Towards a Mean Field dynamo in SPMHD

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Outlook

-Introduction to the mean field theory

-Introduction to SPMHD

-Testing of SPH implementation

-Steps towards Astrophysics (turbulence)

-Conclusions

Physical Motivation.....

$$\vec{V} = \langle \vec{V} \rangle + \vec{v}$$

-Hydrodynamics is complex
-Separation of the problem in parts, is helpful to our understanding

Mean Field Theory.....

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B} - \eta \nabla \times \vec{B})$$

$$\vec{B} = \langle \vec{B} \rangle + \vec{b} \qquad \vec{V} = \langle \vec{V} \rangle + \vec{v}$$

Statistical Properties

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B} - \eta \nabla \times \vec{B} + \vec{\xi})$$

$$\vec{\xi} = \alpha \vec{B} + \gamma \times \vec{B} - \beta \nabla \times \vec{B} - \delta \times \nabla \times \vec{B} + \ldots$$

$$\boxed{\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B} + \alpha \vec{B} - \beta \nabla \times \vec{B})}$$

Nice model and workhorse for theoreticians

Theory

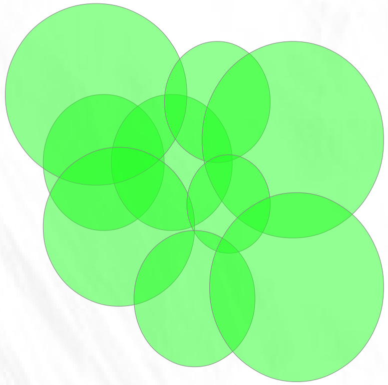
Non Linear Approach

(from B,V get alpha,beta, etc)

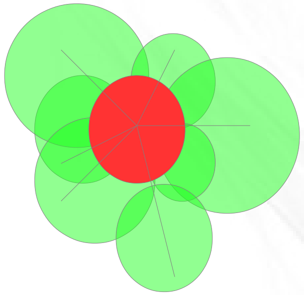
Kinematic approach

(from alpha, beta, etc get B,V)

SPMHD

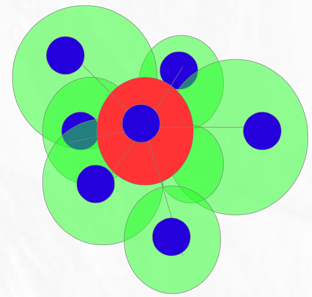


- Lagrangian Scheme
- The mass is discretized, not volume
- Huge dynamical range
- We use a compact Kernel to “smooth” the properties, and build derivatives
- Easy integration with gravity calculation



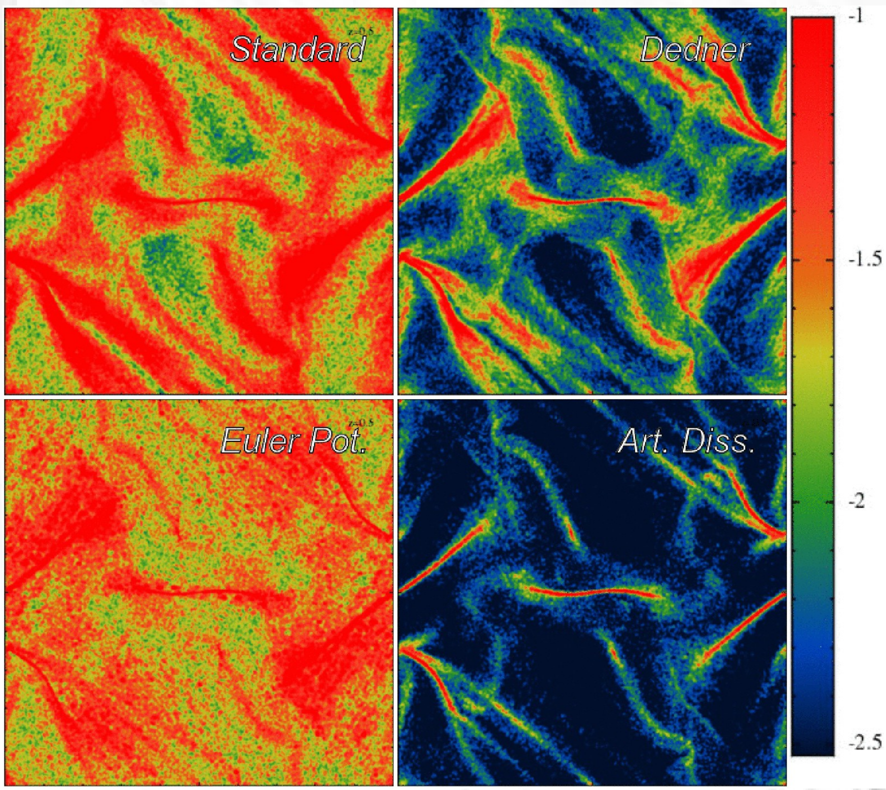
SPMHD

$$\nabla \cdot \vec{B} \approx 0$$



$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B})$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B}) - \nabla \vec{\Phi}$$



$$\frac{\partial \vec{\Phi}}{\partial t} = c_h \nabla \cdot \vec{B} + c_p \vec{\Phi}$$

$$\frac{\partial \vec{\Phi}}{\partial t} = c^2 \nabla \cdot \vec{B} + \frac{c}{\lambda} \vec{\Phi}$$

Stasyszyn et al. 2012
Dedner et al 2002

SPMHD

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\alpha \vec{B} - \beta \nabla \times \vec{B})$$

- Numerical Options:

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\alpha \vec{B} - \beta \nabla \times \vec{B})$$

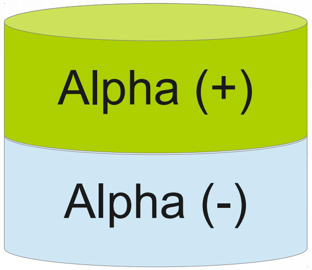
Or?

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\alpha \vec{B}) - \nabla \beta \times \nabla \times \vec{B} + \beta \nabla^2 \vec{B}$$

Therefore we have plenty of possible implementations, with their respective performance and different accuracies

SPMHD - Tests

- Meinel (1990)



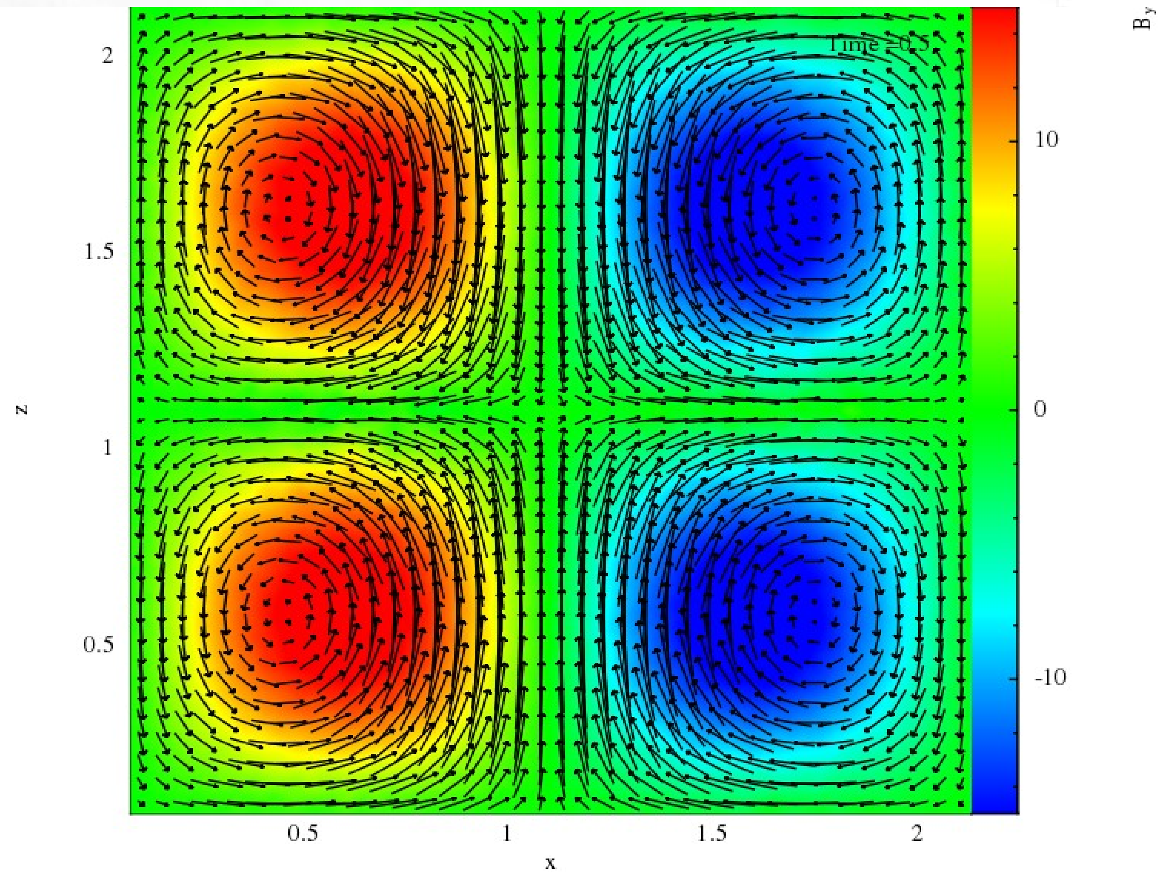
$$
\begin{aligned}
B_r &= -\frac{z'\, r'}{z\, r} J_1(r') \cos(z') \\
B_\phi &= -\beta\, \alpha_{crit} \frac{r'}{r} J_1(r') \sin(z') \\
B_z &= -\sin(z') \frac{r'}{r^2} \left[J_1(r') + r' \left(J_0(r) - J_2(r') \right) \right]
\end{aligned}
$$

Alpha Quenching $\alpha = \frac{\alpha_0}{B/B_{eq} + 1}$

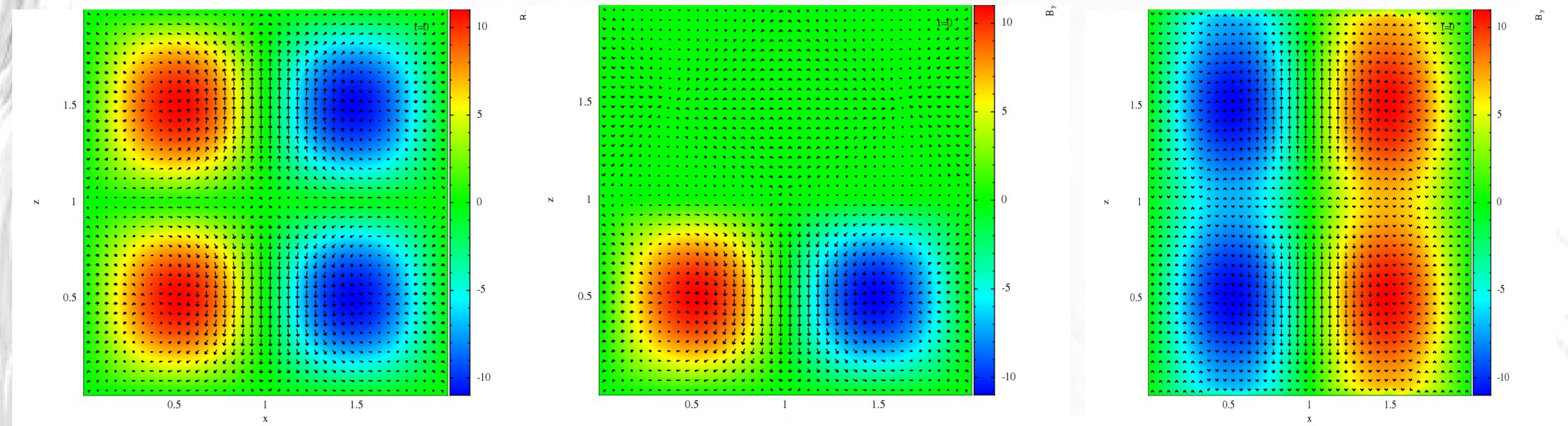
$$\alpha_{crit} = \left(\frac{\pi}{H} + \frac{x_1^1}{R_0} \right)^{1/2}$$

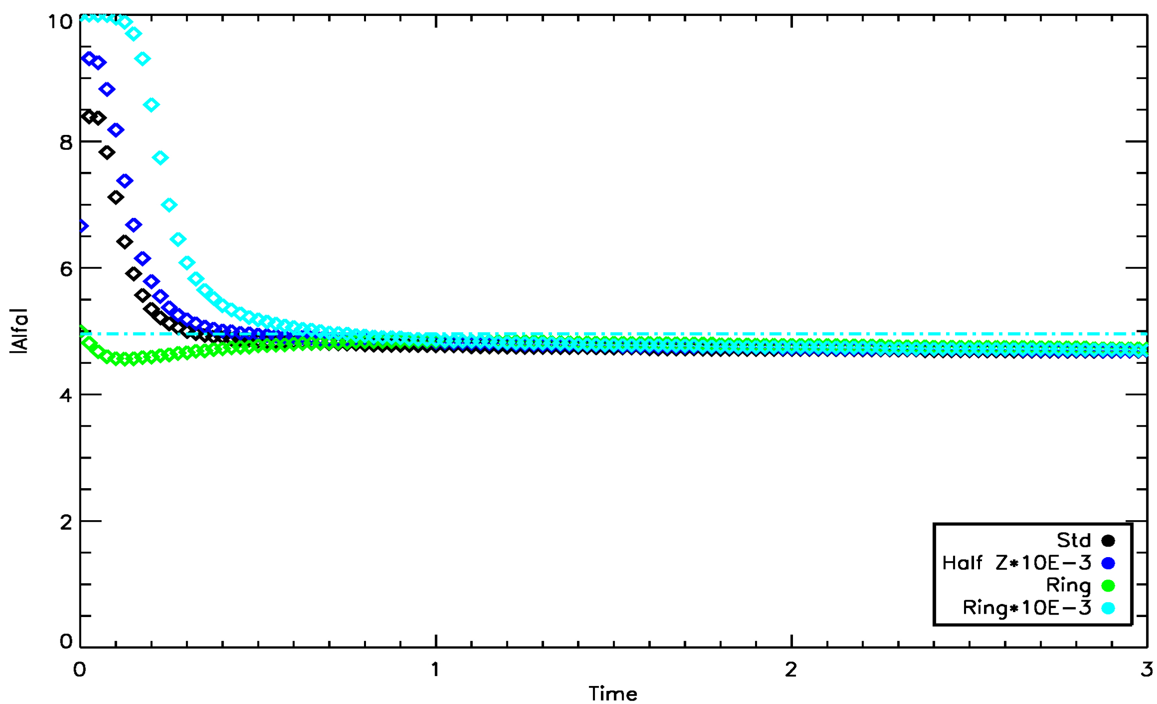
SPMHD - Tests

- Meinel (1990)

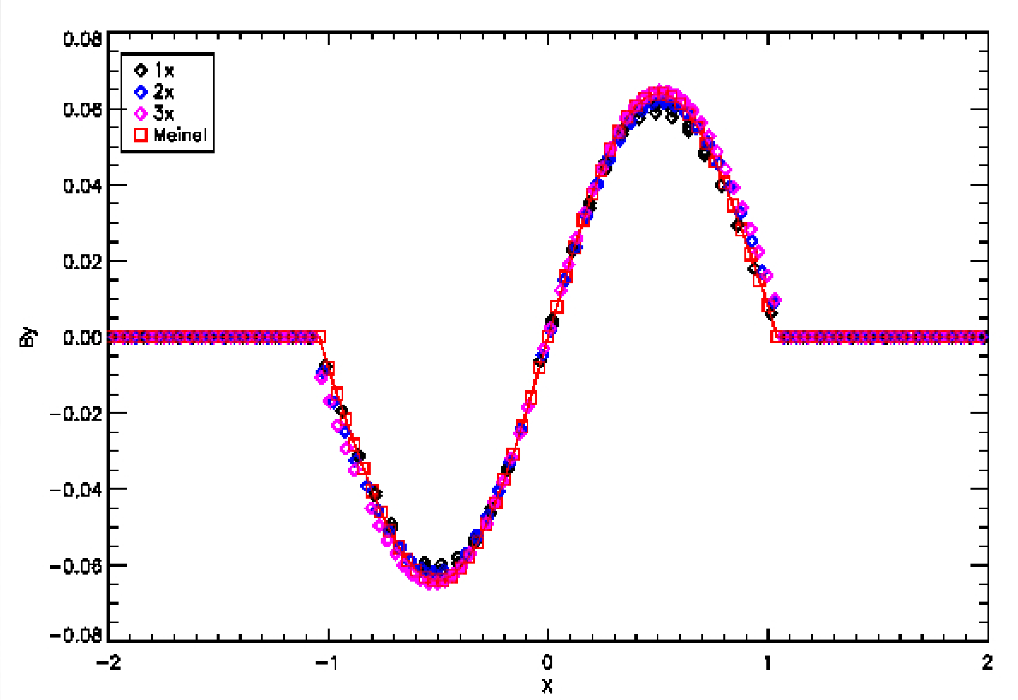


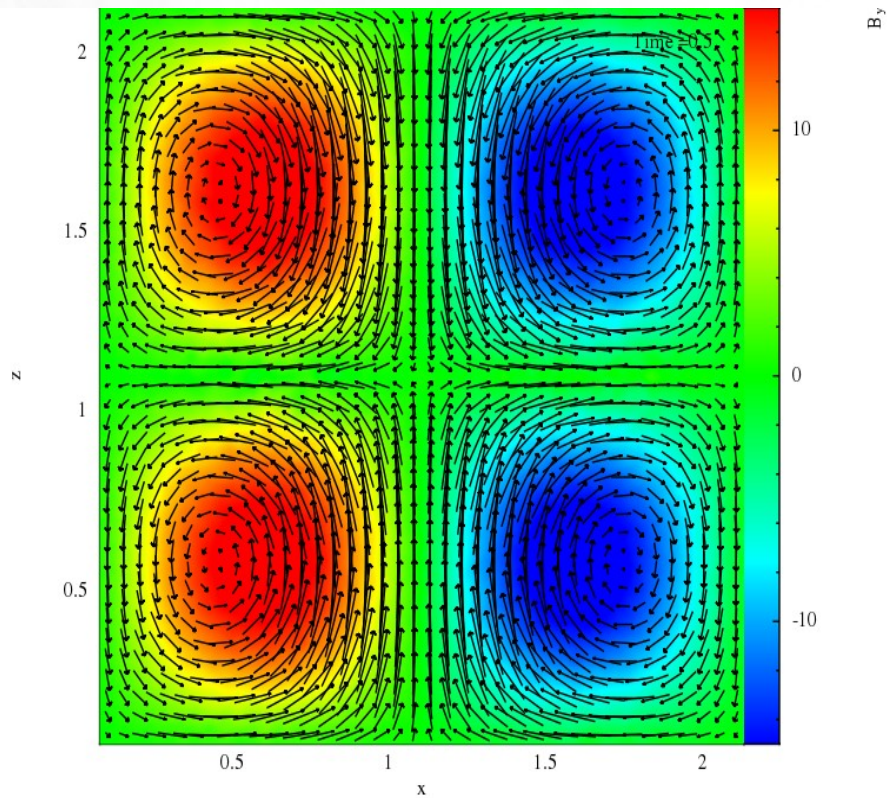
SPMHD - Tests





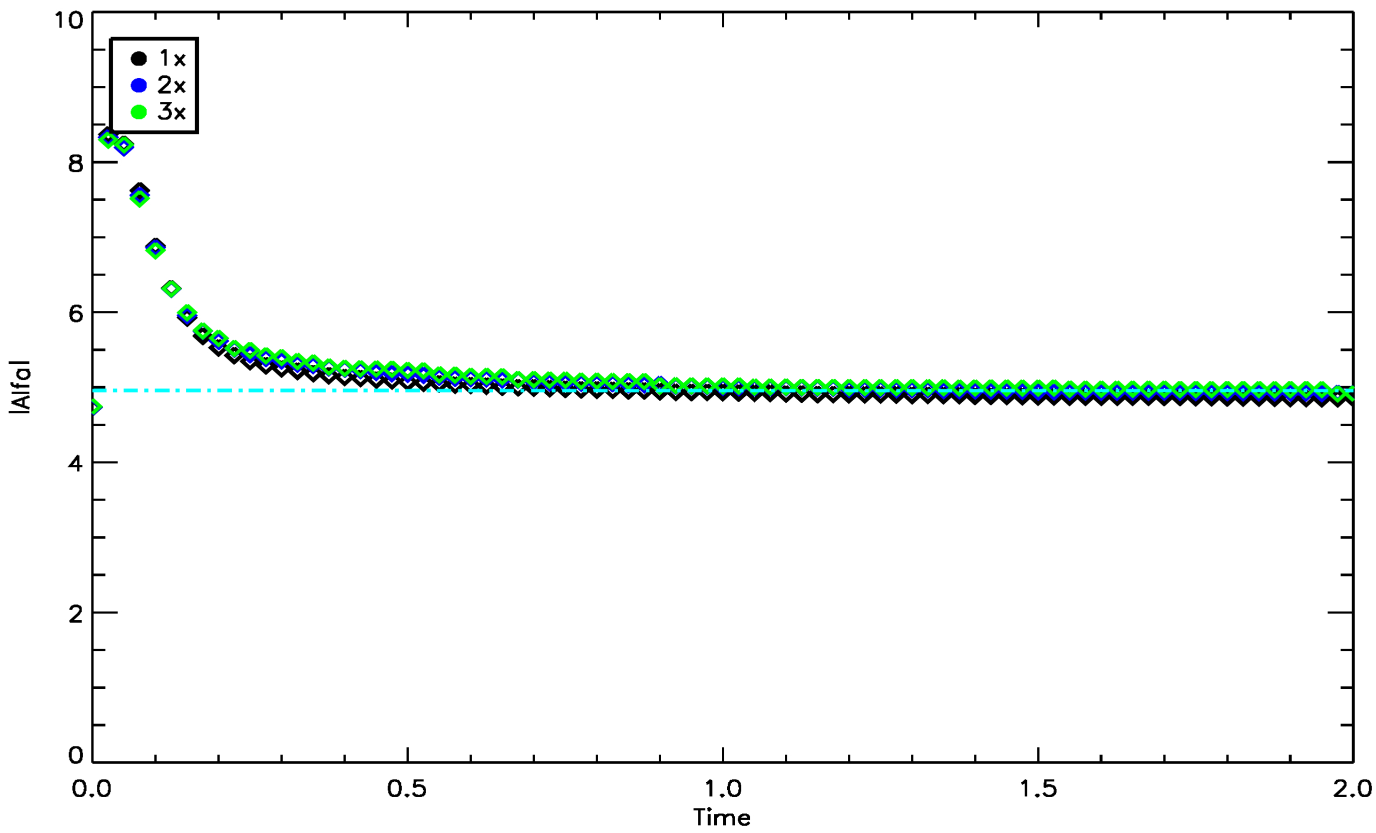
SPMHD - Tests





-Resolution convergence, towards the critical values.

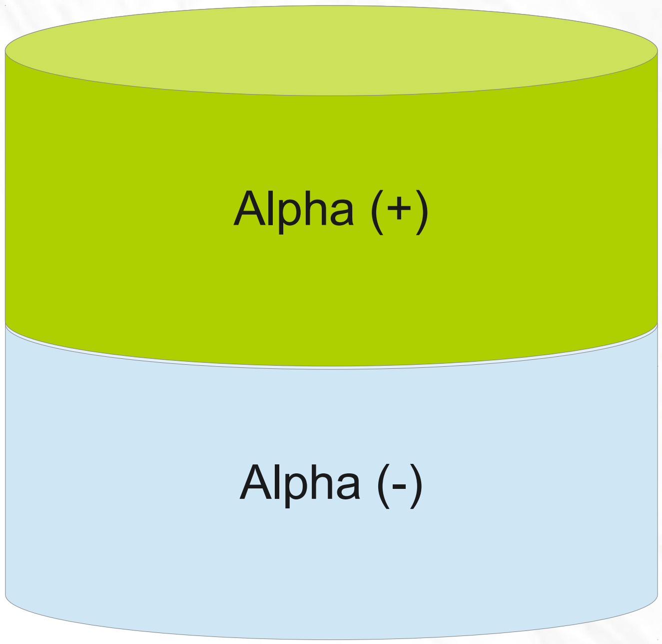
SPMHD - Tests



-Slow resolution convergence towards the critical values. (SPH inherent smoothing)

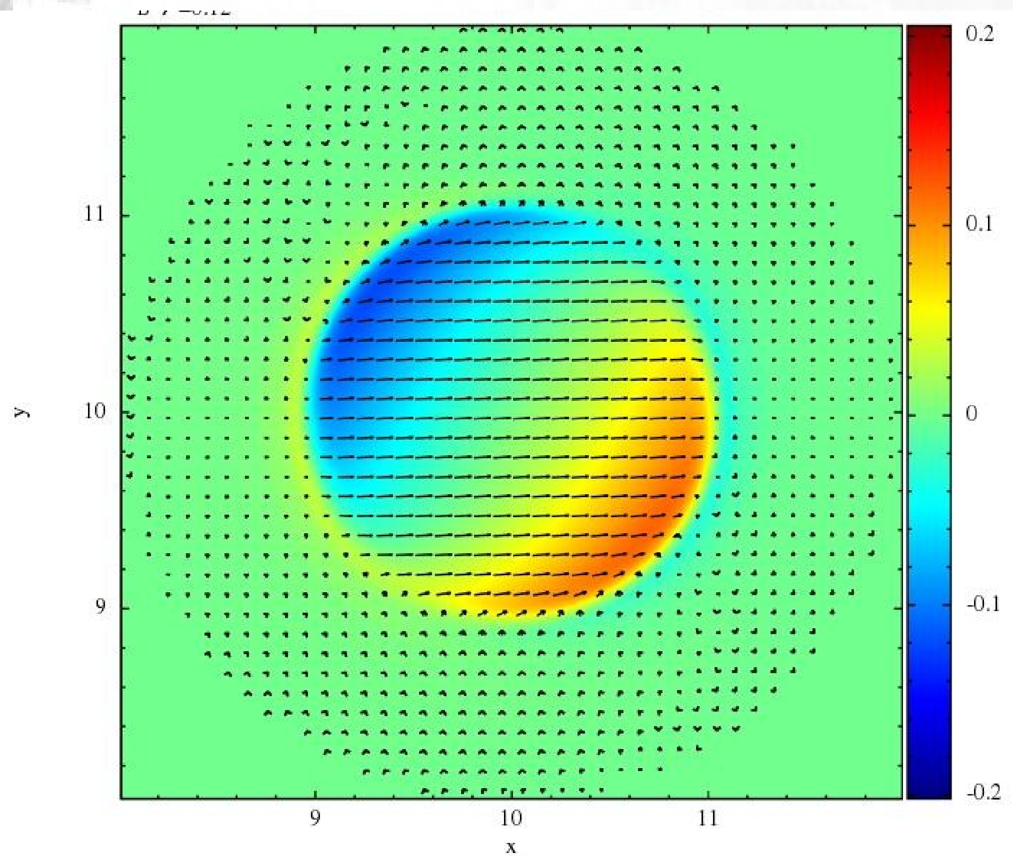
SPMHD - Tests

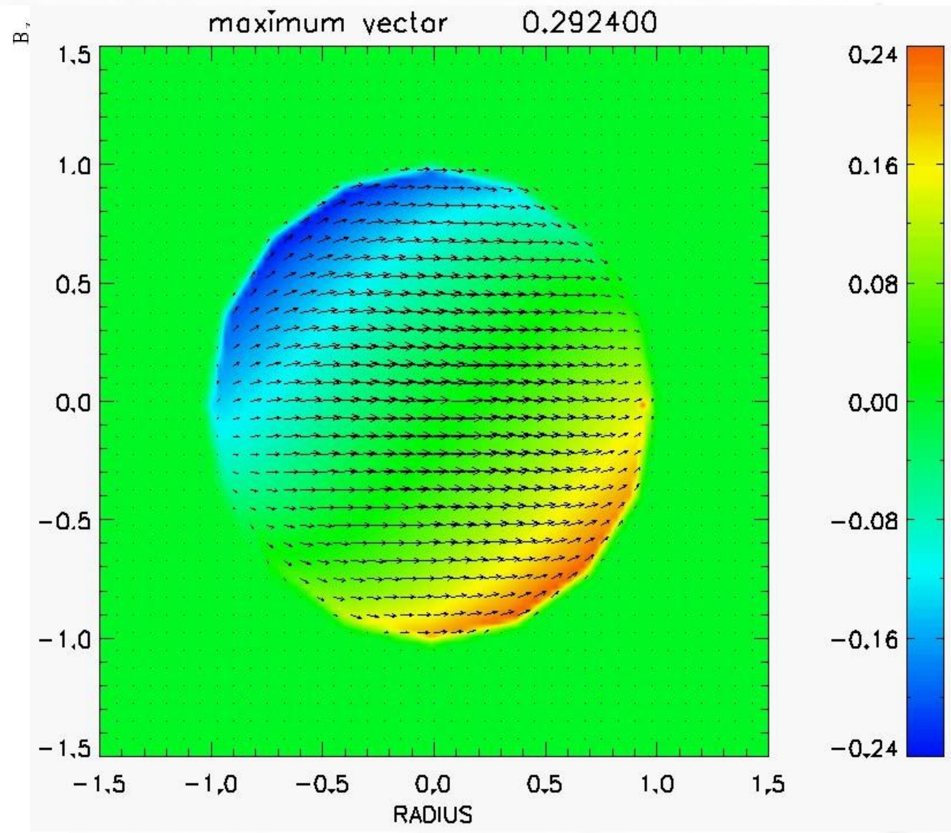
- Anisotropic α



$$\vec{\alpha}=[\alpha,\alpha,0]$$

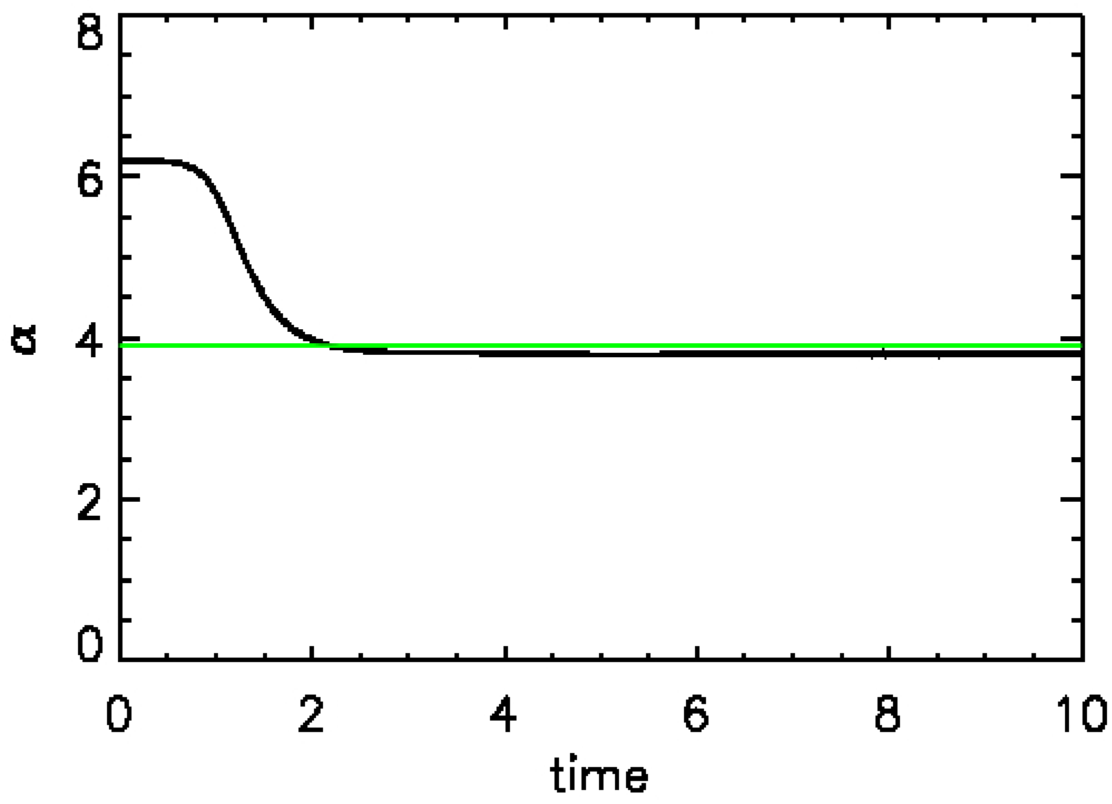
SPMHD - Tests





(Video)

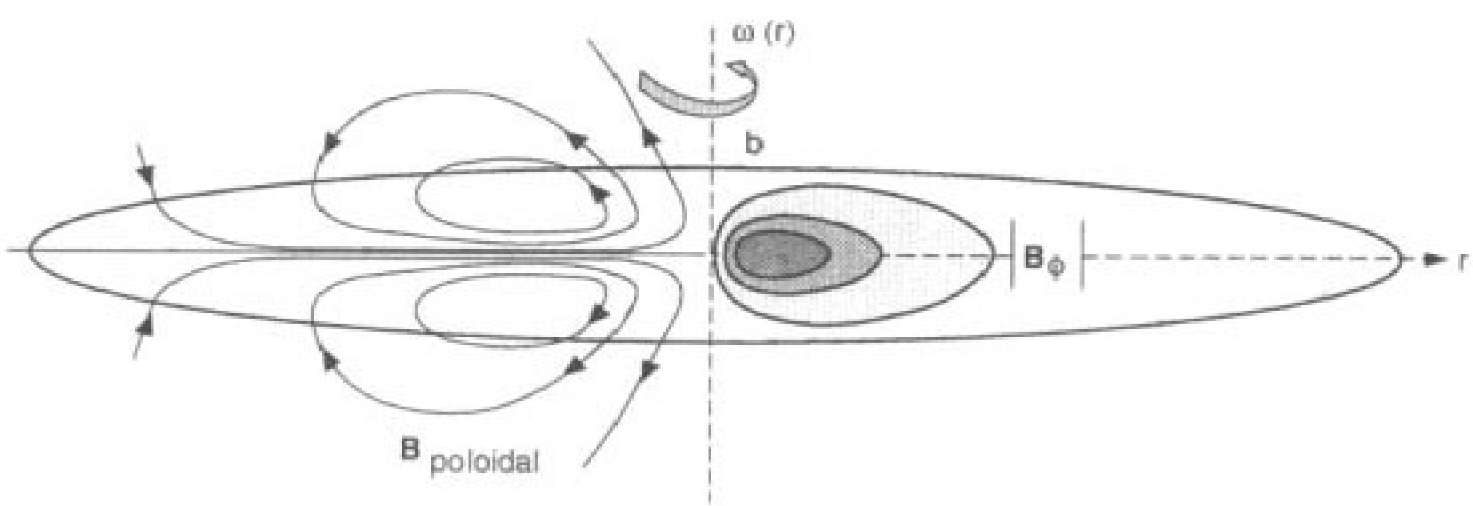
SPMHD - Tests

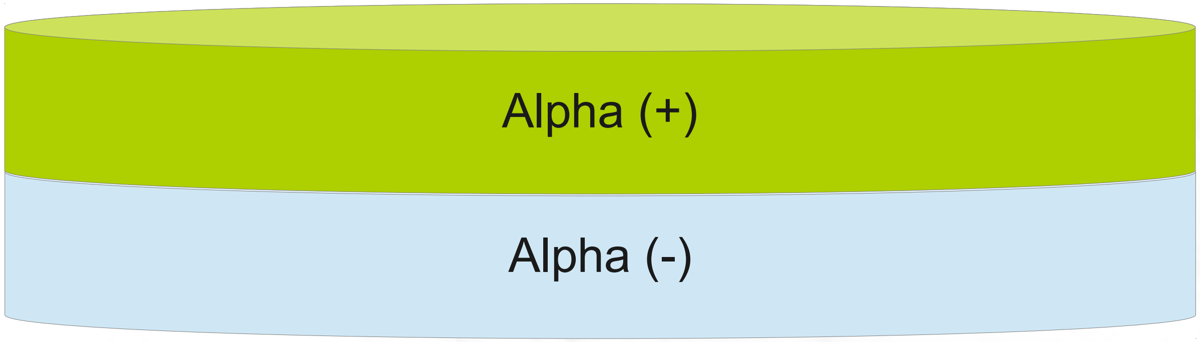


Same Bias as previous tests

SPMHD - Tests

- Alpha-Omega
  - Mean field equations + Differential rotation

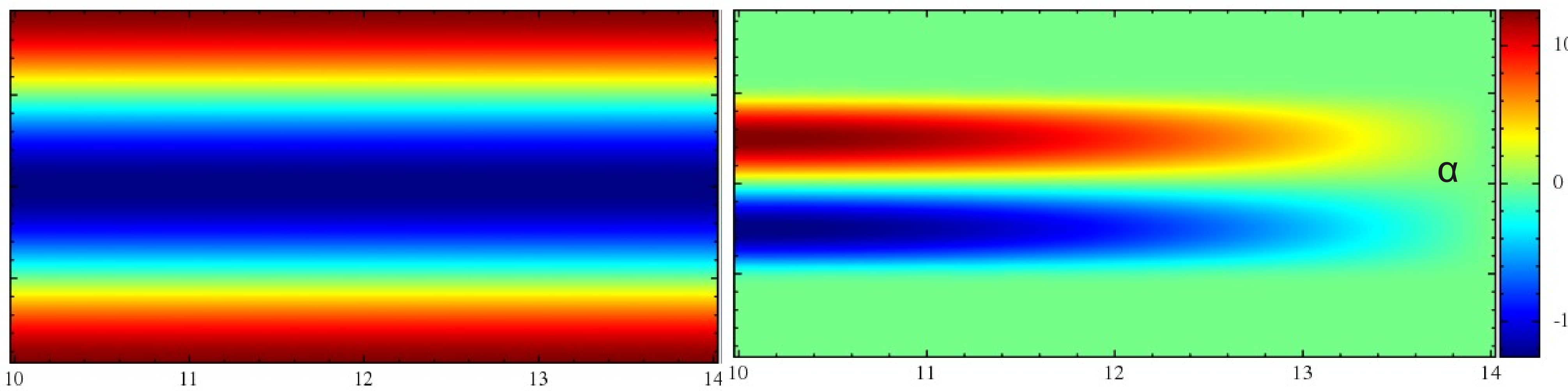


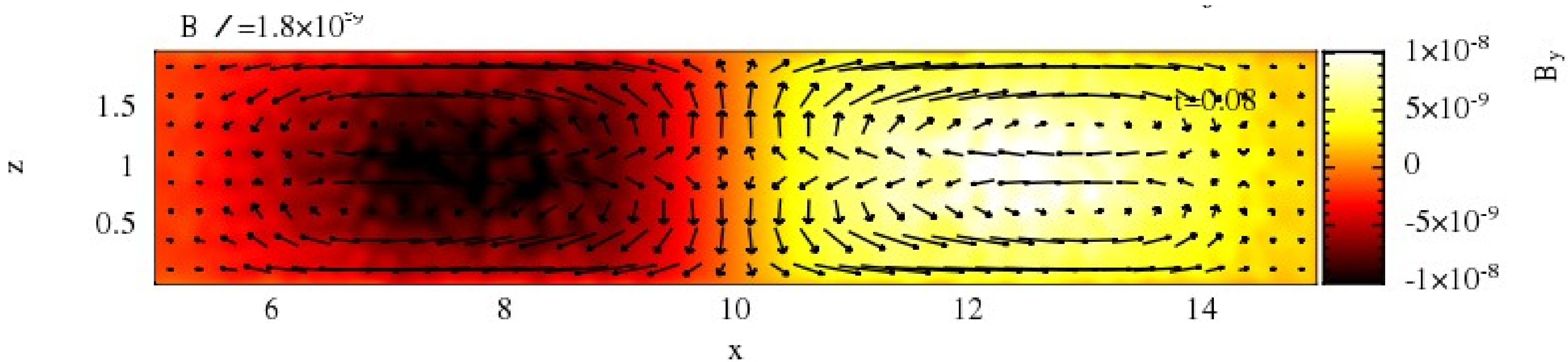


SPMHD - Tests

- Alpha-Omega

η

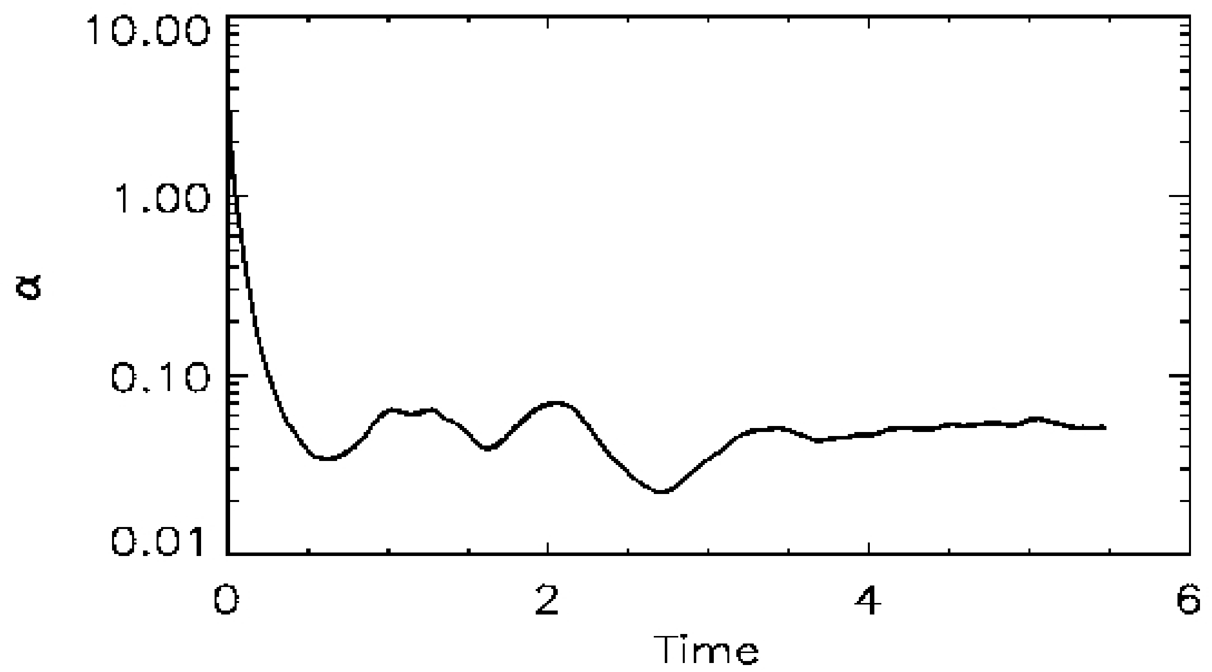




Differential rotation: Brandt Law

SPMHD - Tests

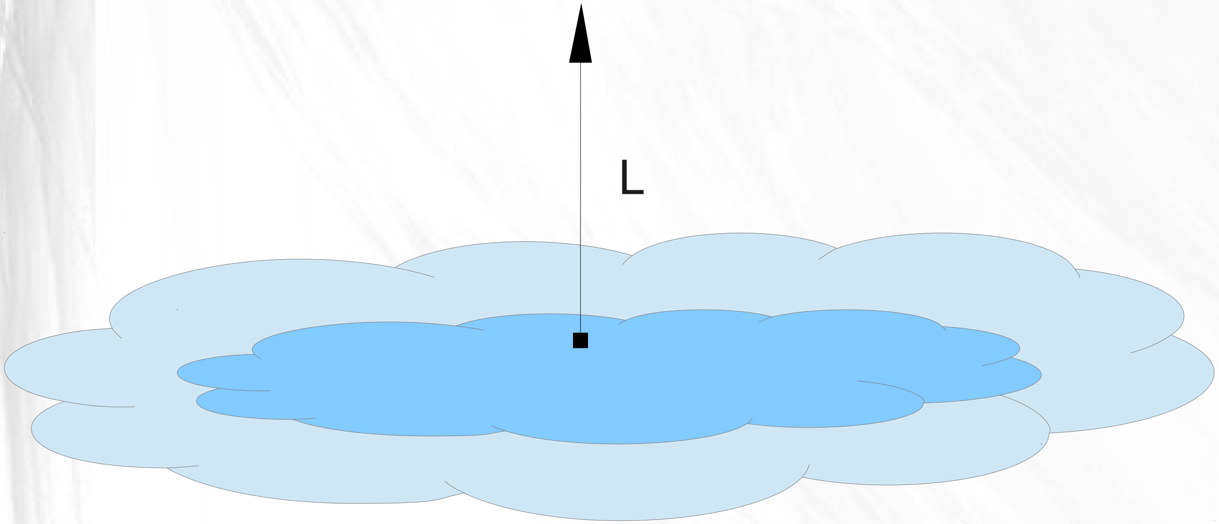
- Alpha Omega



- α evolution not stable
- Critical α, lower than expected.

SPMHD – Proof of Concept

Galaxy Model

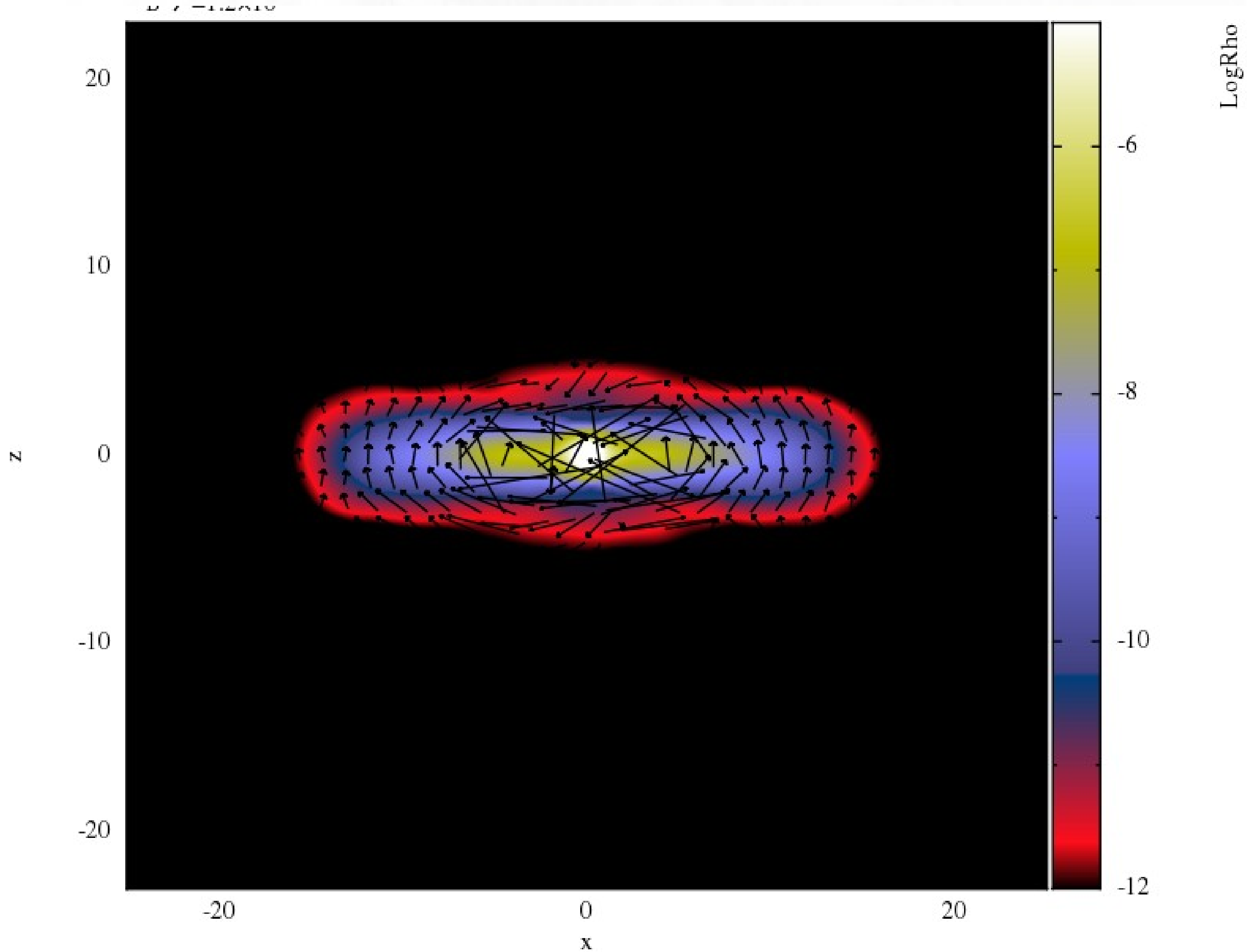


$$\beta \approx \frac{1}{\rho}$$

$$\alpha \approx \pm\rho$$

Sign given by angular momentum

SPMHD – Proof of concept



Important contribution from the IGM, otherwise the B field is truncated.
(wrong boundary conditions) and B field distribution.

Mean Field SPMHD

- Status
  - α and β terms implemented (in tensor form)
  - α^2-dynamo tested
  - α-ω dynamo in testing face
  - Critical α, in general smaller than expected
  - Simple galaxy models developed

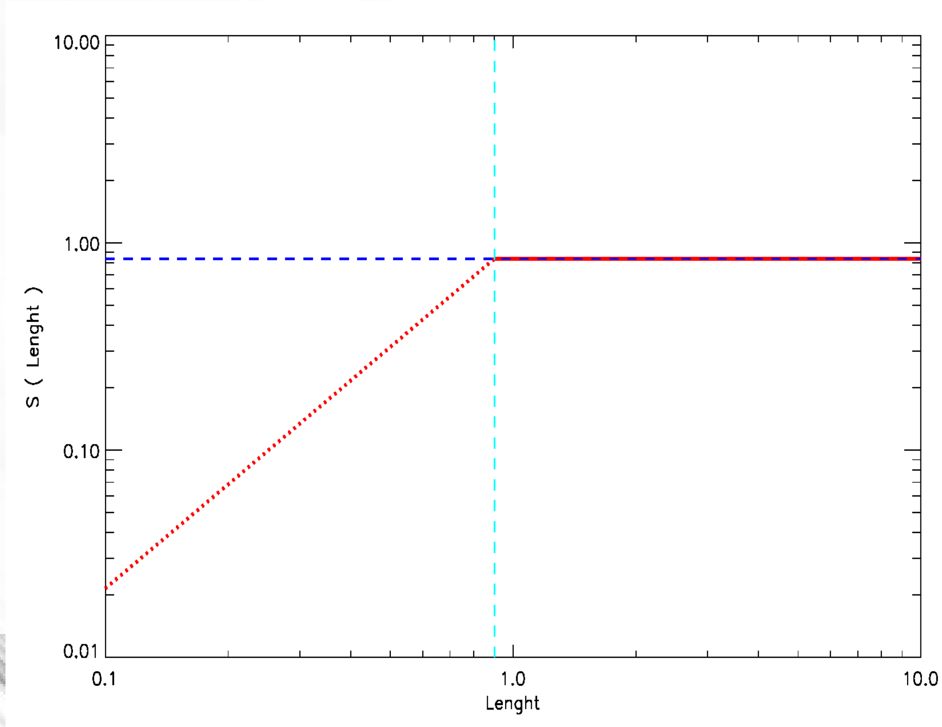
Tests caveats:

  - SPH densities – energies inhomogeneities
  - Long term stability and convergence

Pause: Turbulence

- Turbulence: statistical nature
- Book definitions:
  - Correlation functions
  - Structure functions
- Actual definitions:
  - <RMS>
  - <RMS> Multiscale

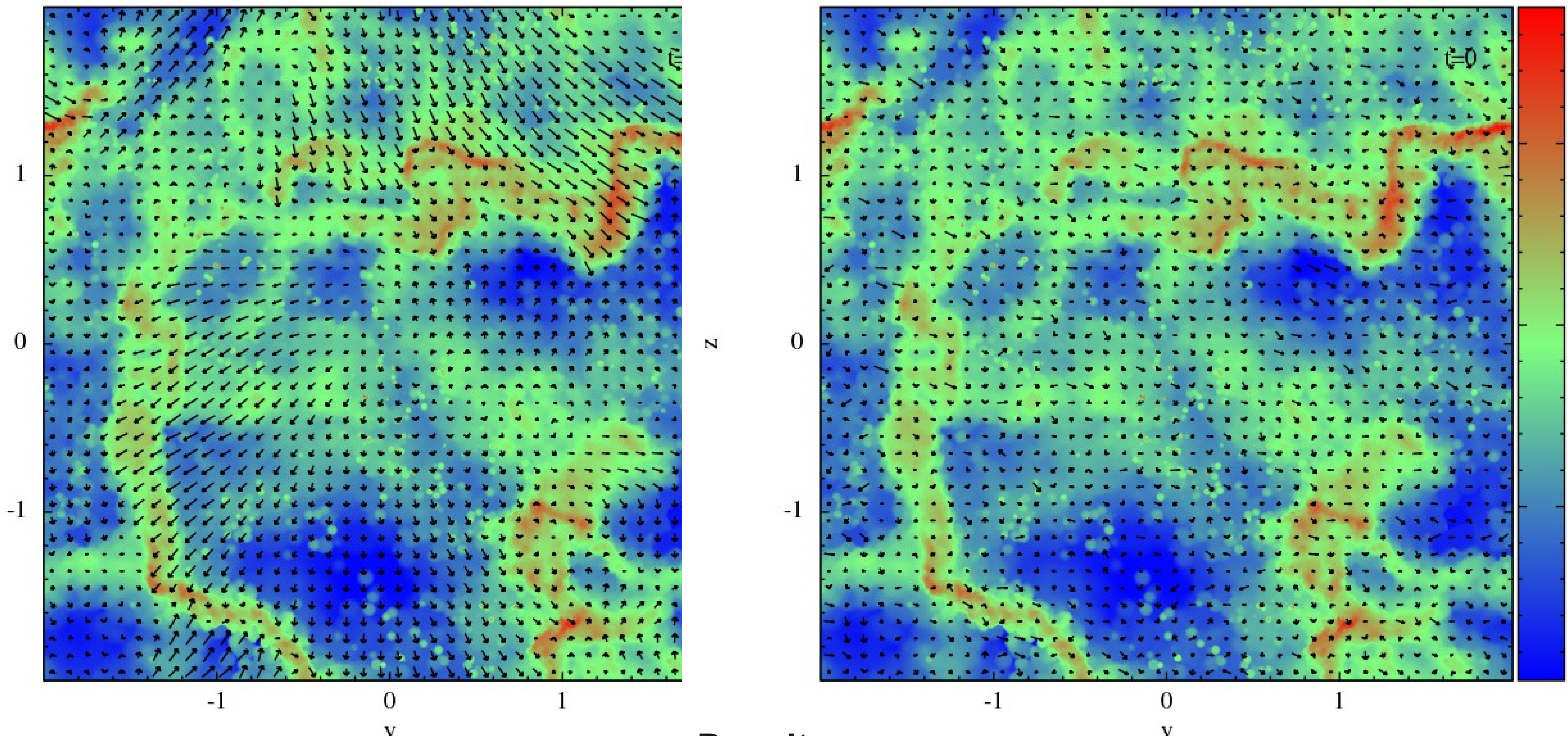
$$S(r)=\langle (f(x-r)-f(x))^2\rangle$$



Stasyszyn 2012, Vazza 2012, Kotarba 2011

Impasse – Turbulence - SF

Turbulent **Random**



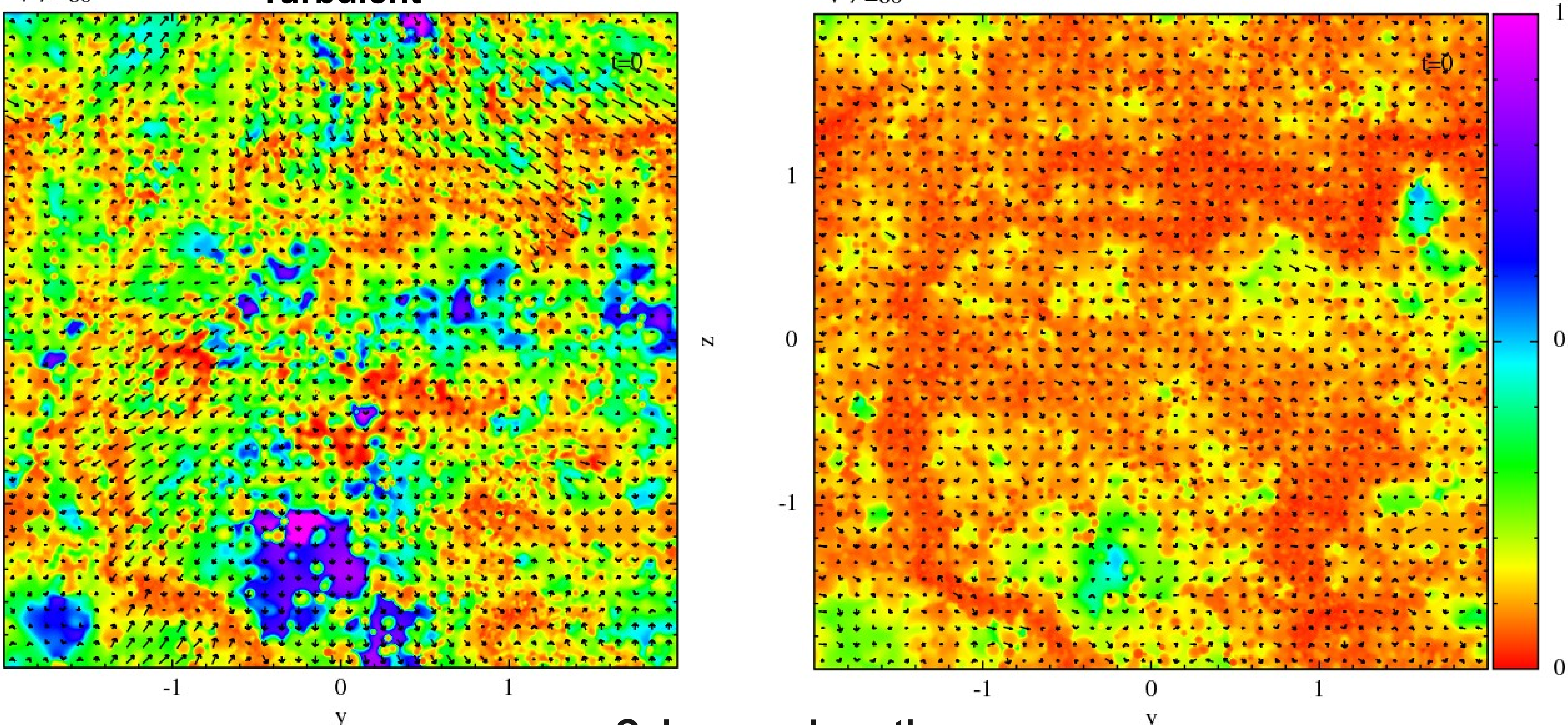
Density

-Turbulent box: 4 pc^3, ~256^3 – initial Kolmogorov spectrum

Gritschneder 2009

Impasse – Turbulence - SF

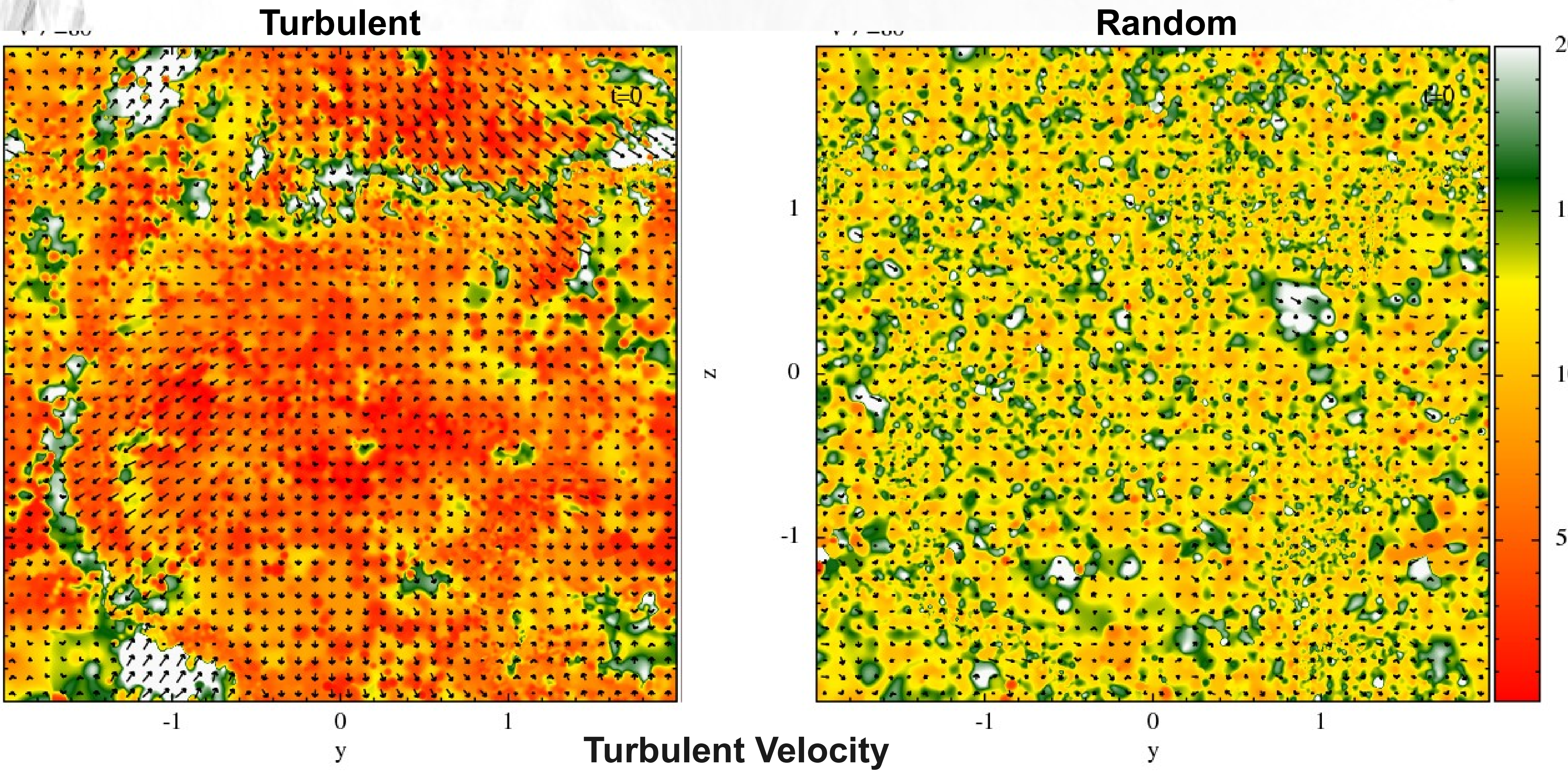
Turbulent **Random**



Coherence Length

Random fields, have an average S***mall*** coherence lengths.

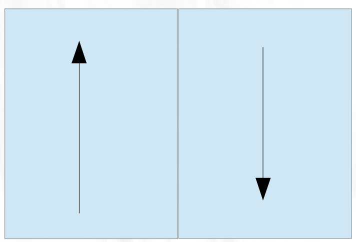
Impasse – Turbulence - SF



Random fields, have an average ***Higher*** turbulent velocity.

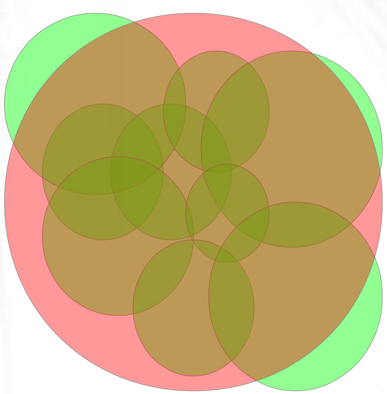
Impasse – Turbulence - SF

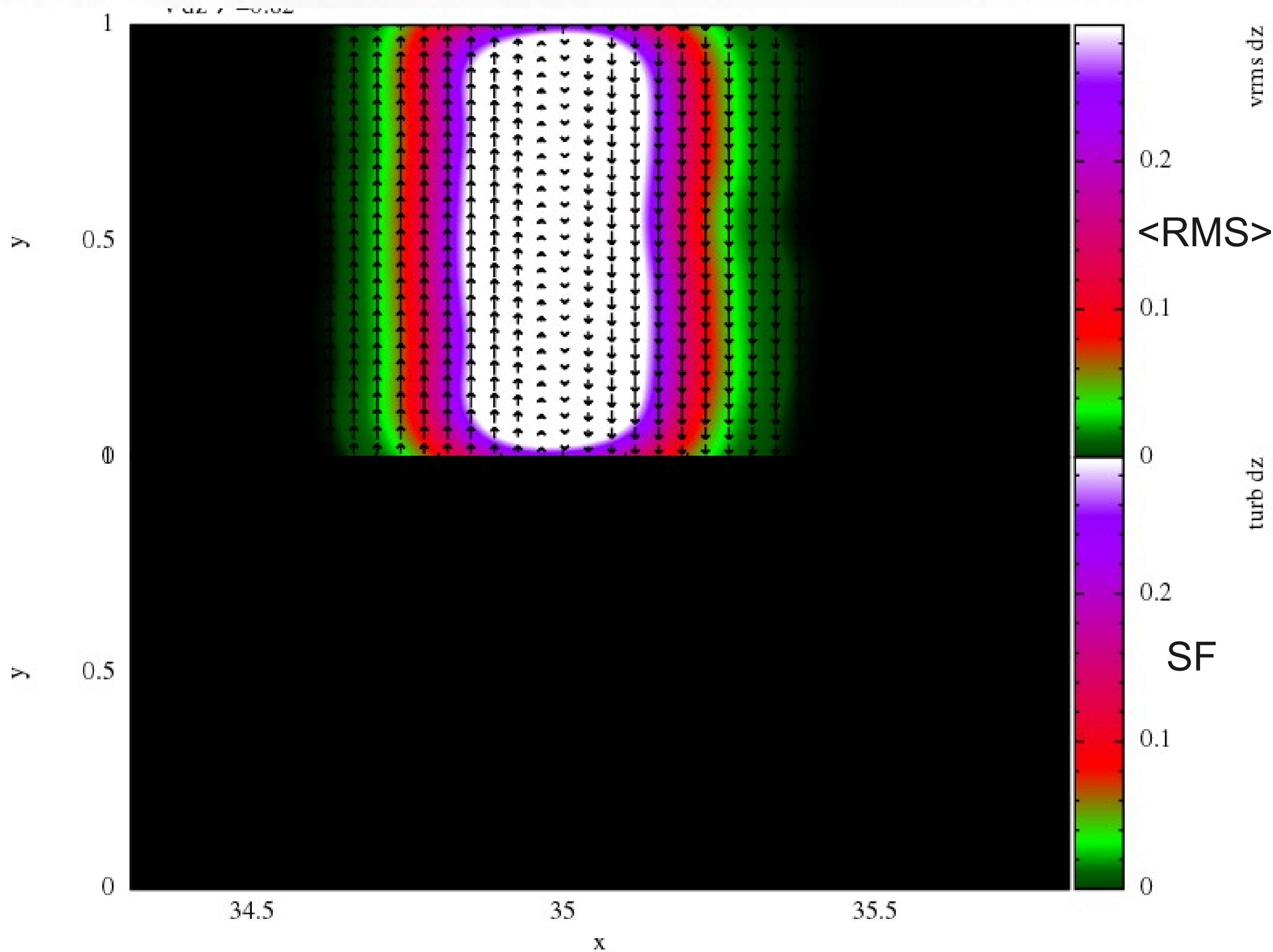
Shear



|V| = 0.5

Box Size: 1x1x20

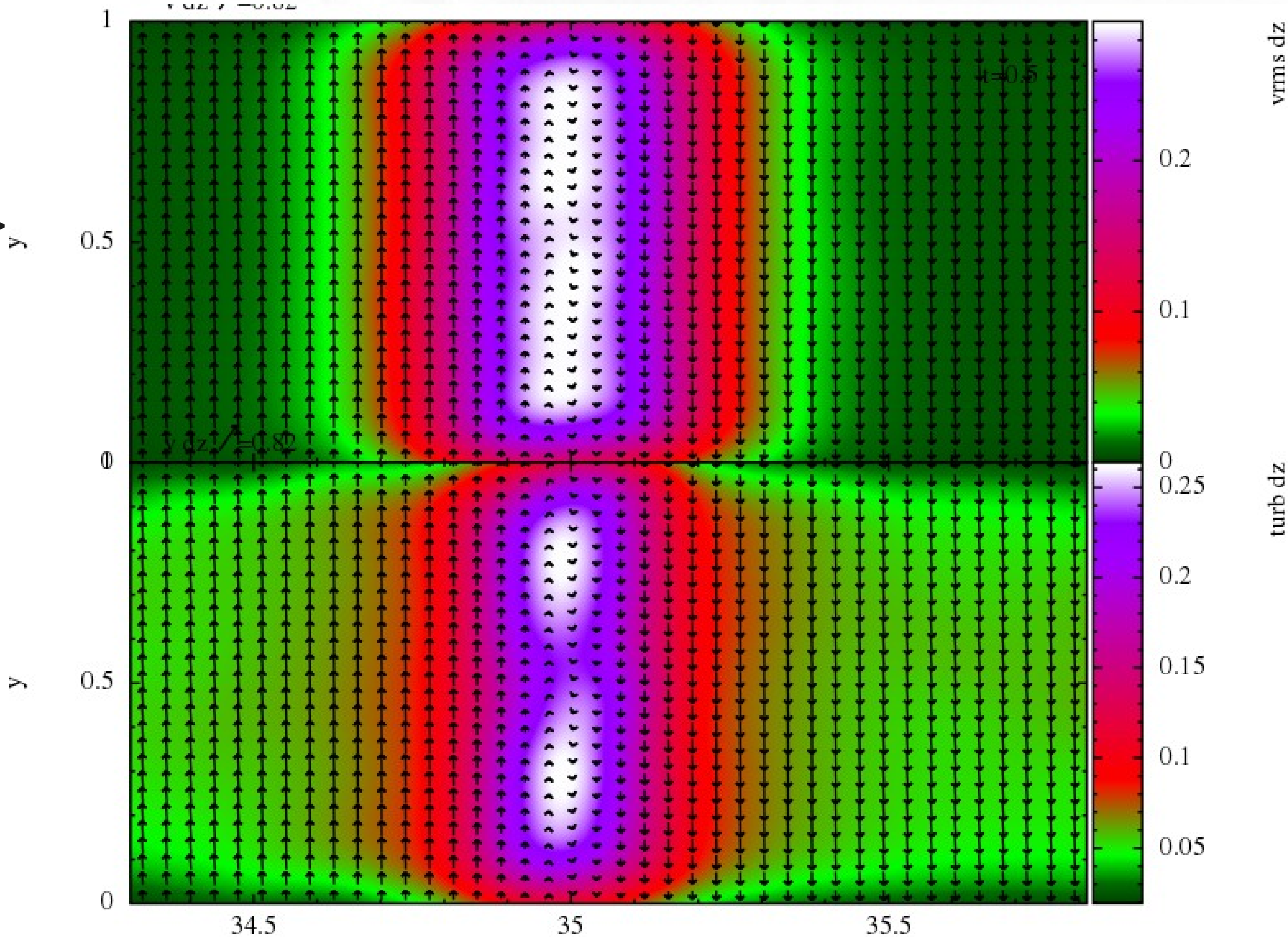




Impasse – Turbulence - SF

<RMS>

SF

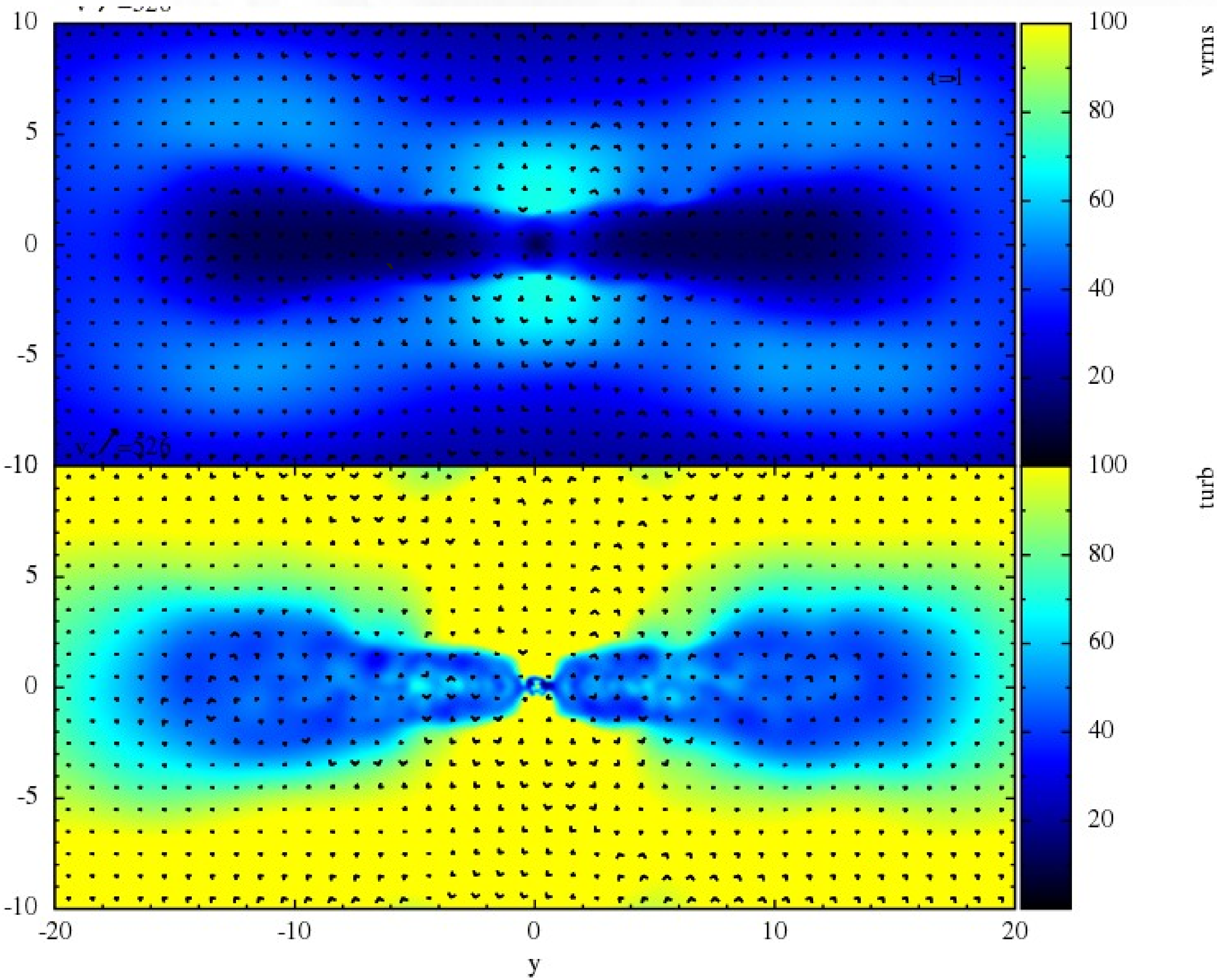


Tub. Velocity expected (from Full K-H instability)~ 0.25, close to measured

Impasse – Turbulence - SF

<RMS>

SF



Disk Galaxy: 3 component + IGM – SF consistent with predictions

Impasse – Turbulence - SF

Conclusions

- Structure Function estimator gives turbulent Velocity and coherence length
- We can identify random field from turbulent fields
- No influence of shear cases in laminar flows
- Values measured are consistent with theory
- No overhead in calculation (inside Gravity)

- Status and Prospects
  - Implemented the mean field equations in SPMHD
  - Consistence checks with other solutions (analytically or/and numerically)
  - Concept proof of Disk Galaxy initial conditions
  - New estimator for Turbulent components

  - Understand the differences between tests
  - Detailed studies of the different α-ω cases
  - Develop more realistic galaxy models (including data from HR simulations, star formation, etc)
  - Synergy between other projects (Cosmic rays, Observables, Galaxy Clusters, etc).