASYMPTOTIC EXPANSIONS.

SMEARED-OUT INCOMING PROFILE

Ditto: SMEARED-OUT CAUSTIC REGION

EVOLUTION OF THE FAR-FIELD CURVED SHOCKS: GRAV^L-WAVE GENERATION

EVOLUTION BY FLAT SPACE-TIME WAVE EQ^N., SUBJECT TO THE USUAL (SCHWARZSCHILD) LOGARITHMIC TIME-DELAY.

GRAV^L. RAD^N. NEAR $\mathscr{I}^+$ DESCRIBED BY THE BONDI NEWS FUNCTION $c_\tau(\tau, \theta)$ (AXI-SYMMETRIC CASE)

RETARDED TIME COORD. (τ) — USUAL POLAR ANGLE (θ)

Rate of MASS LOSS in GRAV^L. WAVES:

$$\frac{d(\text{mass})}{d\tau} = -\frac{1}{2}\int_0^\pi d\theta \, \sin\theta \, \left[c_\tau(\tau,\theta)\right]^2$$

HEAD-ON, EQUAL-MASS, $\gamma \to \infty$ LIMIT:

$$c_\tau(\tau, \theta = 4\gamma^{-1}) = \frac{8\mu}{2\pi}\int_0^\infty dP \int_0^{2\pi} d\phi \, \frac{P^3 \cos(2\phi)}{\left[P^2 + \left(\begin{array}{l} +P\cos\phi + \tau \\ + 8\mu \log P \end{array}\right)\right]^{5/2}}$$

$+ o(1)$ as $\gamma \to \infty$.

The RADIATION PATTERN near the FORWARD AXIS ($\theta = \psi\gamma^{-1}$ with ψ fixed) matches onto a much broader RADIATION PATTERN, spread over the whole CELESTIAL SPHERE, with $\theta = O(1)$:

expect to be able to write (asymptotically, as $\gamma \to \infty$) also

$$\boxed{\zeta_{\tau'}(\tau', \theta) \sim \sum_{n=0}^{\infty} \gamma^{-n} S_n(\tau', \theta)}$$

Leading term, as $\gamma \to \infty$, is expected to admit a convergent series expansion, of the form

$$\boxed{S_0(\tau', \theta) = \sum_{m=0}^{\infty} a_m(\tau') (\sin\theta)^m}$$

Further investigation shows that only EVEN POWERS are present:

$$\boxed{\zeta_\tau\left(\tau, \theta = \frac{\psi}{\gamma}\right) \sim \sum_{n=0}^{\infty} \gamma^{-2n} Q_{2n}(\tau, \psi)} \quad (+)$$

and

$$\boxed{S_0(\tau', \theta) = \sum_{m=0}^{\infty} a_{2m}(\tau') (\sin\theta)^{2m}} \quad (*)$$

SEE LATER (R.N. PAYNE)

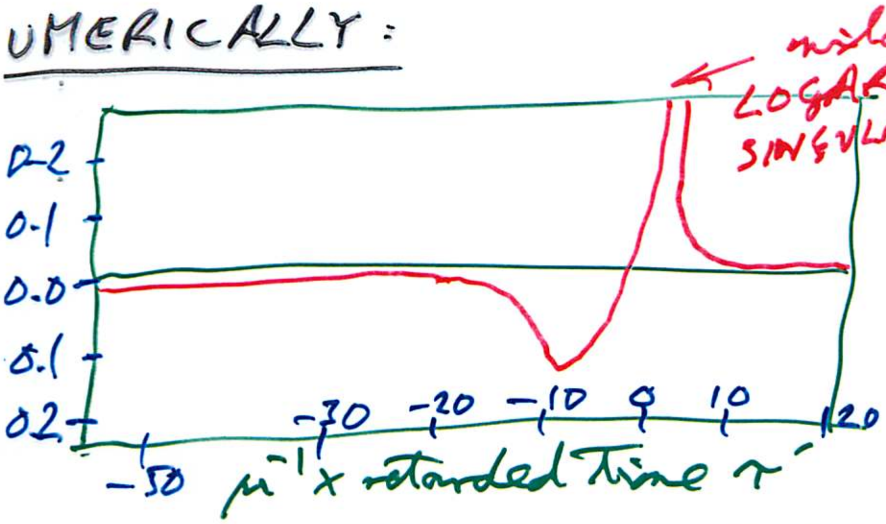
MATCHING outward from (+) at large ψ to (*) at small θ, one finds:

$$\boxed{a_0(\tau') = \frac{4}{\pi} \int_{\mathcal{D}} \frac{d\rho'}{(\rho')^2} \frac{\left[\dfrac{2\left(8\log\rho' + \frac{\tau'}{\mu}\right)^2}{(\rho')^2} - 1\right]}{\left[1 - \dfrac{\left(8\log\rho' + \frac{\tau'}{\mu}\right)^2}{(\rho')^2}\right]^{1/2}}}$$

Here, $\mathcal{D}$ denotes the DOMAIN OF THOSE VALUES ρ' such that

$$\boxed{\left(8\log\rho' - \rho' + \frac{\tau'}{\mu}\right) < 0 < \left(8\log\rho' + \rho' + \frac{\tau'}{\mu}\right)}$$

UMERICALLY:



News function $a_0(\tau')$, assuming that there is no Θ-dependence in Eq. ⊛

$[a_2(\tau') = a_4(\tau') = \ldots = 0]$

Numerically, TOTAL ENERGY radiated in the 'ISOTROPIC PART' $a_0(\tau')$ is

$$\boxed{\Delta E_{(a_0)} = 0.5000\ldots\ \mu}$$

[efficiency of 25%]

Repeat Eq. ⊛:

$$\boxed{S_0(\tau', \Theta) = \sum_{m=0}^{\infty} a_{2m}(\tau')\,(\sin\Theta)^{2m}} \quad ⊛$$

HEAD-ON SPEED-OF-LIGHT COLLISION: FURTHER INFORMATION FROM SECOND-ORDER PERTURBATION THEORY

(P.N. PAYNE, Ph.D. 1978–83; published PNP + PDD 1992)

Apply a very large boost:

for $\lambda \gg 1$.

Study perturbation theory in $\left(\frac{1}{\lambda^2}\right)$.

FIRST-ORDER PERTURBATION THEORY on R.H.S.

$\to a_0(\tau')$ again, as in Eq. ⊛ above.
on L.H.S.

2nd-ORDER PERTURBATION THEORY on R.H.S.

$\to$ next-order term $a_2(\tau')$ in the series ⊛ above for $\gamma = \infty$ collision in C.-M. frame, in an 'analytic' form.

News function known up to

$$\boxed{S_0(\tau', \theta) = a_0(\tau') + a_2(\tau') \sin^2\theta}$$

$[\ldots + a_4(\tau') \sin^4\theta + \ldots]$

as a function over the celestial sphere.

NUMERICAL CALCULATION, using BONDI MASS-LOSS FORMULA

$\Rightarrow$ (using this truncation)

ENERGY LOSS IN GRAV$^{\text{L}}$ WAVES

$\simeq (16.8\%) \times (2\mu)$ INITIAL ENERGY

TURN TO QUANTUM PROPERTIES

SEMI-CLASSICAL QUANTUM AMPLITUDES IN FIELD THEORIES WITH LOCAL SUPERSYMMETRY, SUBJECT TO SUITABLE (FIELD) BOUNDARY DATA

[NAMELY, SUPERGRAVITY OR GAUGE-INVARIANT SUPERGRAVITY (gauge group SU(2) or SU(3), etc..]

NOTE ① WE EXPECT LOCAL S/SY IN NATURE, TO AVOID INFINITIES.

E.g. The BOSONIC STRING HAS DIVERGENT Q. AMPLITUDES ALREADY AT 1-LOOP ORDER.

Today, 'STRING THEORY' means 'SUPER-STRING THEORY'.

SIMILARLY FOR FIELD THEORIES

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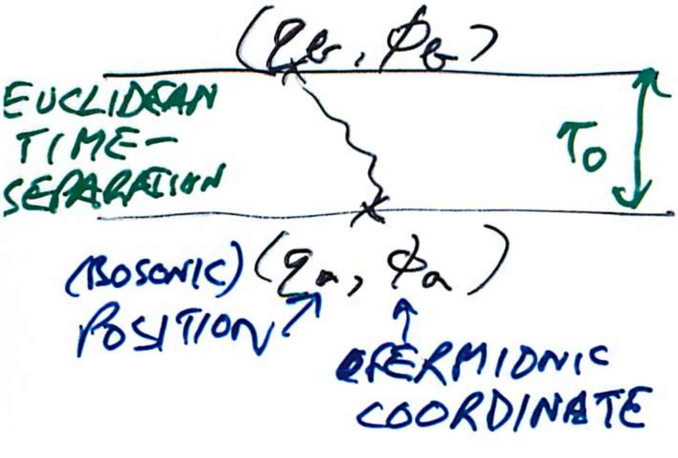
② SEMI-CLASSICAL Q. AMPLITUDES ARE SEEN IN THE SIMPLEST EXAMPLE:

E. WITTEN'S S/SYMMETRIC Q.M. (1981), as made LOCALLY-S/SYMMETRIC BY E. ALVAREZ (1984)

POSE the EUCLIDEAN Q. BDRY-VALUE PROBLEM - find the Q. AMPLITUDE for



One finds

AMPLITUDE = S(s/sy CONSTRAINTS) times $\exp\left(-\frac{I_{class}}{\hbar}\right)$

CLASSICAL EUCLIDEAN ACTION

GENERATORS OF LOCAL (S/)SYMMETRIES

The LOCALLY-S/SYMMETRIC MODEL has INVARIANCE under (local) TIME-REPARAMETRIZATION $t \to f(t)$, with HAMILTONIAN GENERATOR (bosonic) $\mathcal{H}$.

And under 2 local s/symmetries, with FERMIONIC GENERATORS Q_1, Q_2.

Note POISSON BRACKETS, including $[Q_1, Q_2] \sim \mathcal{H}$, etc.

Such a system, with several gauge-like invariances, can be treated QUANTUM-MECHANICALLY by DIRAC's method for 'CONSTRAINED HAMILTONIAN SYSTEMS'.

Classically, all 'constraint generators' must vanish: $Q_1 = 0$, $Q_2 = 0$, $\mathcal{H} = 0$ at a classical solution.

In the QUANTUM THEORY, any PHYSICALLY-ALLOWED STATE ψ (coordinate variables) must obey the QUANTUM CONSTRAINTS

$$\hat{Q}_1 \psi = 0, \quad \hat{Q}_2 \psi = 0, \quad \hat{\mathcal{H}} \psi = 0.$$

In the ABOVE EXAMPLE, the quantum (anti-commutator) bracket relation

$\Rightarrow$ IT IS SUFFICIENT TO SOLVE ONLY THE (FERMIONIC) LOCAL-S/SY Q. CONSTRAINTS.

$\hat{Q}_1 \psi = 0$ and $\hat{Q}_2 \psi = 0$.

SIMILAR 'SEMI-CLASSICAL' Q. AMPLITUDES 66
FOUND FOR S/GRAVITY
and for GAUGE-INVARIANT S/GRAVITY
Might be PHYSICALLY REALISTIC, for some SU(n).

NOTE (COUNTER-INTUITIVE?):
ABOVE APPROACH RELIES ON INITIAL AND FINAL FIELD STATES

BUT WORKERS USING PARTICLE IN- AND OUT-STATES FIND INFINITIES IN FEYNMAN DIAGRAMS!

Possible Explanation: There may be NO UNITARY TRANSFORMATION relating the FIELD and PARTICLE 'BASES'.

COMPARE HAAG'S THEOREM (1955) IN QUANTUM FIELD THEORY —
— THIS CAN EASILY HAPPEN WHEN THERE ARE AN INFINITE No. OF DEGREES OF FREEDOM

QUANTUM THEORY
POSSIBLE IMPLICATIONS FOR BLACK HOLES

SERIES OF PAPERS DURING 2004–10.

For example:
A.N. St. J. FARLEY + P.D. D'EATH (2010)
'QUANTUM AMPLITUDES IN BLACK-HOLE EVAPORATION: COMPLEX APPROACH AND SPIN-0 AMPLITUDES'
(arXiv gr-qc/1002.3979)

ONE HAS QUANTUM STATES AND QUANTUM AMPLITUDES (NOT JUST PROBABILITIES)

FURTHER (AS ON p.2 ABOVE), THE QUANTUM PROBLEM OF THE DECAY OF A PLANCK-MASS BLACK HOLE WOULD BE REPLACED BY A CLASSICAL PROBLEM,

IF ONE COULD FIND SOLITONIC (PARTICLE-LIKE) CLASSICAL SOLUTIONS FOR GAUGE-INVARIANT SUPERGRAVITY / POSSIBLY CONTAINING NON-ZERO FERMIONIC PARTS, SO AS TO GIVE A SOLUTION OF THE FULL NON-LINEAR FIELD EQUATIONS

GIDDINGS + EARDLEY (2001/02)

In dimension $D = 4$, they essentially reproduced the results of R. PENROSE (1973/4, UNPUBLISHED).

for $D = 4$, they find a trapped surface, provided $\boxed{\text{Impact parameter} \leq b_{MAX} = 3.219\,\mu}$

[where μ = energy of each incoming particle in the CENTRE-OF-MASS frame.]

This gives a LOWER BOUND on the CROSS-SECTION:

$$\sigma_{BH\ production} \geq \pi (b_{MAX})^2 \simeq 32.6\,\mu^2$$

For dimension $D > 4$, Giddings + Eardley only studied the HEAD-ON CASE $b = 0$ IN DETAIL. (MODEST numerical difference)

Subsequent work, 2002 → present

Large number of papers, NECESSARILY 'MULTI-DIMENSIONAL'.