Diffusive Shock Acceleration and magnetic fields in SNR

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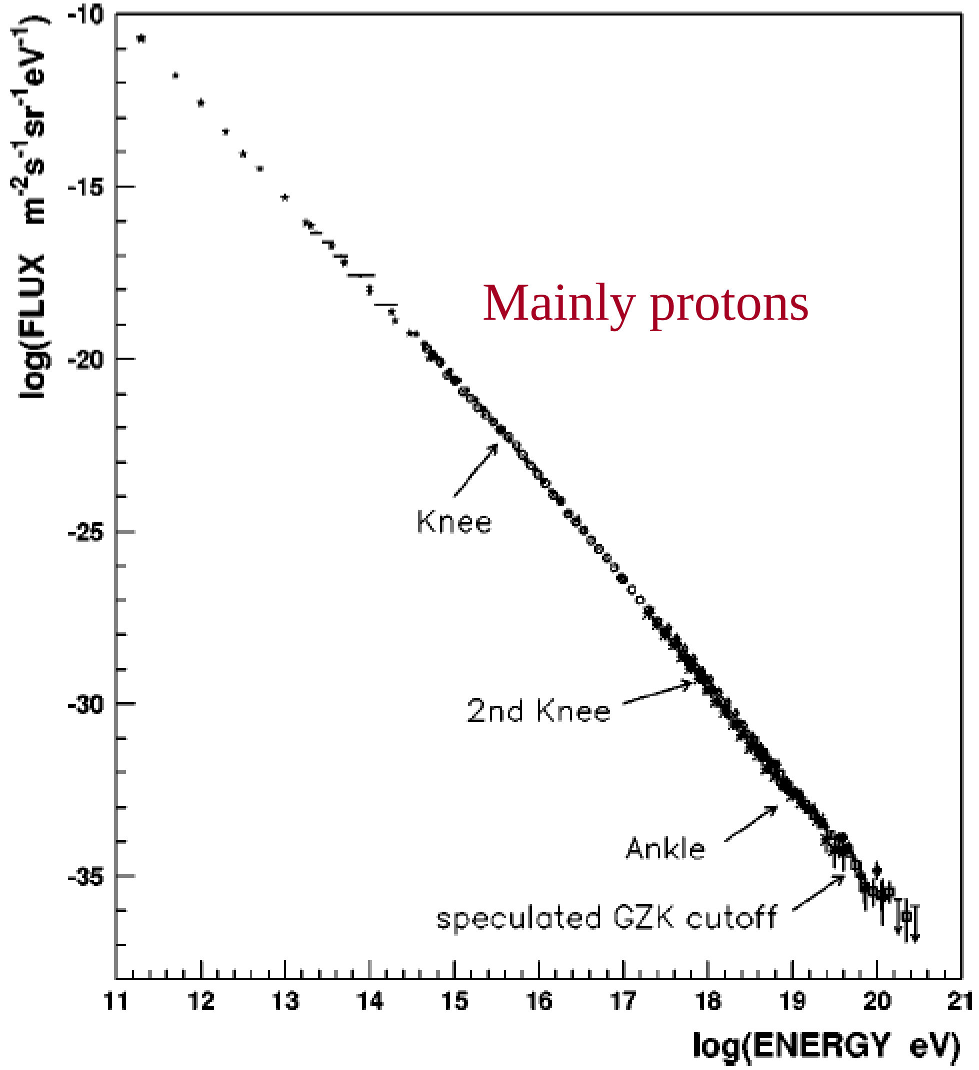
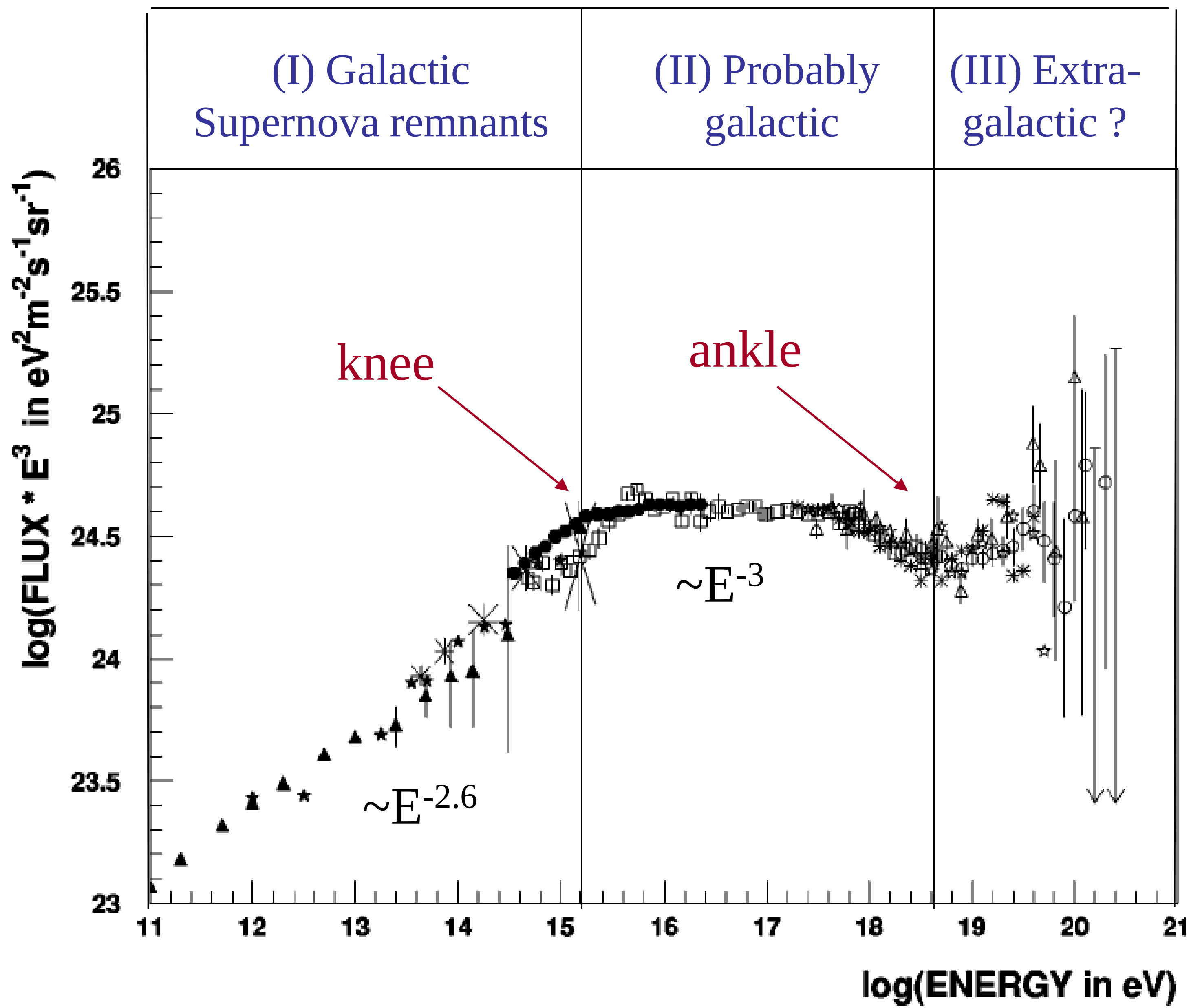
SN1006: A supernova remnant 7,000 light years from Earth
X-ray (blue): NASA/CXC/Rutgers/G.Cassam-Chenai, J.Hughes et al; Radio (red): NRAO/AUI/GBT/VLA/Dyer, Maddalena & Cornwell; Optical (yellow/orange): Middlebury College/F.Winkler. NOAO/AURA/NSF/CTIO Schmidt & DSS

Cosmic ray spectrum arriving at earth

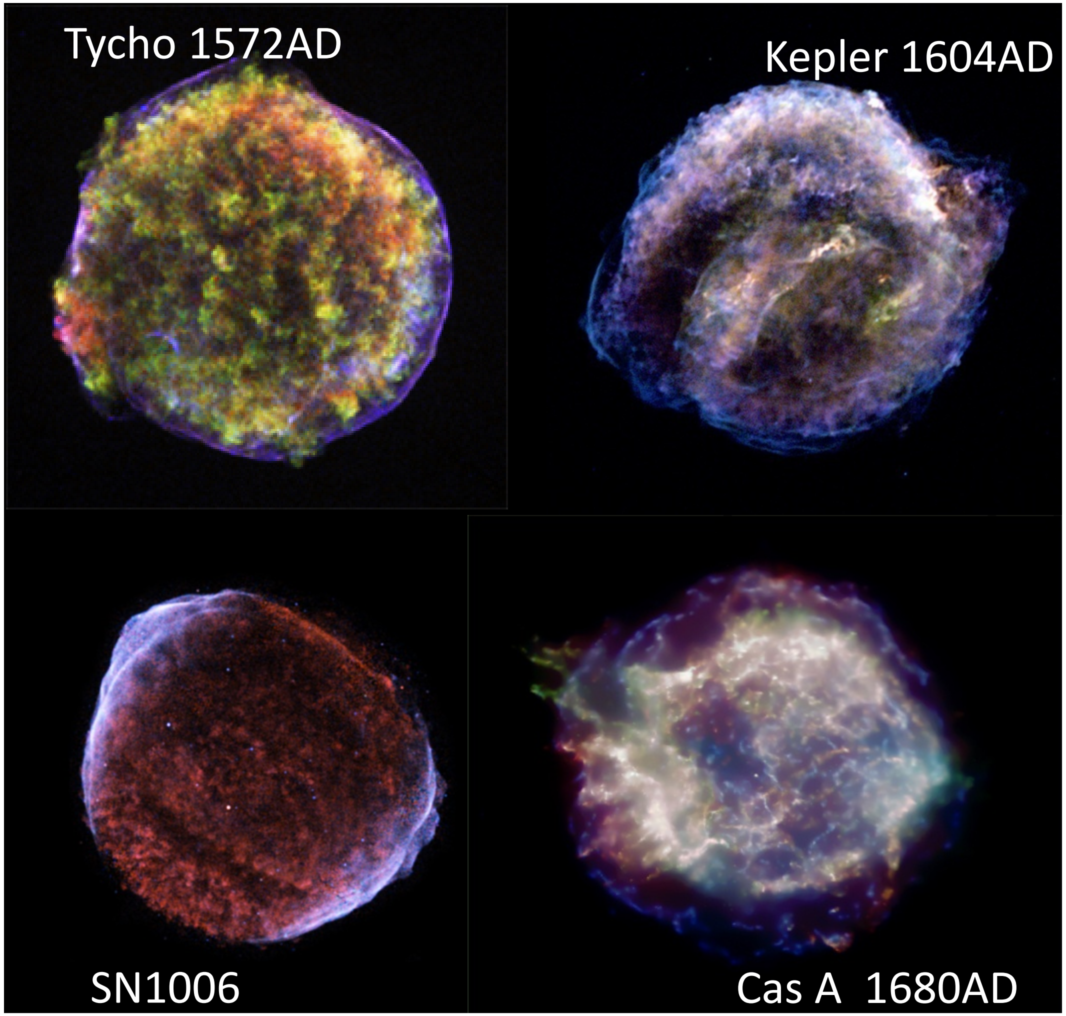


CR populations



Why shocks?

Historical shell supernova remnants



Chandra observations

NASA/CXC/Rutgers/ J.Hughes et al.

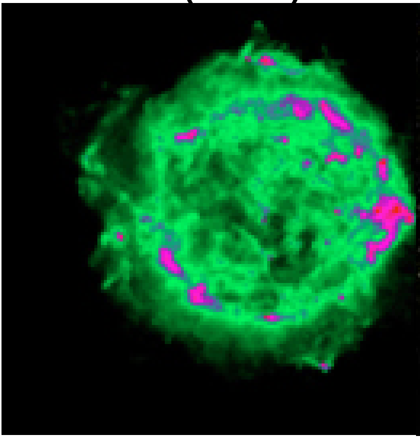
NASA/CXC/Rutgers/ J.Warren & J.Hughes et al.

NASA/CXC/NCSU/ S.Reynolds et al.

NASA/CXC/MIT/UMass Amherst/ M.D.Stage et al.

Cassiopeia A

Radio
(VLA)



chandra.harvard.edu/photo/
0237/0237_radio.jpg

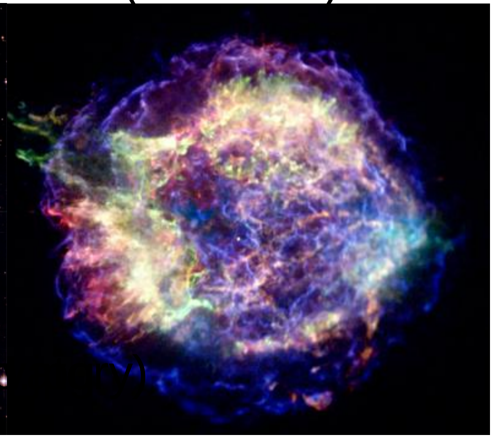
Infrared
(Spitzer)

NASA/JPL-Caltech/
O Krause(Steward Obs)

Optical
(Hubble)

NASA/ESA/
Hubble Heritage
(STScI/AURA))

X-ray
(Chandra)



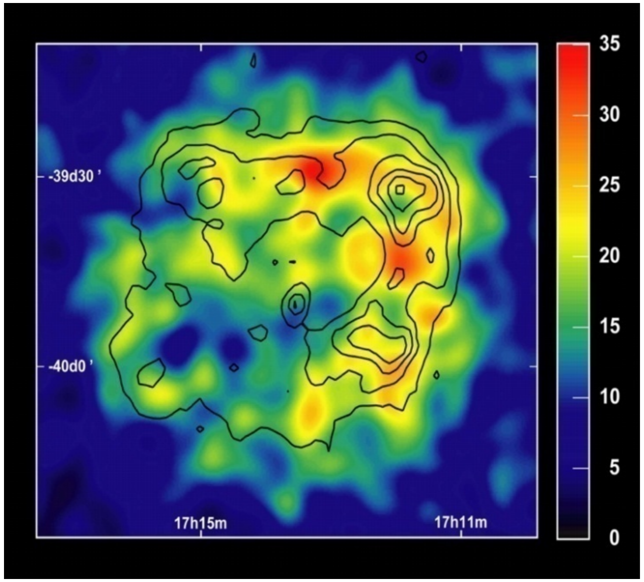
NASA/CXC/MIT/UMass Amherst/
M.D.Stage et al.

Mixture of line radiation
& synchrotron continuum

Synchrotron in magnetic field ~ 0.1-1mG
Radio ($h\nu \sim 10^{-5}$eV): electron energy ~1 GeV
X-ray ($h\nu \sim 10^{3}$eV): electron energy ~ 10 TeV

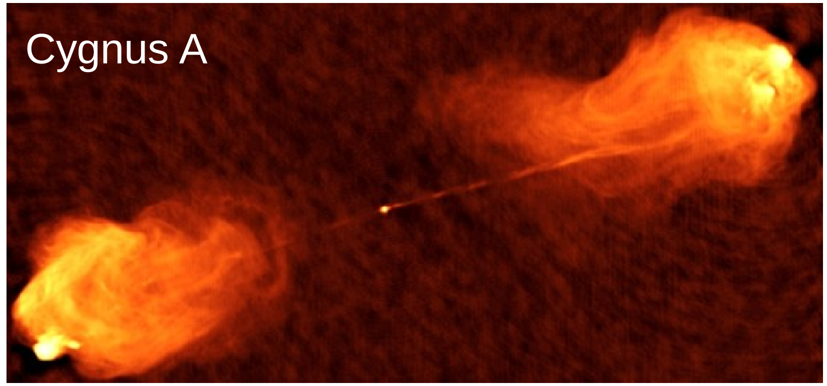
HESS: γ-rays directly produced by TeV particles

SNR RX J1713.7-3946



Aharonian et al
Nature (2004)

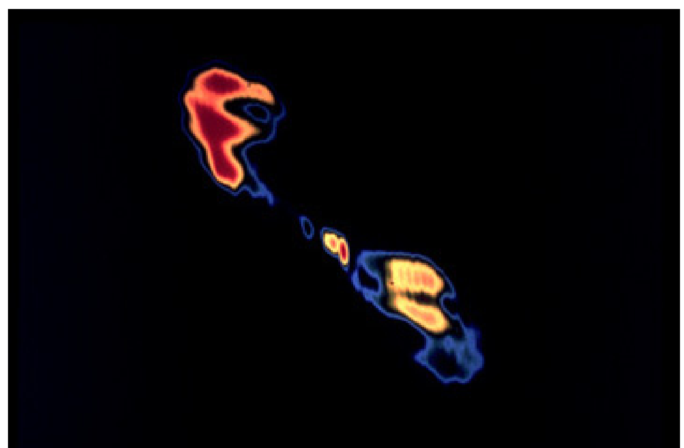
Active galaxies



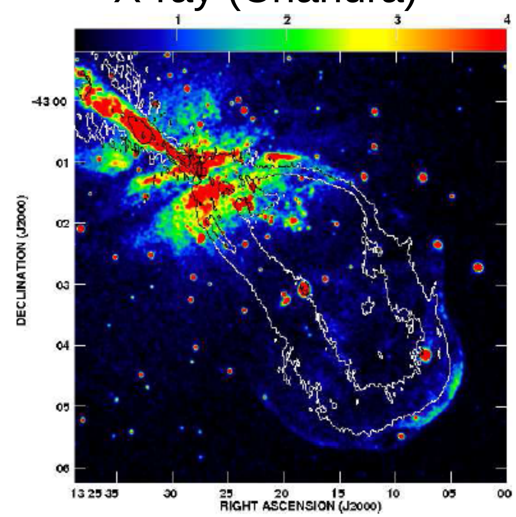
Centaurus A is the closest powerful radio galaxy (5Mpc)

optical

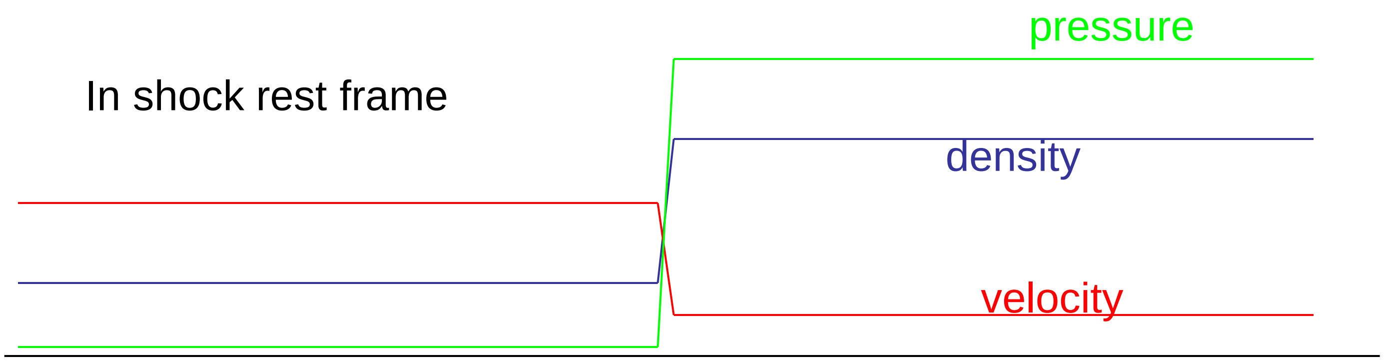
Radio jets



X-ray (Chandra)



Strong shock: high Mach number



Upstream	Downstream
$u = u_{shock}$ $\quad \rho = \rho_0$	$u = \frac{1}{4} u_{shock}$ $\quad \rho = 4\rho_0$
$kT = 0$ $\quad P = 0$	$kT = \frac{3}{16}\left(\frac{\overline{A}}{1+\overline{Z}}\right) m_p u_{shock}^2$ $\quad P = \frac{3}{4}\rho u_{shock}^2$

Conserved across shock (Rankine Hugoniot relations)

Mass flux $\quad \rho u$

Momentum flux $\quad P + \rho u^2$

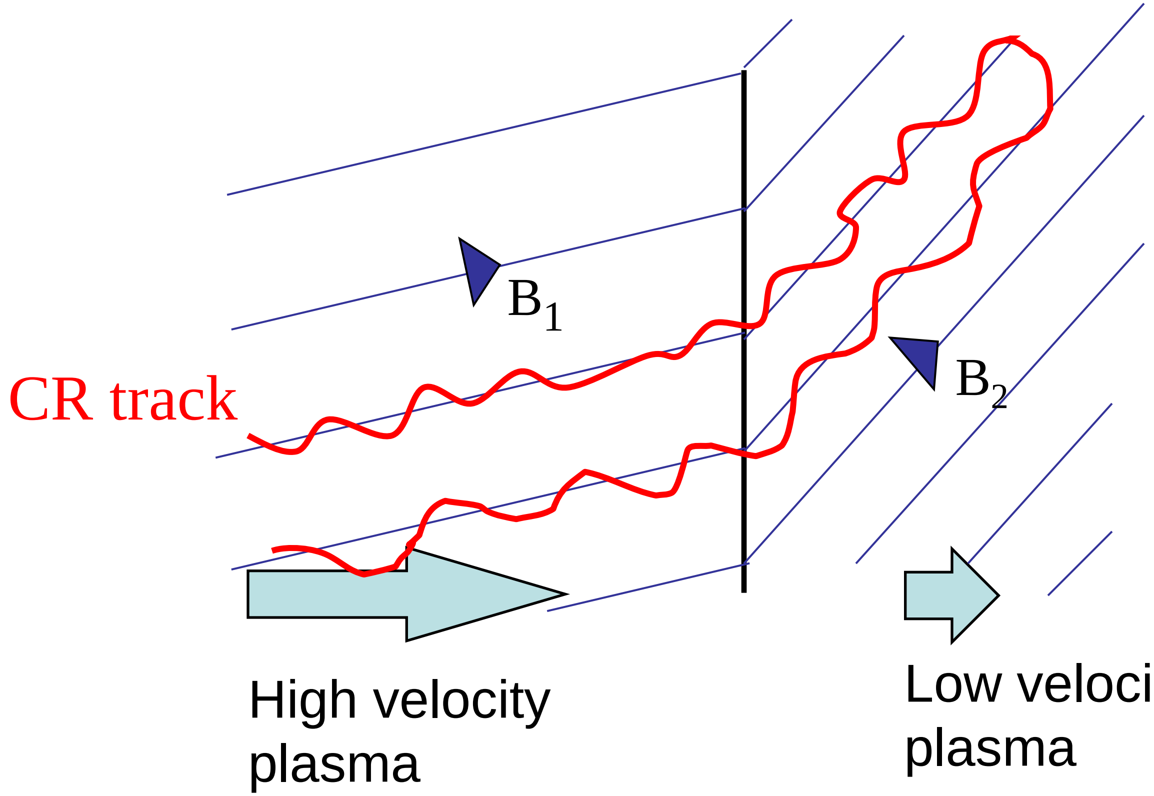
Energy flux $\quad \frac{5}{2} P u + \frac{1}{2}\rho u^3$

Shock turns kinetic streaming energy
Into random thermal energy

Divert part of thermal energy
Into high energy particles

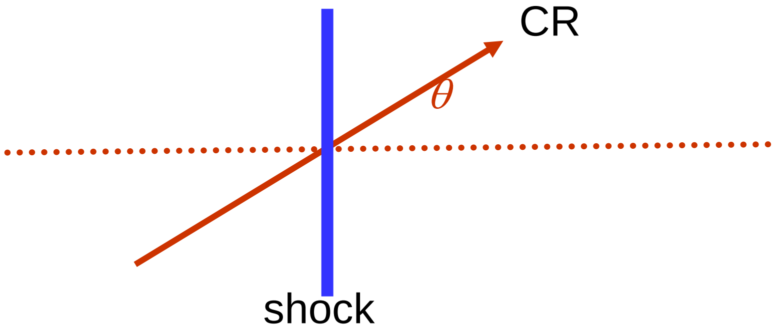
Cosmic ray acceleration by shocks

Cosmic ray acceleration



Due to scattering, CR recrosses shock many times
Gains energy at each crossing

Shock acceleration energy spectrum: energy gain



Change in fluid velocity across shock

$$\Delta \mathrm{v} = \mathrm{v}_s - \frac{\mathrm{v}_s}{4} = \frac{3\mathrm{v}_s}{4}$$

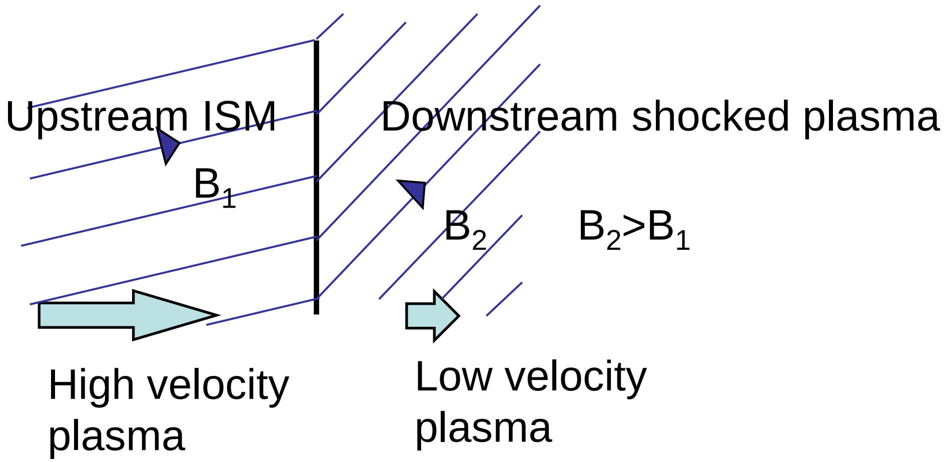
Change in momentum from upstream to downstream $\Delta p_1 = p' - p = p \dfrac{\Delta \mathrm{v}}{c} \cos \vartheta$

Mean increase in momentum $< \Delta p_1 >= \dfrac{\int_0^{\pi/2} \Delta p_1 \cos \vartheta \sin \vartheta d\vartheta}{\int_0^{\pi/2} \cos \vartheta \sin \vartheta d\vartheta} = (\mathrm{v}_s / 2c) p$

Similar increase in momentum on recrossing into upstream $< \Delta p_2 >= (\mathrm{v}_s / 2c) p$

Average fractional energy gained at each crossing is $\boxed{\dfrac{\Delta \varepsilon}{\varepsilon} = \dfrac{< \Delta p_1 > + < \Delta p_2 >}{p} = \dfrac{\mathrm{v}_s}{c}}$

Shock acceleration energy spectrum: loss rate



Shock velocity: v_s

CR density at shock: n

CR cross from upstream to downstream at rate $nc/4$

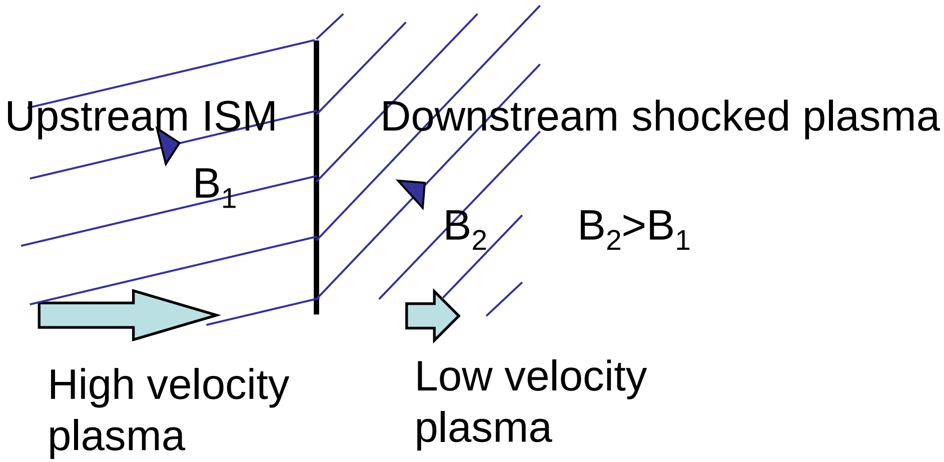
CR carried away downstream at rate $nv_{\mathrm{downstream}} = nv_s/4$

Mean number of shock crossings = $(nc/4)/(nv_s/4) = c/v_s$

Fraction lost at each shock crossing is v_s/c

$$\boxed{\frac{\Delta n}{n} = -\frac{\mathrm{v}_s}{c}}$$

Shock acceleration energy spectrum



Shock velocity: v_s

CR density at shock: n

Fractional CR loss per shock crossing $$\frac{\Delta n}{n} = -\frac{v_s}{c}$$

Fractional energy gain per shock crossing $$\frac{\Delta \varepsilon}{\varepsilon} = \frac{v_s}{c}$$

Turn into differential equation $$\frac{dn}{d\varepsilon} \approx \frac{\Delta n}{\Delta \varepsilon} = -\frac{n}{\varepsilon} \Longrightarrow n \propto \varepsilon^{-1}$$

integrated spectrum

Differential energy spectrum $$\boxed{N(\varepsilon)d\varepsilon \propto \varepsilon^{-2} d\varepsilon}$$

Derivation from Boltzmann equation

Krimskii 1977

Axford, Leer & Skadron 1977

Blandford & Ostriker 1978

The Vlasov-Fokker-Planck (VFP) equation

$$\underbrace{\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{r}} - e(\mathbf{E} + \mathbf{v} \times \mathbf{B}).\frac{\partial f}{\partial \mathbf{p}}}_{\text{Vlasov equation (collisioness)}} = \underbrace{C(f)}_{\text{Collisions Fokker-Planck}}$$

$$f(x, y, z, p_x, p_y, p_z, t)\, dx\, dy\, dz\, dp_x\, dp_y\, dp_z$$

= number of CR in phase space volume $dx\, dy\, dz\, dp_x\, dp_y\, dp_z$

VFP equation:

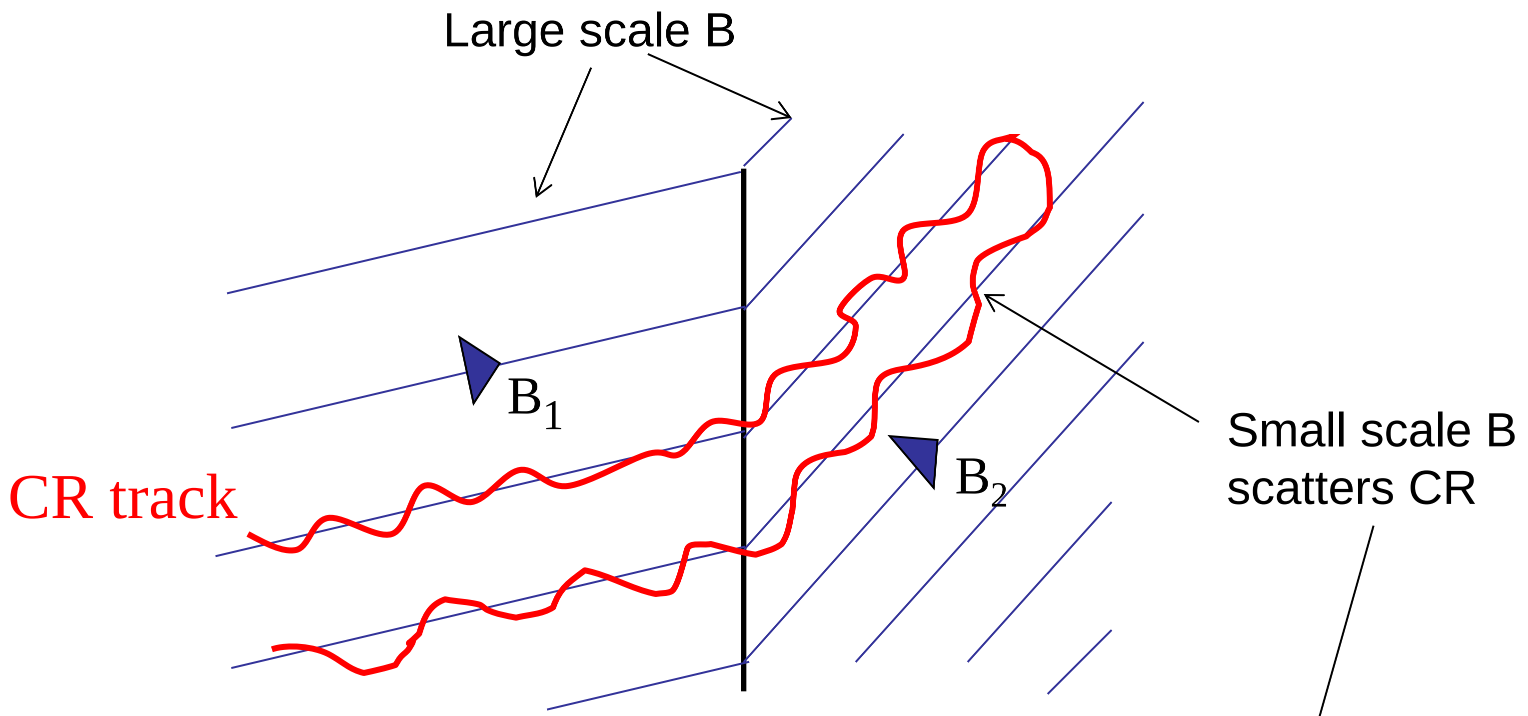
1) Advection: at velocity **v** in **r**-space
at velocity e(E+vxB) in **p**-space

B on scale > CR Larmor radius

2) Collisions: small angle scattering

Due to B on scale < CR Larmor radius

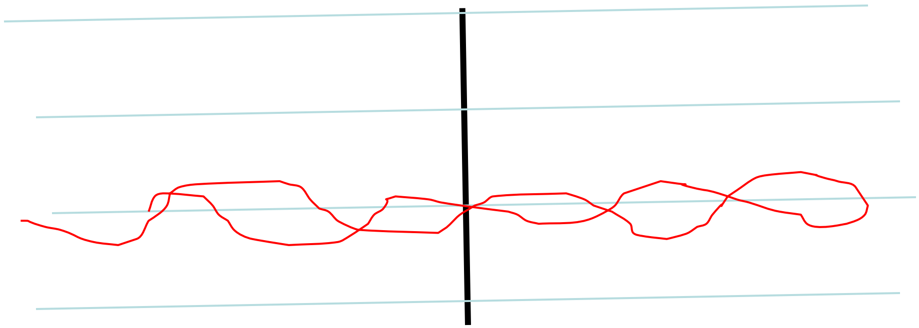
Cosmic ray acceleration



$$\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{r}} - e(\mathbf{E} + \mathbf{v} \times \mathbf{B}).\frac{\partial f}{\partial \mathbf{p}} = C(f)$$

Parallel shock

CR track



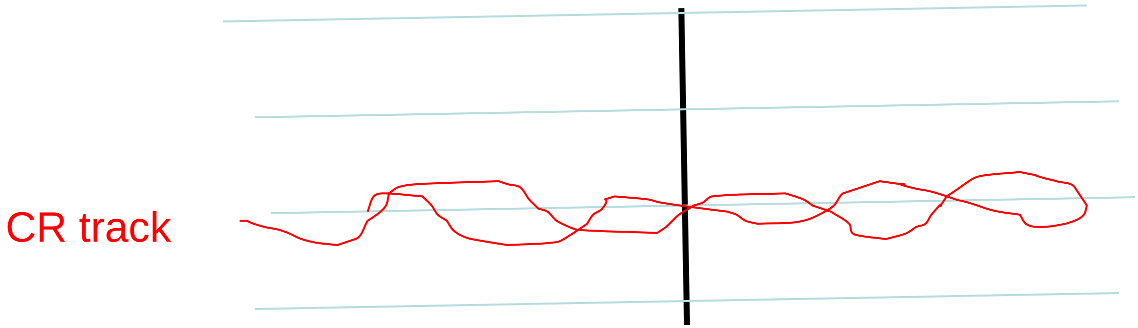
$$\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{r}} - e\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right).\frac{\partial f}{\partial \mathbf{p}} = C(f) \quad \longrightarrow \quad \frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{r}} = C(f)$$

Only diffusion along B matters

Large scale field irrelevant

Same is if no large scale field

Redefine *f* in local fluid rest frame



f fluid rest frame

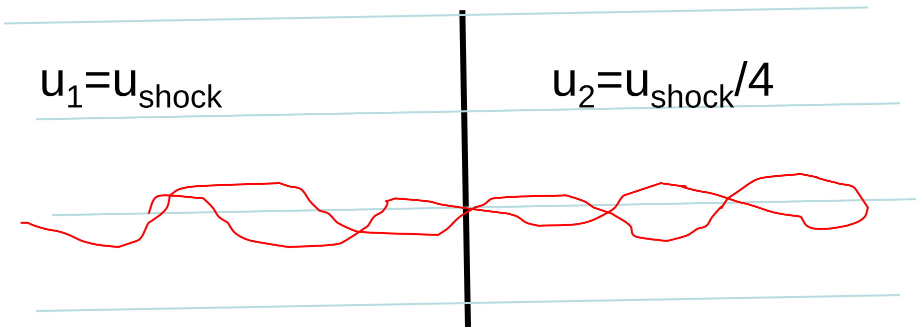
Fluid moves at velocity u

$$\frac{\partial f}{\partial t}+(\mathrm{v}_x+u)\frac{\partial f}{\partial x}-\frac{\partial u}{\partial x}p_x\frac{\partial f}{\partial p_x}=C(f)$$

advection with fluid

Frame transformation

Sub-relativistic shocks: small u/c



To first approximation in u/c

$$f = f_0(p) + f_1(p)\frac{p_x}{p}$$

isotropic

drift

VFP equation reduces to

$$\frac{\partial f_0}{\partial t} + u\frac{\partial f_0}{\partial x} + \frac{\mathrm{v}}{3}\frac{\partial f_1}{\partial x} - \frac{1}{3}\frac{\partial u}{\partial x}p\frac{\partial f_0}{\partial p} = 0$$

$$\mathrm{v}\frac{\partial f_0}{\partial x} = -\nu f_1$$

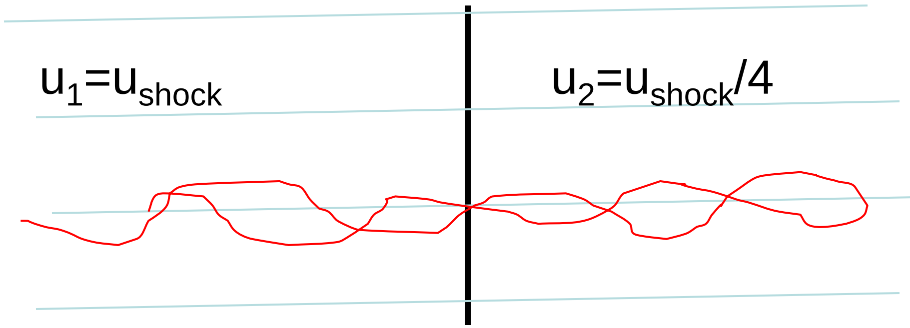
Scattering frequency

$$\frac{\partial f_0}{\partial t} + u\frac{\partial f_0}{\partial x} - \frac{\partial}{\partial x}\left(\frac{\mathrm{v}\nu}{3}\frac{\partial f_0}{\partial x}\right) - \frac{1}{3}\frac{\partial u}{\partial x}p\frac{\partial f_0}{\partial p} = 0$$

Advection Diffusion adiabatic compression

Steady state solution

$$f = f_0(p) + f_1(p)\frac{p_x}{p}$$



No escape upstream:

$$u_1 f_0 + \frac{c}{3} f_1 = 0$$

Downstream: no drift relative to background

$$f_1 = 0$$

Boundary condition at shock $\left[f_1 - \frac{u_1}{c} p \frac{\partial f_0}{\partial p} \right]_{upstream} = \left[f_1 - \frac{u_2}{c} p \frac{\partial f_0}{\partial p} \right]_{downstream}$

$$\Rightarrow (u_1 - u_2) p \frac{\partial f_0}{\partial p} = -3 u_1 f_0 \quad \Rightarrow \quad \boxed{f_0 \propto p^{-4}}$$

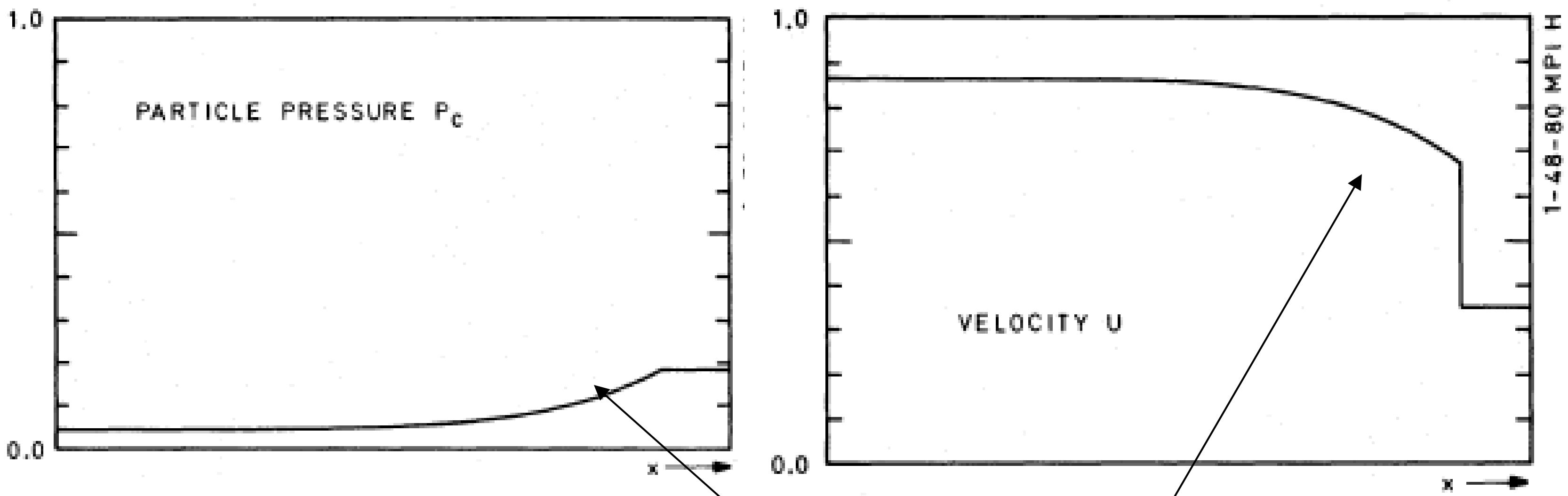
Acceleration efficiency

Efficiency

- Has to be efficient (10-50%) to explain galactic CR energy density
- Solar wind shocks can be >10% efficient
- Shock processes produce many suprathermal protons

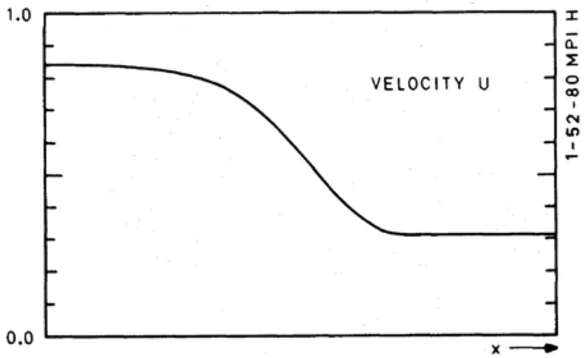
At high efficiency: non-linear feedback onto shock

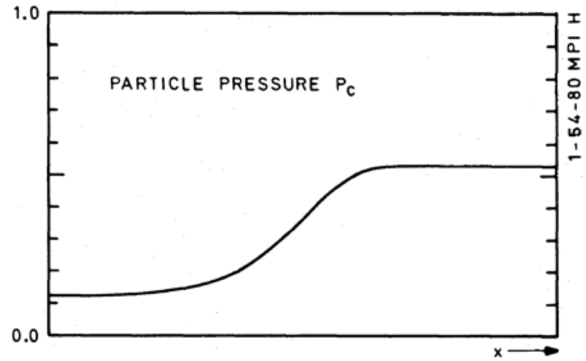
Drury & Voelk (1981)

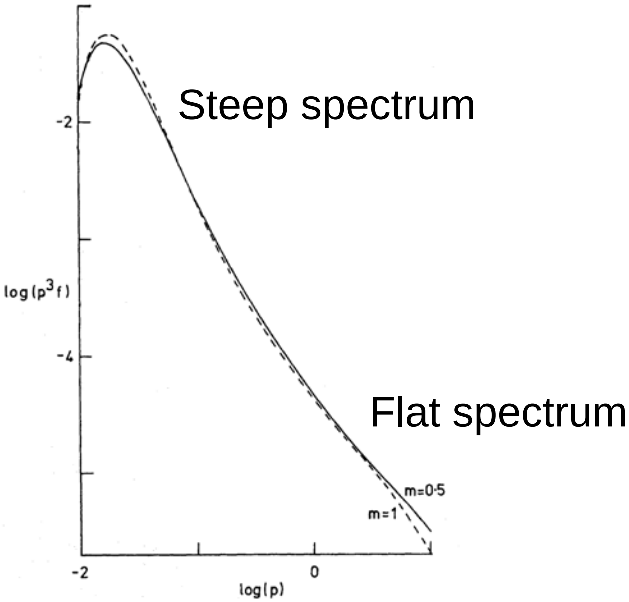


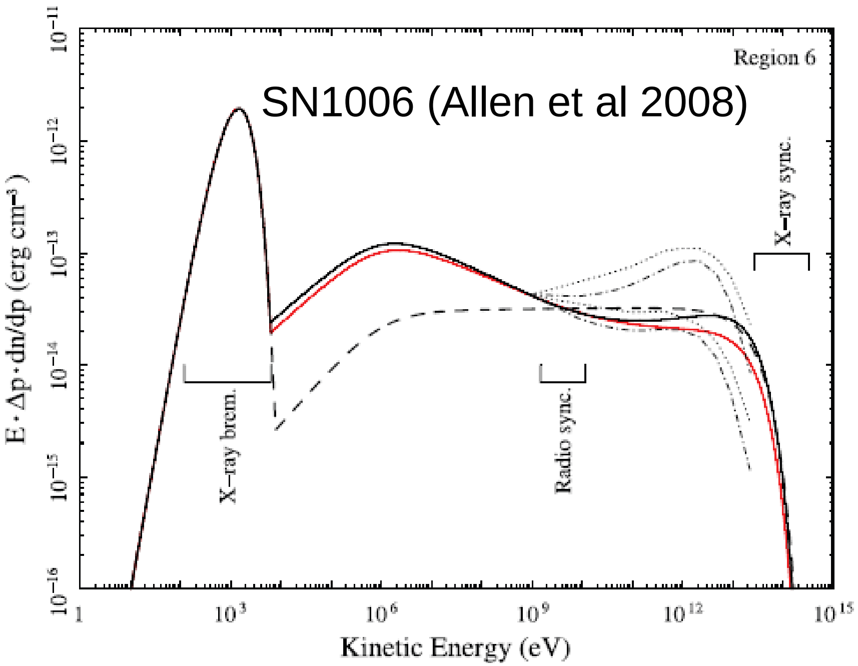
CR pressure decelerates flow into shock

High efficiency: concave spectrum



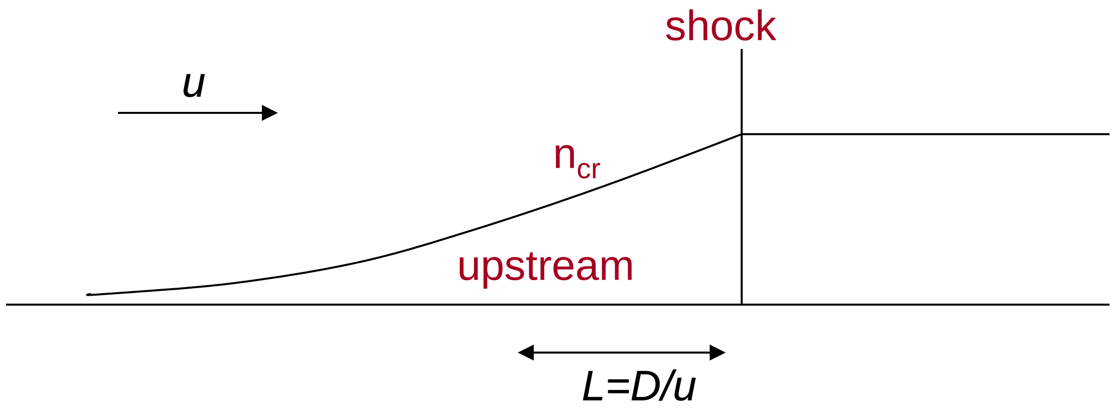






Maximum CR energy

CR upstream of shock



Balance between:

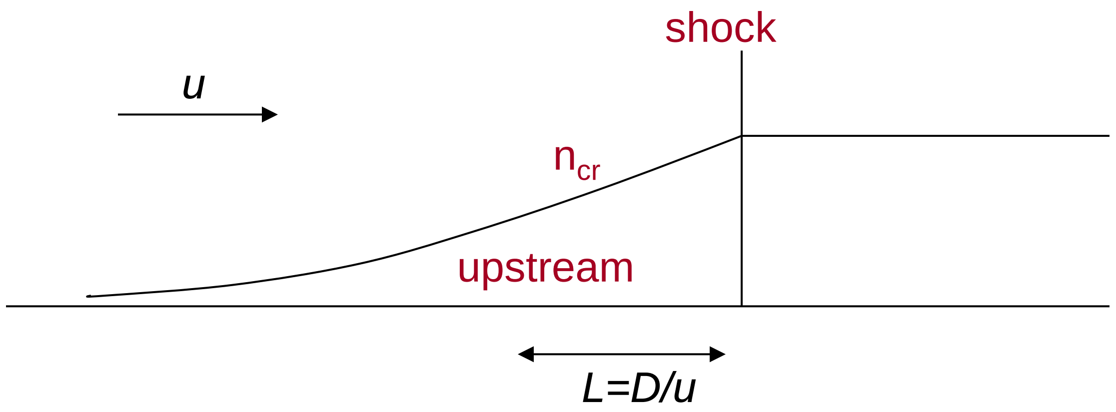
- flow into shock
- diffusion away from shock

$$\frac{\partial n_{cr}}{\partial t} = -u\frac{\partial n_{cr}}{\partial x} - D\frac{\partial^2 n_{cr}}{\partial x^2} = 0$$

Exponential density $$n_{cr} = n_0 e^{ux/D}$$

Scaleheight $$L = \frac{D}{u}$$

CR acceleration time



Number of CR upstream: $n_{cr}\dfrac{D_{upstream}}{u_{shock}}$ Rate CR cross shock: $\dfrac{1}{4}n_{cr}c$

Average time spent upstream: $\Delta t = \dfrac{4D_{upstream}}{cu_{shock}}$ (neglect time spent downstream)

Average energy gain per shock crossing: $\dfrac{\Delta\varepsilon}{\varepsilon} = \dfrac{u_{shock}}{c}$

Acceleration rate: $\dfrac{\Delta\varepsilon}{\Delta t} = \dfrac{\varepsilon}{4}\dfrac{u^2_{shock}}{D_{upstream}}$

Time needed for acceleration (Lagage & Cesarsky) $\tau = \dfrac{4D_{upstream}}{u^2_{shock}} + \dfrac{4D_{downstream}}{(u_{shock}/4)^2}$

downstream time

Maximum CR energy

Acceleration time $\tau = \dfrac{8D}{u_{shock}^2}$

Diffusion coefficient $D = \dfrac{\lambda c}{3}$

$$\lambda \le \frac{3}{8}\frac{u_{shock}}{c}R$$

where $R = u_{shock}\tau$

SNR radius

Smallest possible mfp: $\lambda = \dfrac{p_{cr}}{eB}$

Limit on CR momentum: $p_{cr} = \dfrac{3}{8}\dfrac{u_{shock}}{c}eBR$

Limit on CR energy in eV: $\dfrac{cp_{cr}}{e} = \dfrac{3}{8}Bu_{shock}R$

Typically for young SNR

ISM mag field: few μG

$u_{shock}=c/30$

$R \sim 10^{17}$m

Max CR energy ~ 10^{14}eV

under favourable assumptions

Hillas diagram

(condition on *RuB*)

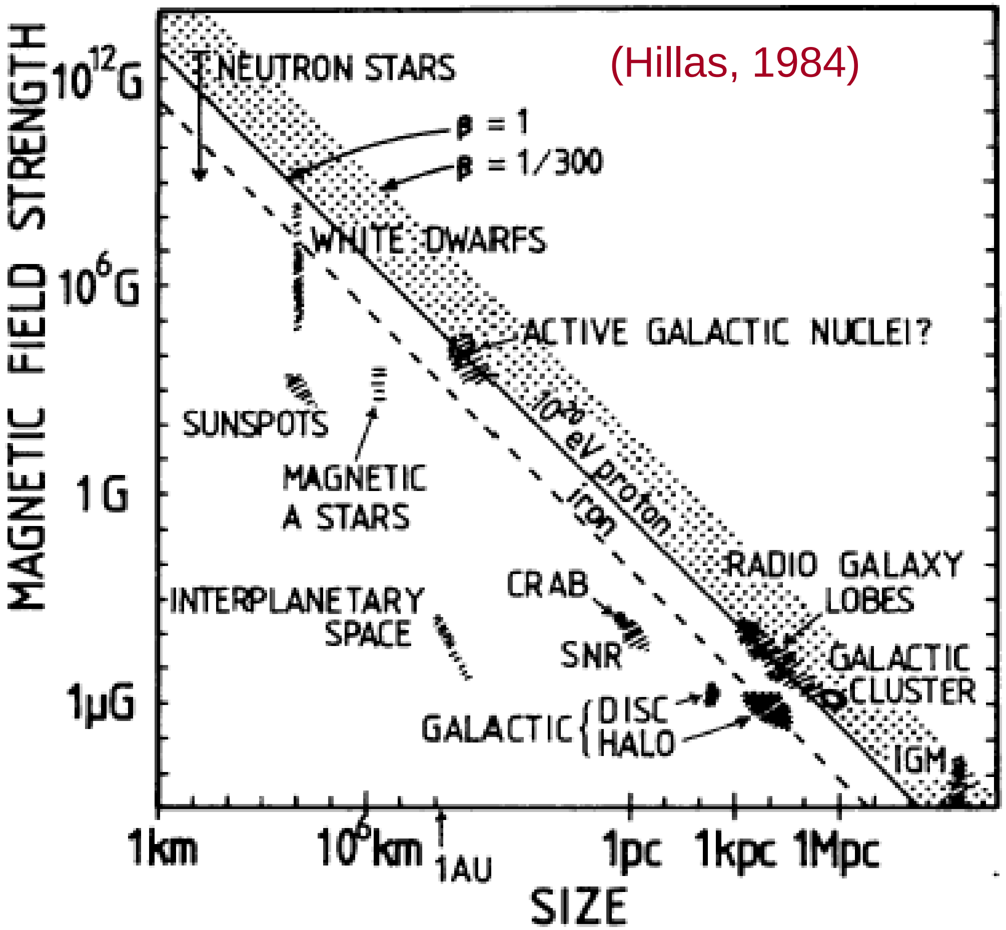


Figure 1 Size and magnetic field strength of possible sites of particle acceleration. Objects below the diagonal line cannot accelerate protons to 10^{20} eV.

Perpendicular shocks

(Jokipii 1982, 1987)

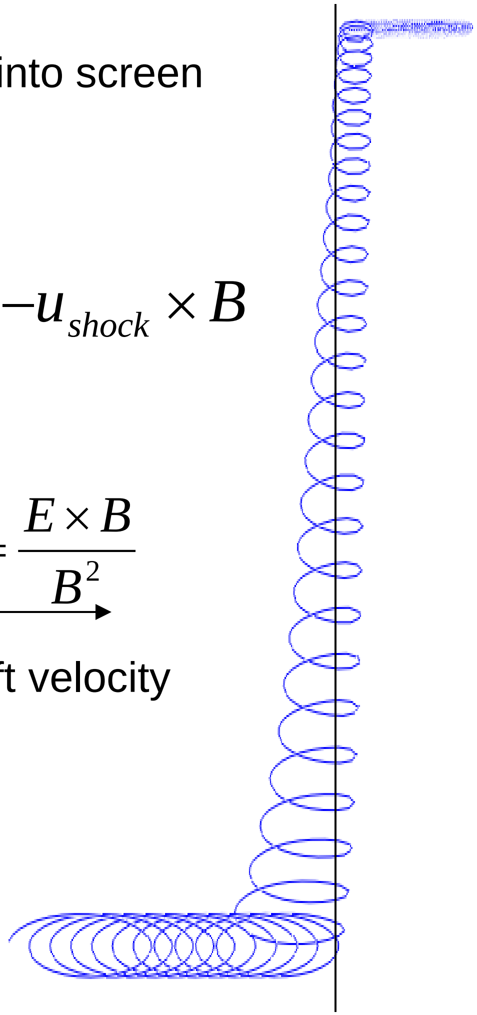
CR trajectory at perpendicular shock (no scattering)

B into screen

$$E = -u_{shock} \times B$$

$$\mathrm{v}_{drift} = \frac{E \times B}{B^2}$$

CR drift velocity



shock

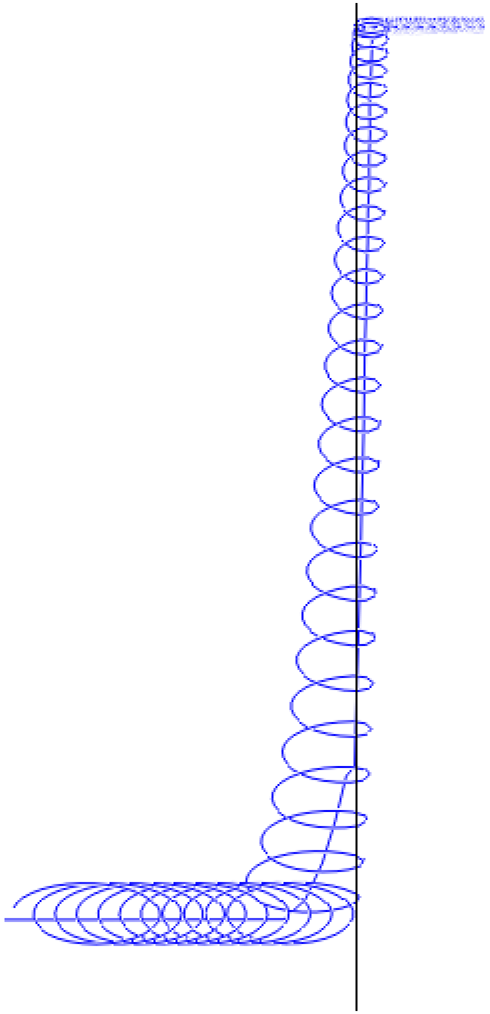
CR gain energy by
drifting in E field

CR acceleration at perpendicular shock

CR trajectory divides into

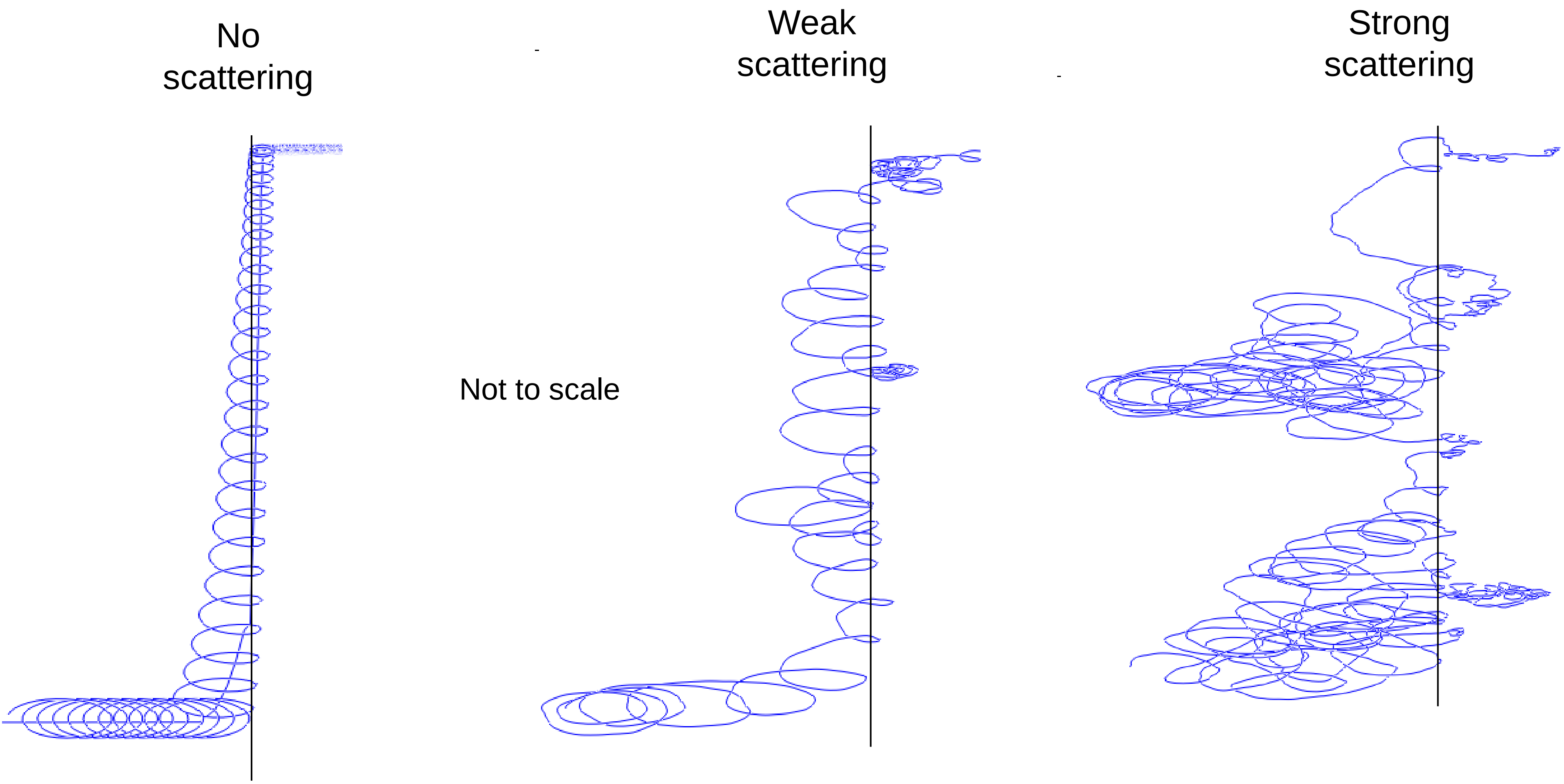
- Motion of gyrocentre
- Gyration about gyrocentre

Without diffusion:
Every CR gets small adiabatic gain
due to compression at shock

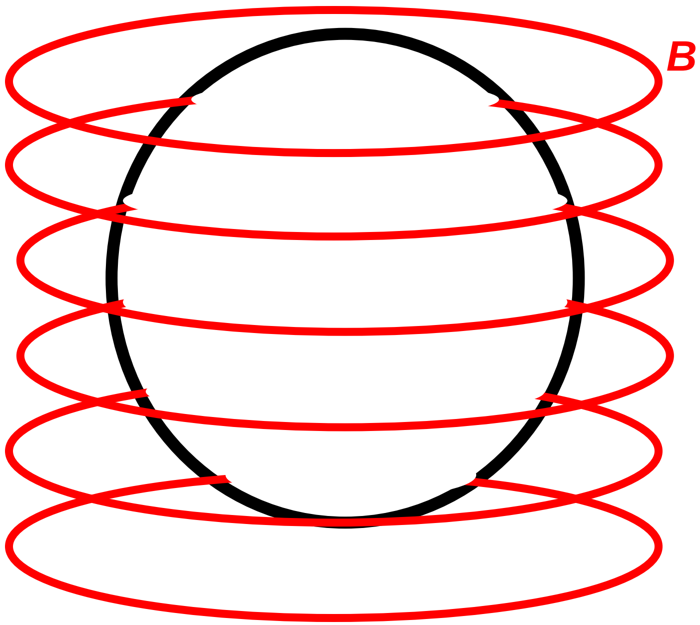


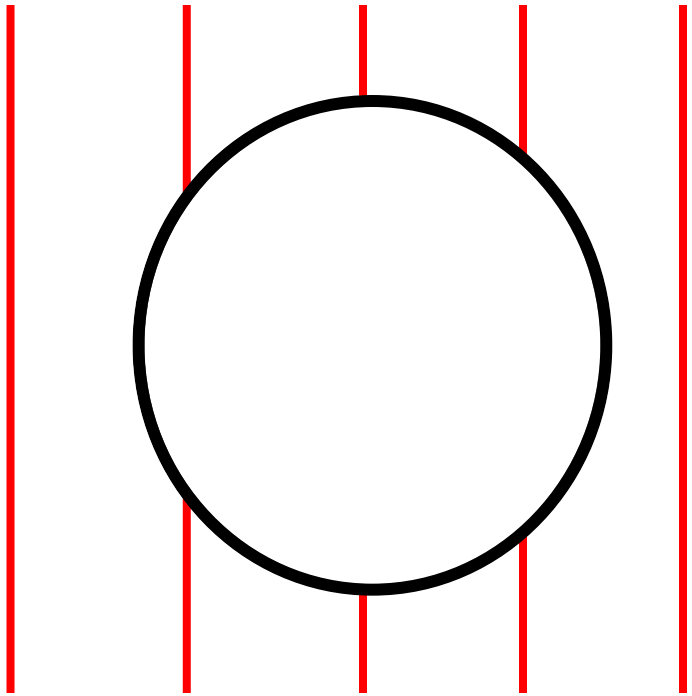
CR acceleration at perpendicular shock: with scattering

Diffusive shock theory applies
Provided gyrocentre diffuses over distances
greater than Larmor radius during shock transit
Same power law (see later)



CR acceleration at perpendicular shock



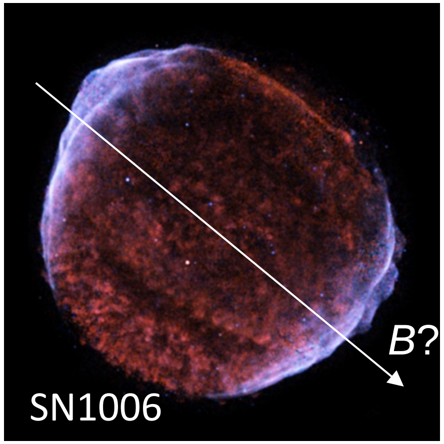


$$E = -u_{shock} \times B$$

Transit between pole & equator: energy gain ~ $eER = eu_{shock}BR$

Hillas parameter as with parallel shock: similar max CR energy

The case of SN1006



Polar x-ray synchrotron emission?

(Rothenflug et al 2004)

At perpendicular shocks

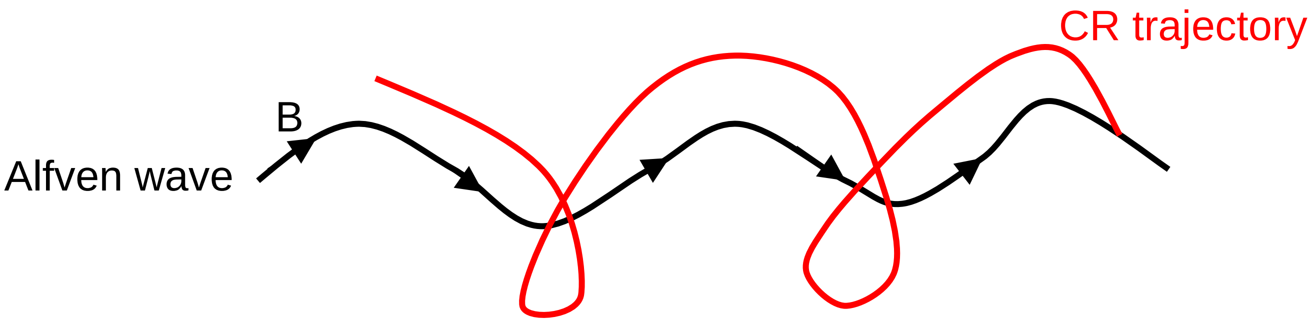
- Acceleration is faster – potentially higher CR energy
- CR energy limited to $euBR$ (Hillas) by space rather than time
- Injection is more difficult at a perpendicular shock
- CR scattering frequency has to be in right range

Room for discussion!

CR scattering

what is the mean free path?

CR drive a ‘resonant’ instability



Spatial resonance between wavelength and CR Larmor radius

wave deflects CR ↔ CR current drives wave

Skilling (1975)

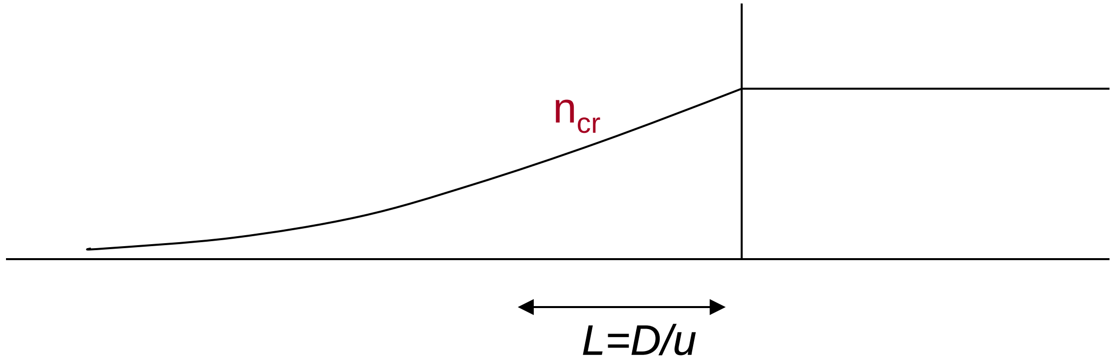
Wave growth (energy density *I*)

$$\frac{\partial I}{\partial t} + u\frac{\partial I}{\partial x} = \frac{\mathrm{v}_A pc}{U_M}\frac{\partial n_{cr}}{\partial x} \qquad I = \frac{\delta B^2}{B^2}$$

CR scattering

$$\frac{\partial n_{cr}}{\partial t} + u\frac{\partial n_{cr}}{\partial x} = \frac{\partial}{\partial x}\left(D\frac{\partial n_{cr}}{\partial x}\right) \qquad D = \frac{4}{3\pi}\frac{cr_g}{I}$$

Turbulence upstream of shock



Skilling (1975)

Wave growth (amplitude I)

$$\frac{\partial I}{\partial t} + u\frac{\partial I}{\partial x} = \frac{\mathrm{v}_A pc}{U_M}\frac{\partial n_{cr}}{\partial x}$$

CR scattering

$$\frac{\partial n_{cr}}{\partial t} + u\frac{\partial n_{cr}}{\partial x} = \frac{\partial}{\partial x}\left(D\frac{\partial n_{cr}}{\partial x}\right)$$

$$D = \frac{4}{3\pi}\frac{cr_g}{I}$$

Solution

$$I_{shock} = M_A \frac{U_{cr}}{\rho u_{shock}^2}$$

$$\text{mfp} = \frac{4}{\pi}\frac{r_g}{I}$$

$$L = \frac{4}{3}\frac{c}{u_{shock}}\frac{r_g}{I}$$

Alfven Mach number ~1000

CR efficiency ~0.1

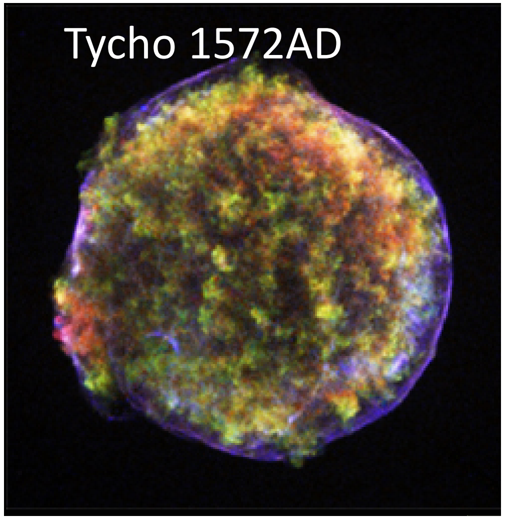
Question: What does I > 1 tell us?

?Implies?: mfp < Larmor radius

Waves non-linear: $I \gg 1$

Evidence that magnetic field exceeds typical interstellar value

1) x-ray observations



2) Needed ti accelerate galactic CR to a few PeV

3) Theory breaks down because of large wave growth

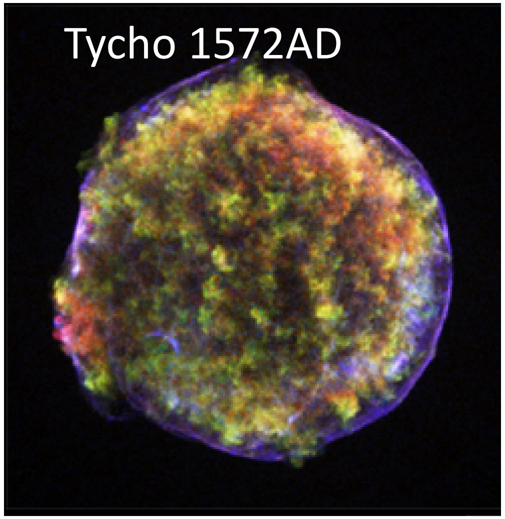
$$I_{shock} = M_A \frac{U_{cr}}{\rho u_{shock}^2}$$

Alfven Mach number ~1000

CR efficiency ~0.1

Evidence that magnetic field exceeds typical interstellar value

1) x-ray observations



2) Needed ti accelerate galactic CR to a few PeV

Need to look more closely at CR interaction with magnetic field

3) Theory breaks down because of large wave growth

$$I_{shock} = M_A \frac{U_{cr}}{\rho u_{shock}^2}$$

Alfven Mach number ~1000

CR efficiency ~0.1

Diffusive Shock Acceleration and magnetic fields in SNR

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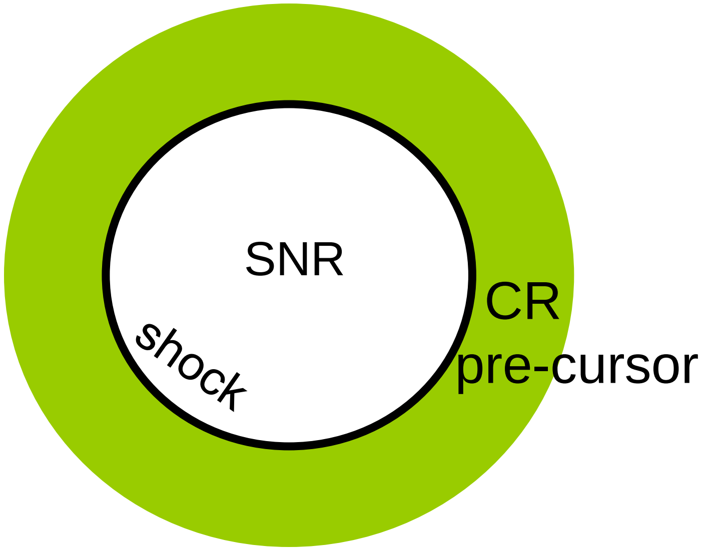
University of Oxford
Rutherford Appleton Laboratory

SN1006: A supernova remnant 7,000 light years from Earth
X-ray (blue): NASA/CXC/Rutgers/G.Cassam-Chenai, J.Hughes et al; Radio (red): NRAO/AUI/GBT/VLA/Dyer, Maddalena & Cornwell; Optical (yellow/orange): Middlebury College/F.Winkler. NOAO/AURA/NSF/CTIO Schmidt & DSS

magnetic field amplification
and
cosmic ray scattering

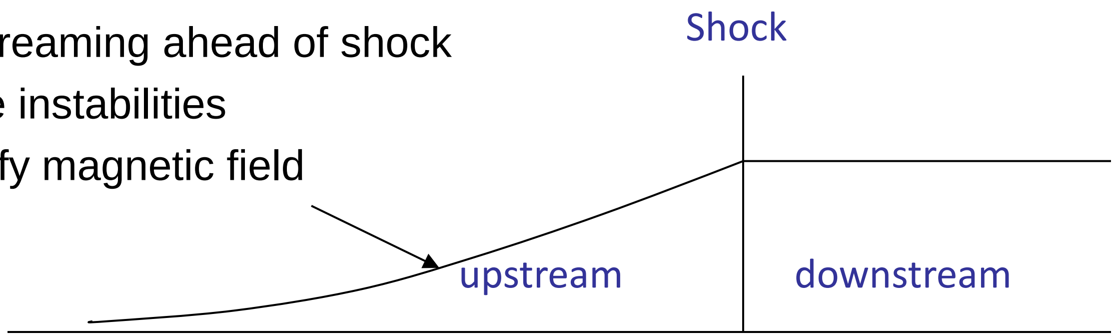
Streaming CR excite instabilities



CR streaming ahead of shock

Excite instabilities

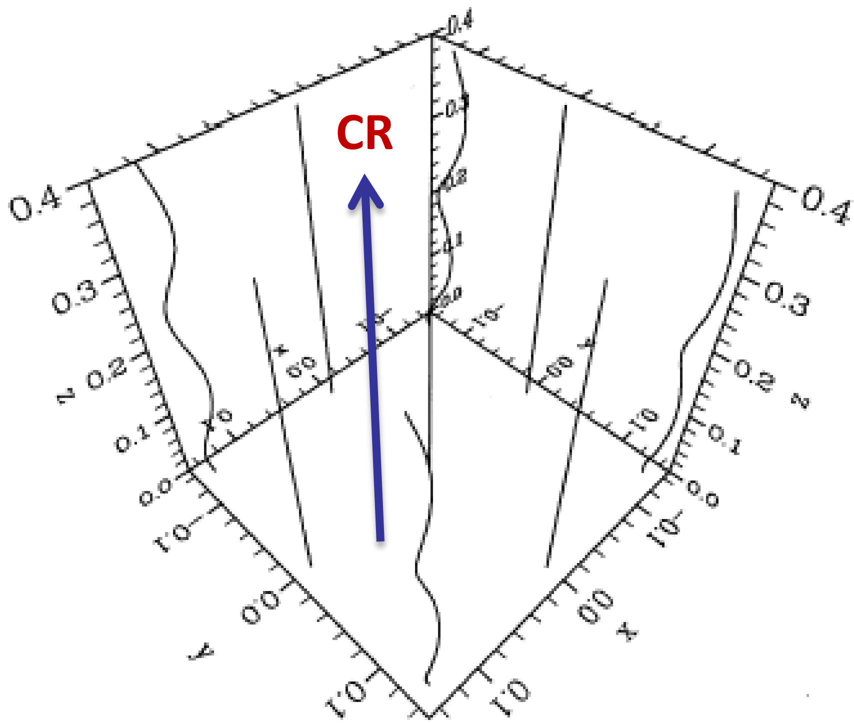
Amplify magnetic field



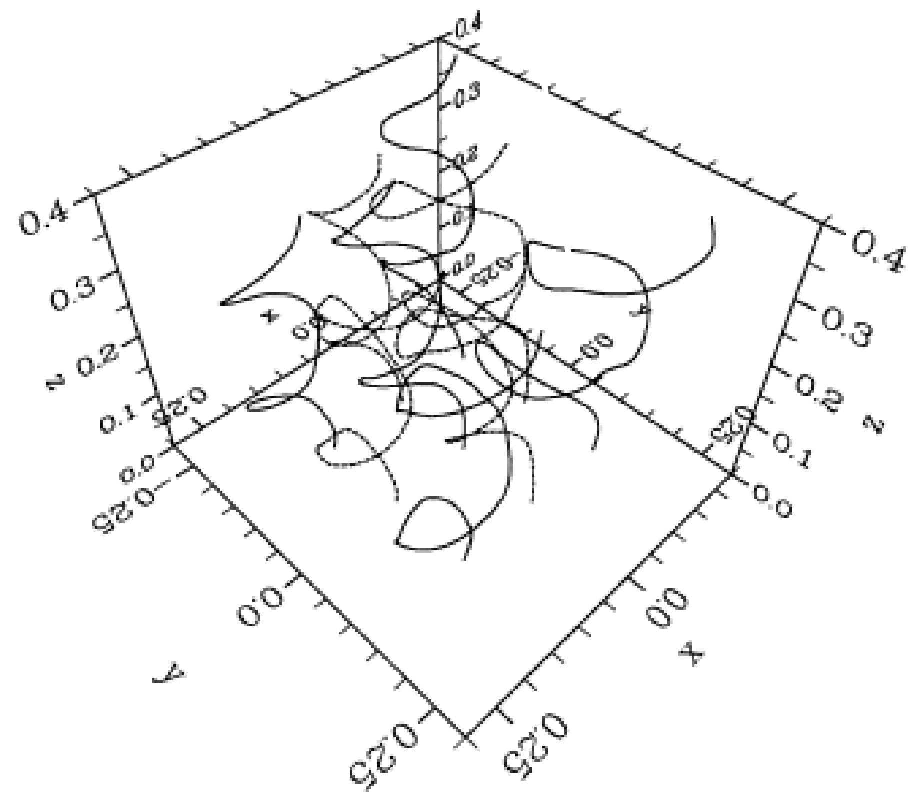
Streaming instabilities amplify magnetic field

Lucek & Bell (2000)

B field lines, t = 0



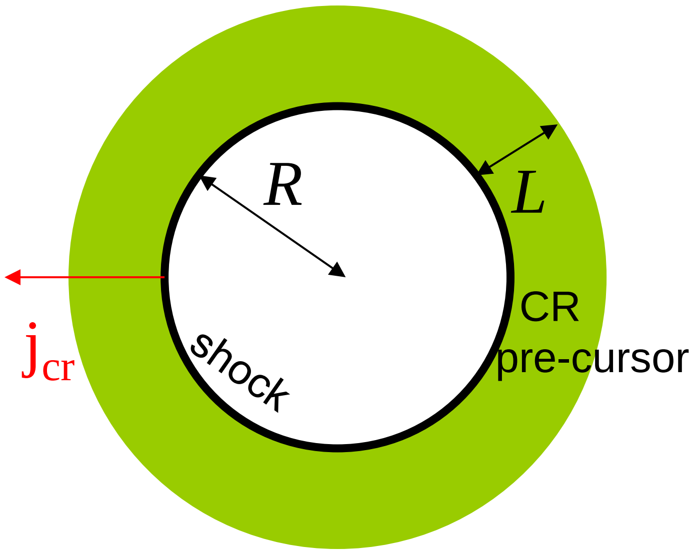
B field lines, t = 2



CR treated as particles

Thermal plasma as MHD

Electric currents carried by CR and thermal plasma



Density of 10^{15}eV CR: $\sim 10^{-12}$ cm^{-3}
Current density: $j_{cr} \sim 10^{-18}$ Amp m^{-2}

CR current must be balanced by current carried by thermal plasma

$$j_{thermal} = -j_{cr}$$

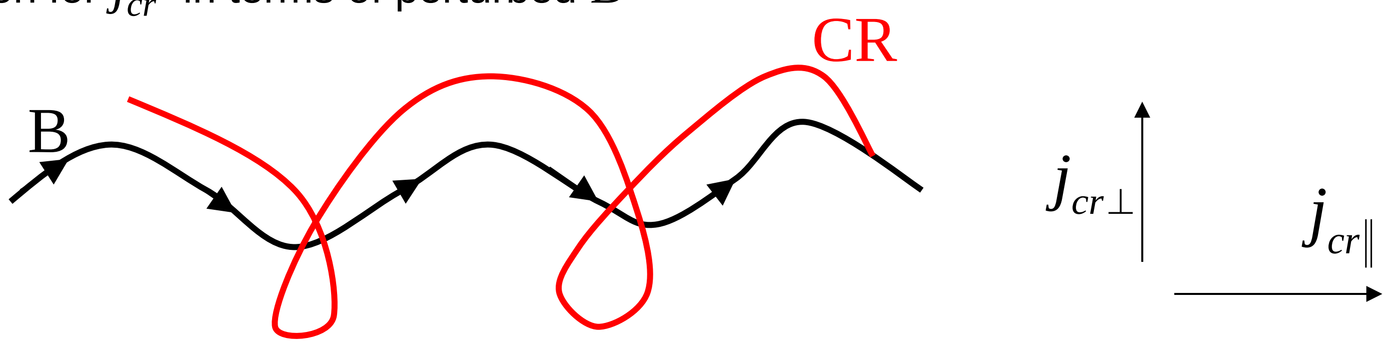
$j_{thermal}$xB force acts on plasma to balance j_{cr}xB force on CR

Three equations control the instability

1) $$\rho\frac{du}{dt} = -\nabla p - \frac{1}{\mu_0}B\wedge(\nabla\wedge B) - j_{cr}\wedge B$$

2) $$\frac{\partial B}{\partial t} = \nabla\wedge(u\wedge B)$$

3) Equation for j_{cr} in terms of perturbed B

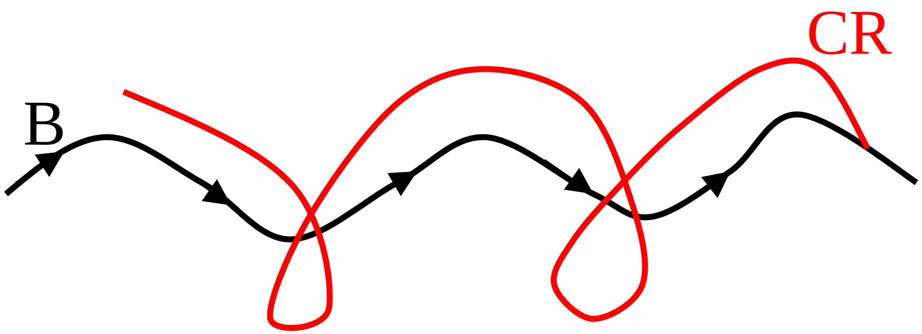


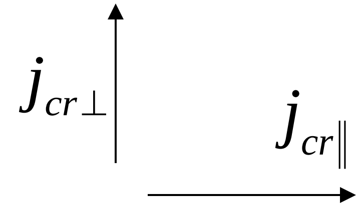
*j*x*B* driving force splits into two parts:

$$\rho\frac{du}{dt} = -\nabla p - \frac{1}{\mu_0}B\times(\nabla\times B) - j_{cr\perp}\times B - j_{cr\|}\times B$$

Resonant Alfven instability

$$\rho \frac{\partial u}{\partial t} = -\nabla p - \frac{1}{\mu_0} B \times (\nabla \times B) - \boxed{j_{cr\perp} \times B} - j_{cr\parallel} \times B$$



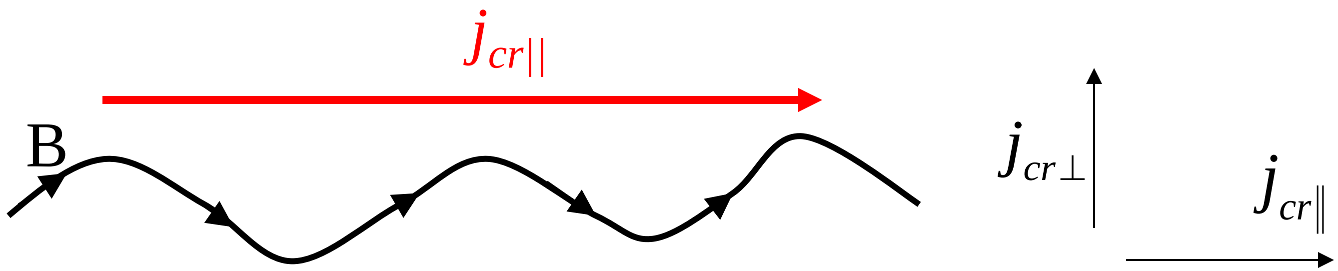


$j_{cr\perp} \times B$ drives Alfven waves

Perturbed cosmic ray current

Non-resonant instability

$$\rho \frac{\partial u}{\partial t} = -\nabla p - \frac{1}{\mu_0} B \times (\nabla \times B) - j_{cr\perp} \times B - \boxed{j_{cr\|} \times B}$$

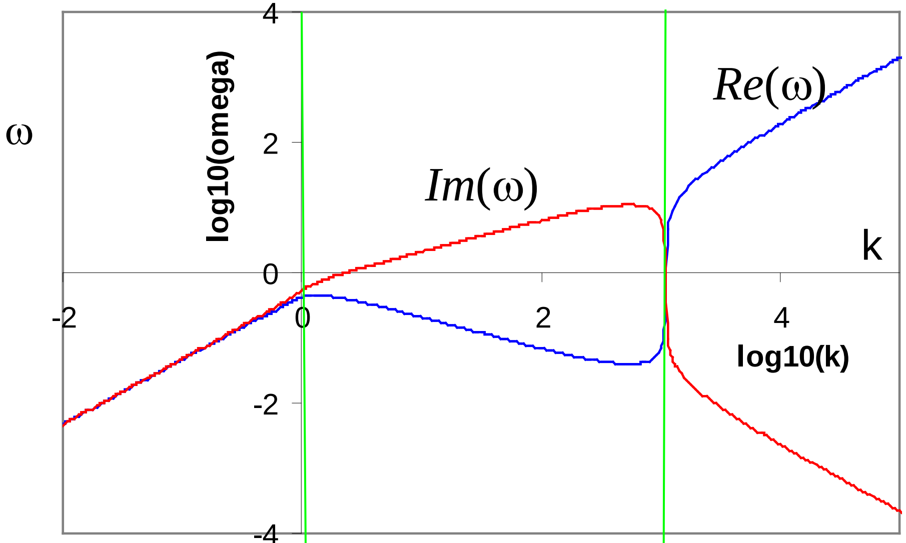


$j_{cr\|} \times B$ dominates for shock acceleration in SNR

Perturbed magnetic field

Dispersion relation

Red line is growth rate



k in units of r_g^{-1}
ω in units of v_S^2/cr_g

Wavelength longer than Larmor radius
CR follow field lines.
jxB drives weak instability

$$\rho \frac{d\mathbf{u}}{dt} = -\mathbf{j}_{CR} \times \mathbf{B}$$

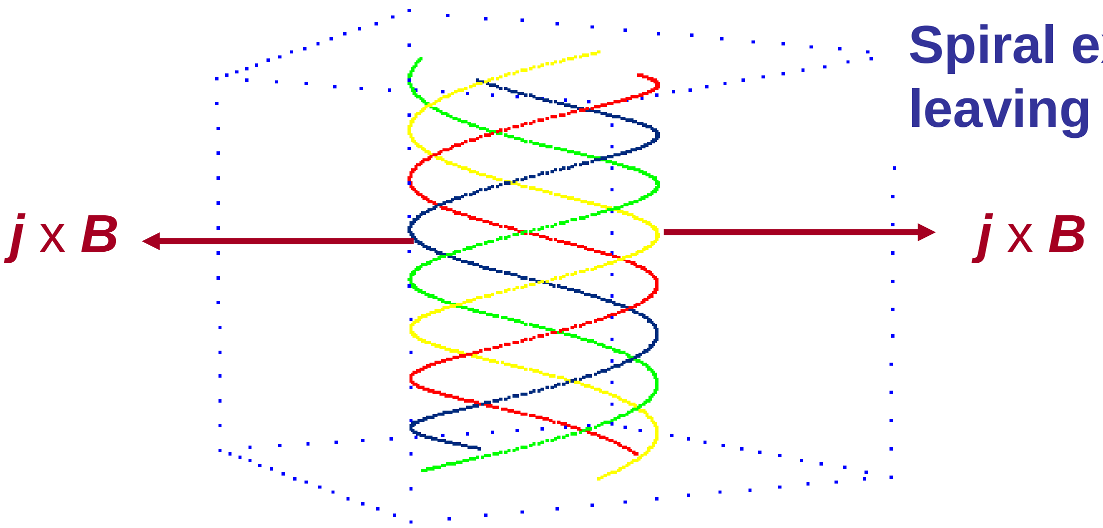
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\gamma = \left(\frac{k B_0 j_{CR}}{\rho} \right)^{1/2}$$

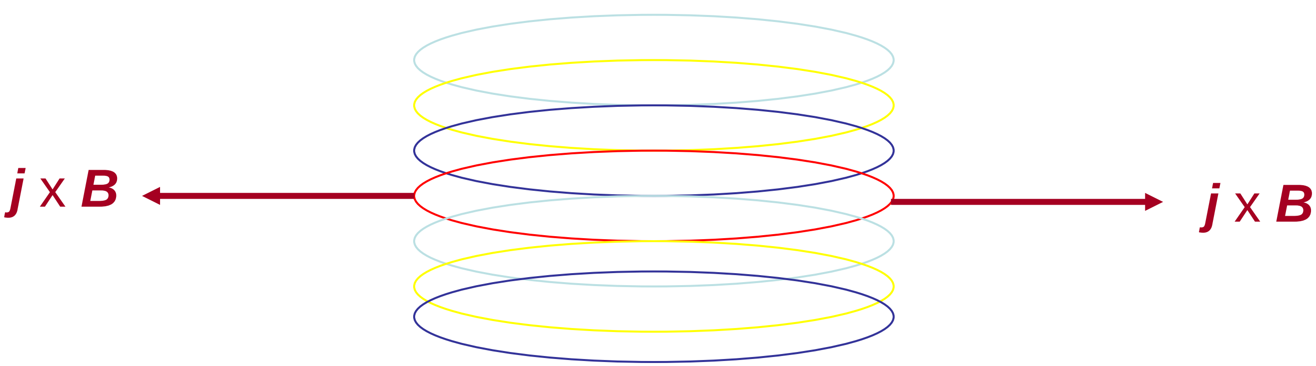
Magnetic tension inhibits instability

The essence of the non-resonant instability

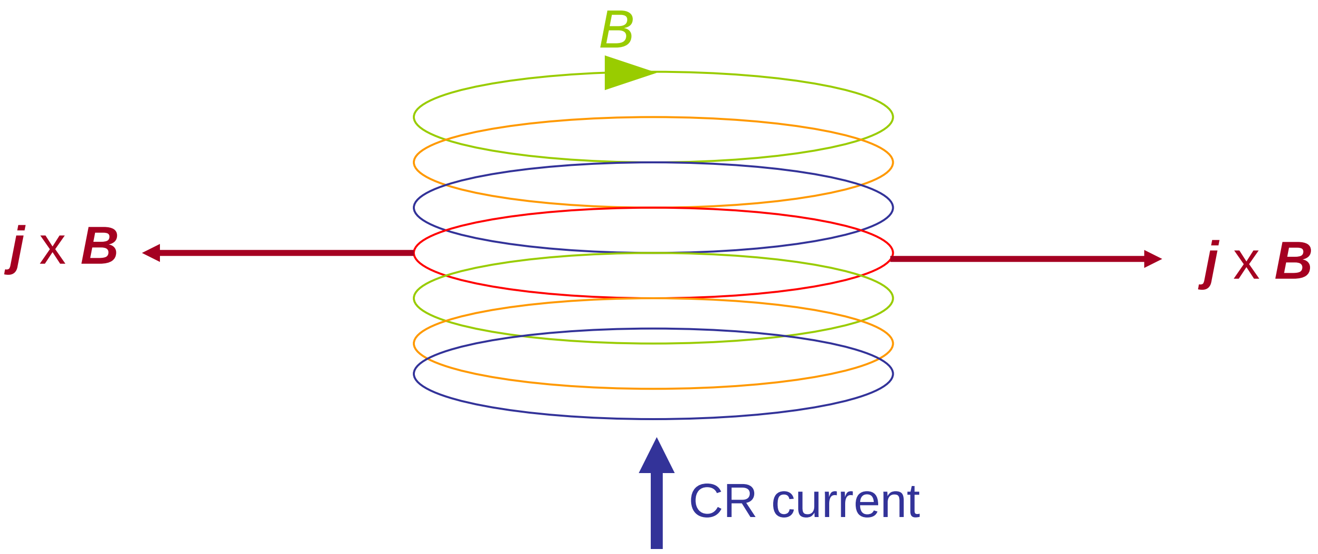
Spiral expands leaving central cavity



Same without vertical field



Simplest form: expanding loops of B



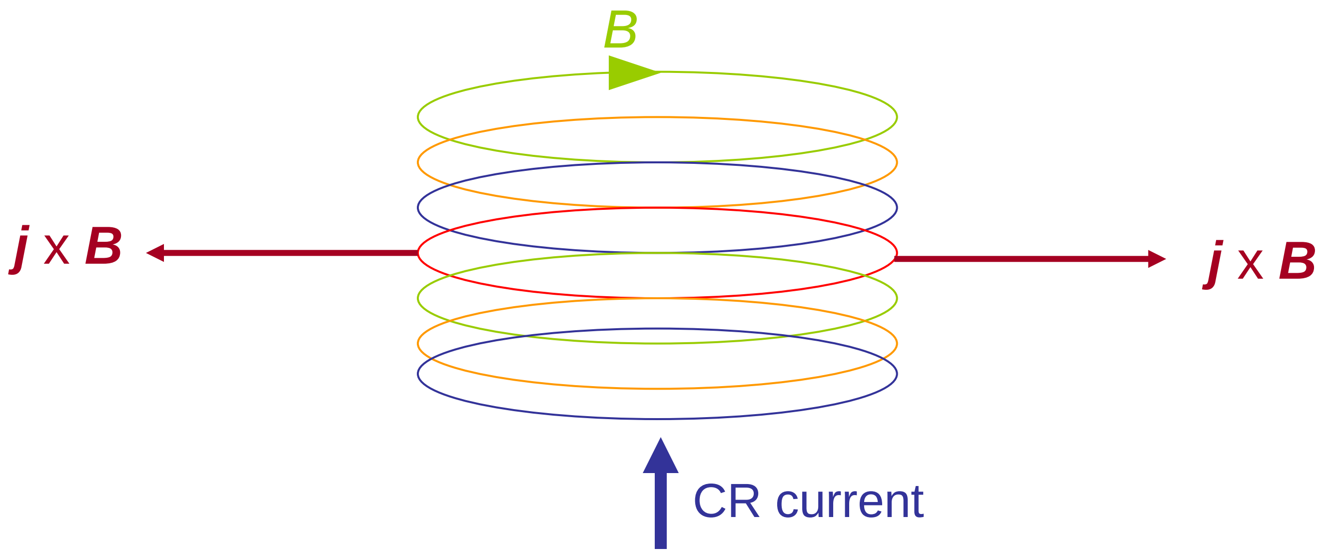
jxB expands loops

$\longrightarrow$ stretches field lines

$\longrightarrow$ more B

$\longrightarrow$ more jxB

Simplest form: expanding loops of B



$j\mathrm{x}B$ expands loops

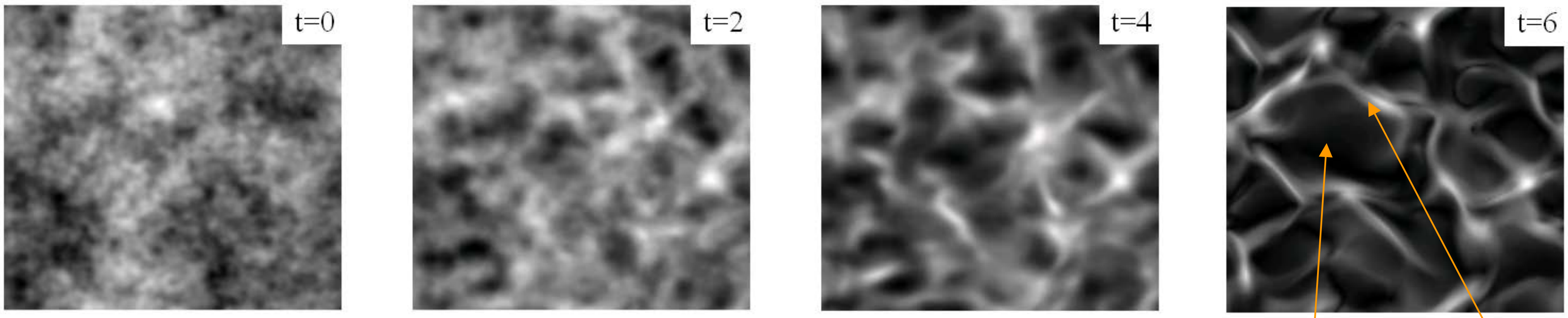
→ stretches field lines

→ more B

→ more $j\mathrm{x}B$

Non-linear growth – expanding loops

Slices through $|B|$ - time sequence (fixed CR current)

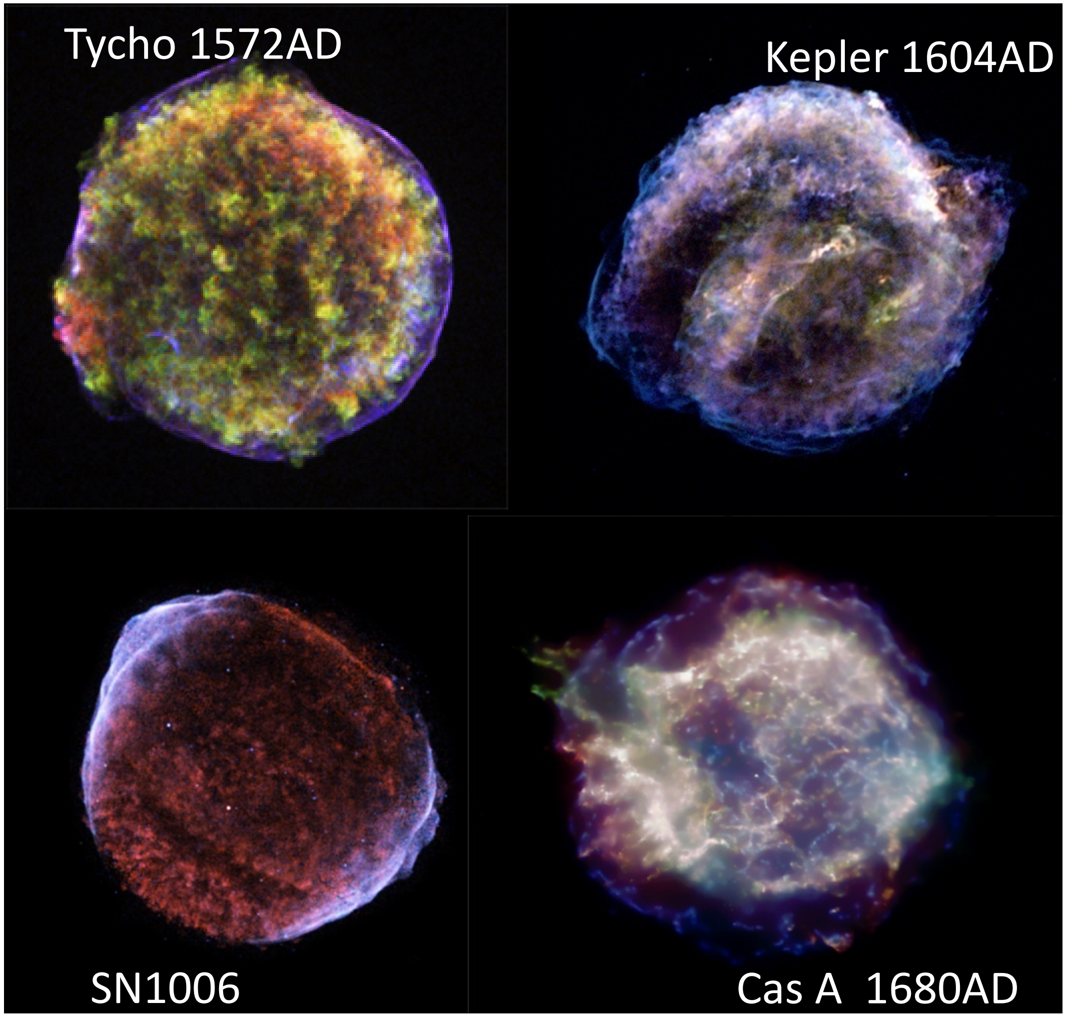


Cavities and walls in $|B|$ & ρ

Field lines: wandering spirals

How large does the magnetic field grow?

Historical SNR



Chandra observations

NASA/CXC/Rutgers/ J.Hughes et al.

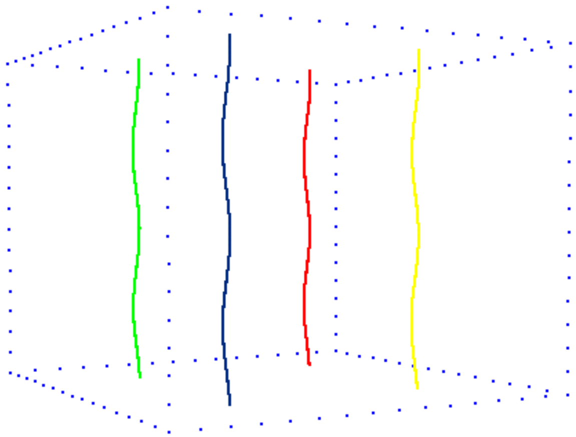
NASA/CXC/Rutgers/ J.Warren & J.Hughes et al.

NASA/CXC/NCSU/ S.Reynolds et al.

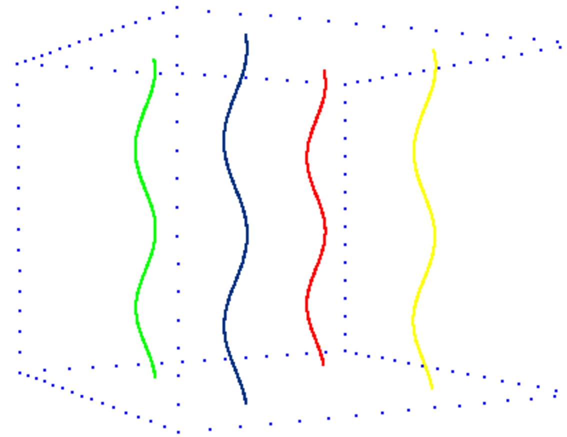
NASA/CXC/MIT/UMass Amherst/ M.D.Stage et al.

Instability growth

a)

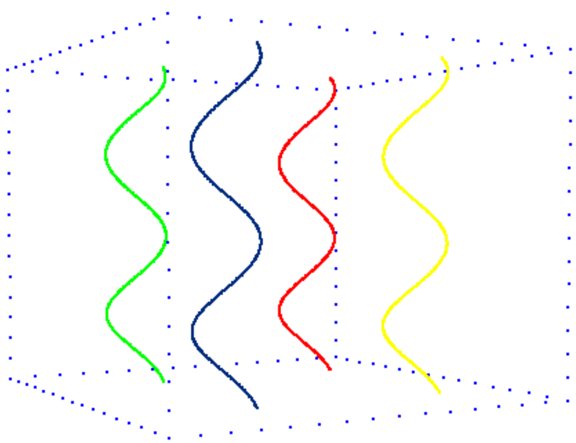


b)

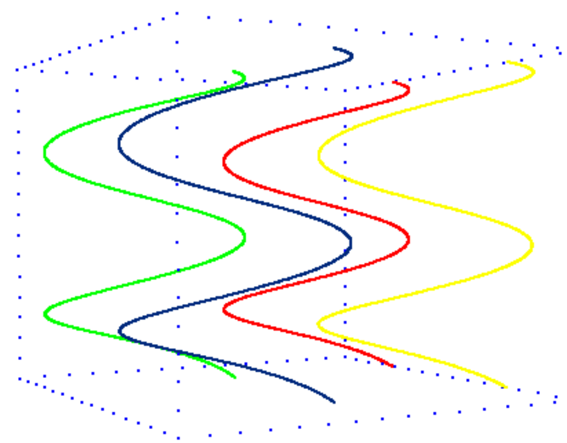


No reason for non-linear saturation of a single mode

c)



d)



Saturation (back of envelope)

Magnetic field grows until

1) $$\frac{1}{\mu_0} B \times \left(\nabla \times B \right) \approx j_{CR} \times B$$

Magnetic tension CR driving force

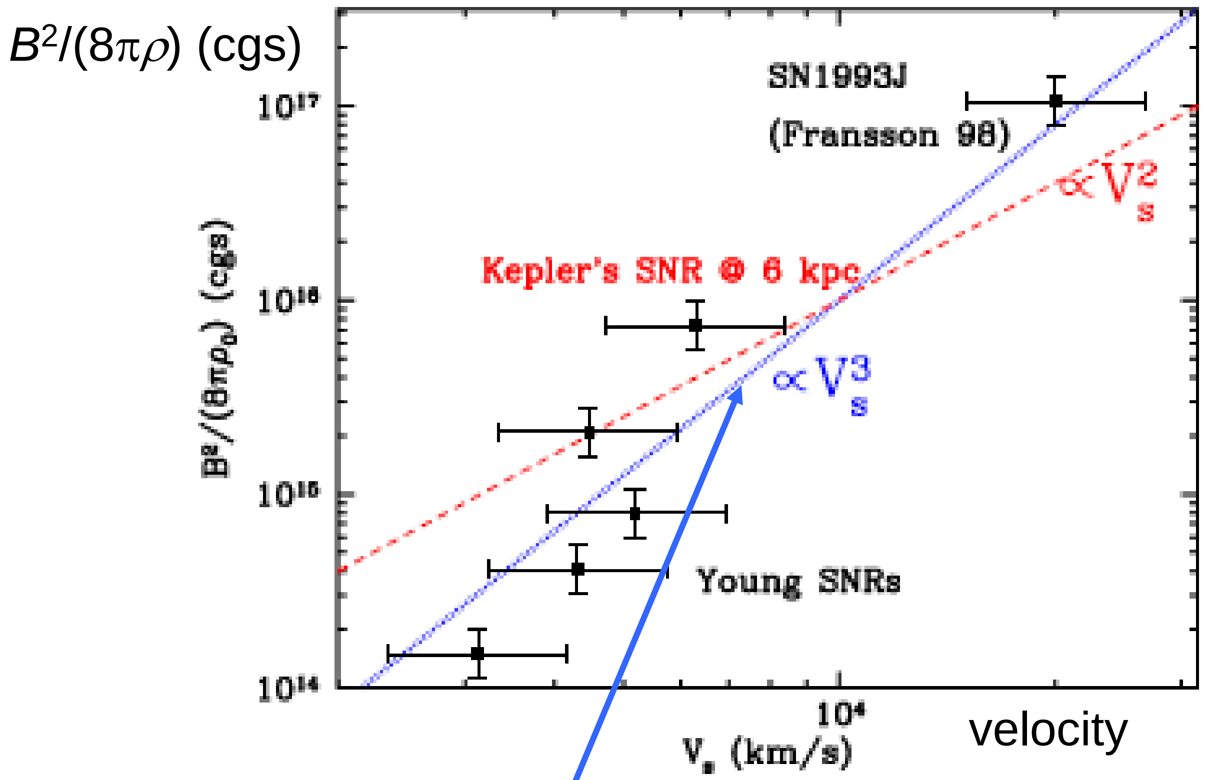
2) $$\frac{p}{eB} \approx L$$

CR Larmor radius scalelength

Set $\nabla = \frac{1}{L}$ **and eliminate** L **between 1) & 2)**

$$\frac{B^2}{\mu_0} \approx \frac{p j_{CR}}{e} \approx \text{efficiency} \times \frac{\rho u_{shock}^3}{c}$$

Inferred downstream magnetic field (Vink 2008)



Data for RCW86, SN1006, Tycho, Kepler, Cas A, SN1993J

Fit to obs (Vink):

$$B \approx 700 \left(\frac{u}{10^4\,\mathrm{kms}^{-1}} \right)^{3/2} \left(\frac{n_e}{\mathrm{cm}^{-3}} \right)^{1/2} \mu\mathrm{G}$$

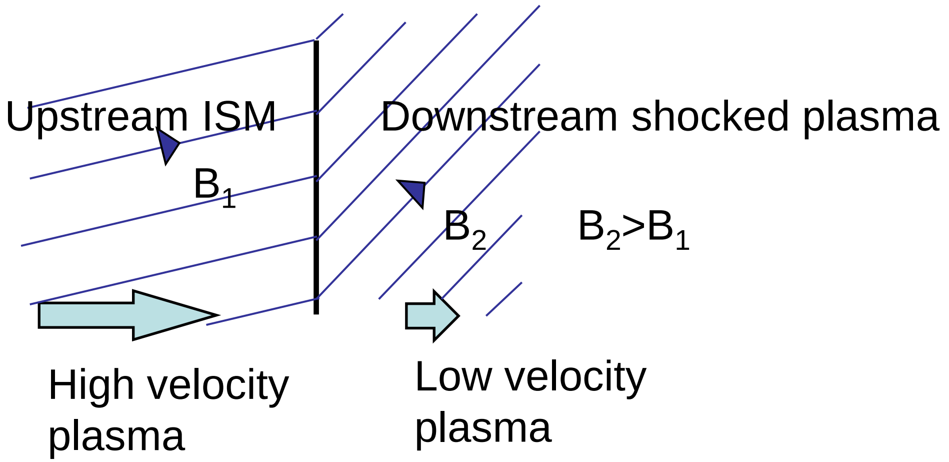
Theory:

$$B \approx 400 \left(\frac{u}{10^4\,\mathrm{kms}^{-1}} \right)^{3/2} \left(\frac{n_e}{\mathrm{cm}^{-3}} \right)^{1/2} \left(\frac{\eta}{0.1} \right)^{1/2} \mu\mathrm{G}$$

The cosmic ray spectrum

revision from p^{-4}

Shock acceleration energy spectrum: loss rate



CR cross from upstream to downstream at rate $n_{shock}c/4$

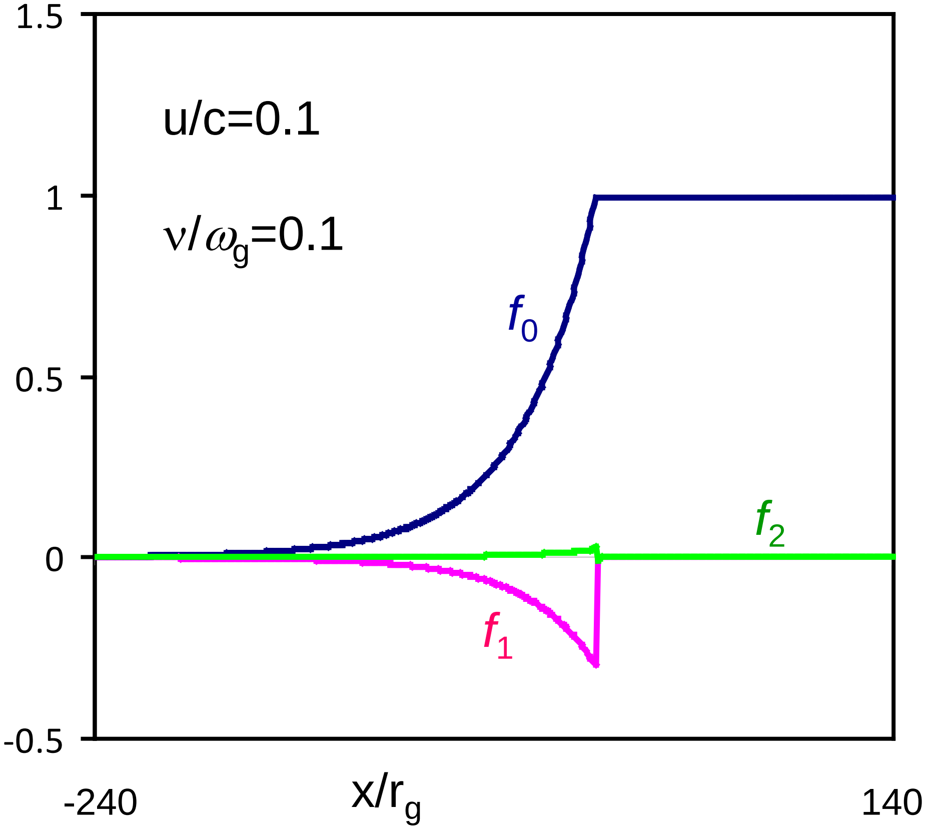
CR carried away downstream at rate = $n_{\mathrm{downstream}}\,\mathrm{v}_s/4$

Fraction lost at each shock crossing

$$\boxed{\frac{\Delta n}{n} = -\frac{\mathrm{v}_s}{c}\frac{n_{downstream}}{n_{shock}}}$$

In diffusive limit $n_{downstream} = n_{shock}$

Parallel shock



Vlasov-Fokker-Planck (VFP) analysis for oblique magnetic field

$$\frac{\partial f}{\partial t}+(\mathrm{v}_x+u)\frac{\partial f}{\partial x}-\frac{\partial u}{\partial x}p_x\frac{\partial f}{\partial p_x}+e\mathbf{v}\times\mathbf{B}.\frac{\partial f}{\partial \mathbf{p}}=C(f)$$

Extra term

Equivalent form of solution for small u/c

$$f = f_0(p)+f_x(p)\frac{p_x}{p}+f_y(p)\frac{p_y}{p}+f_z(p)\frac{p_z}{p}$$

Upstream solution

$$f_x=-\frac{3u_1}{c}f_0 \qquad f_y=\frac{3u_1}{c}\frac{\nu\omega_z}{\nu^2+\omega_x^2}f_0 \qquad f_z=\frac{3u_1}{c}\frac{\omega_z^2}{\nu^2+\omega_x^2}f_0$$

$$\boldsymbol{\omega}=\frac{e\mathbf{B}}{\gamma m_p}$$

Downstream solution

$$f_x=f_y=f_z=0$$

Cannot match f_y & f_z across the shock

$$\frac{\partial f}{\partial t}+(\mathrm{v}_x+u)\frac{\partial f}{\partial x}-\frac{\partial u}{\partial x}p_x\frac{\partial f}{\partial p_x}+e\mathbf{v}\times\mathbf{B}.\frac{\partial f}{\partial \mathbf{p}}=C(f)$$

Extra term

Equivalent form of solution for small u/c

$$f = f_0(p)+f_x(p)\frac{p_x}{p}+f_y(p)\frac{p_y}{p}+f_z(p)\frac{p_z}{p}$$

NOT VALID SOLUTION

Upstream solution

$$f_x=-\frac{3u_1}{c}f_0 \qquad f_y=\frac{3u_1}{c}\frac{\nu\omega_z}{\nu^2+\omega_x^2}f_0 \qquad f_z=\frac{3u_1}{c}\frac{\omega_z^2}{\nu^2+\omega_x^2}f_0$$

Downstream solution

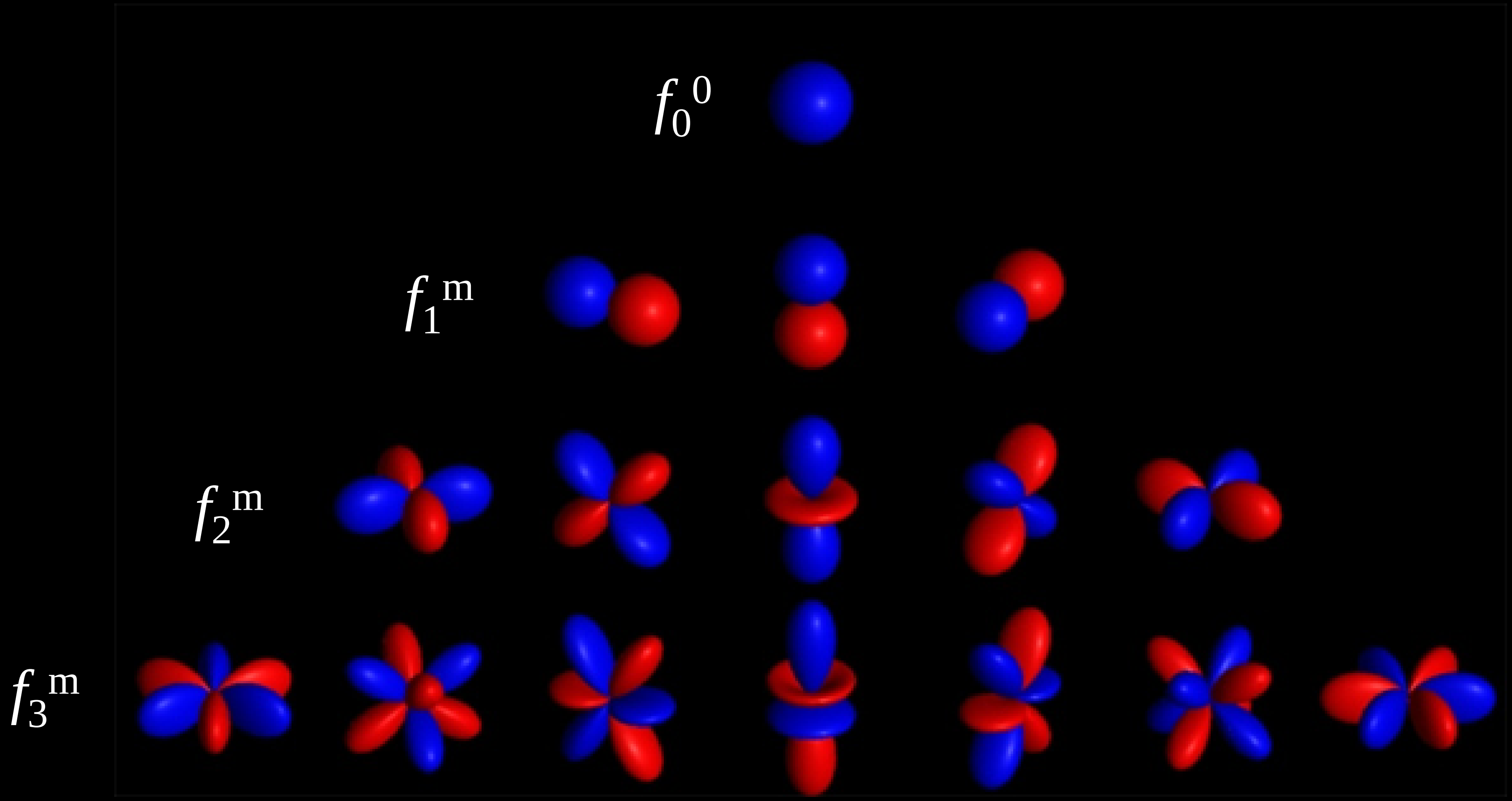
$$f_x=f_y=f_z=0$$

$$\boldsymbol{\omega}=\frac{e\mathbf{B}}{\gamma m_p}$$

Cannot match f_y & f_z across the shock

Expand in spherical harmonics

www.trinnov-audio.com/images/sphericalHarm.jpg



$$f(x,\mathbf{p},t) = \sum_{l,m} f_l^{\,m}\left(x,|\mathbf{p}|,t\right) P_l^m(\cos\vartheta)\exp\left(im\phi\right)$$

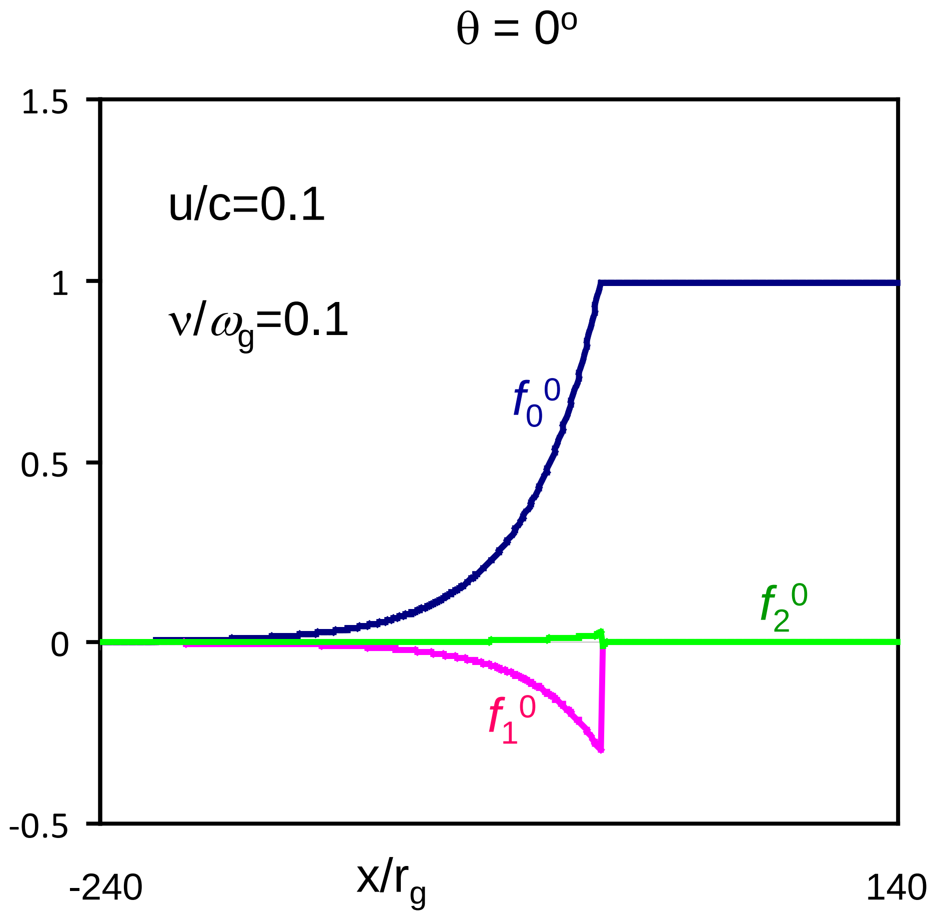
Equation for evolution of each spherical harmonic $f(x,\mathbf{p},t)=\sum_{l,m} f_l^m(x,p,t)P_l^m(\cos\theta)e^{im\phi}$

$$\frac{\partial f_l^m}{\partial t}+u\frac{\partial f_l^m}{\partial x}+c\left[\frac{l-m}{2l-1}\frac{\partial f_{l-1}^m}{\partial x}+\frac{l+m+1}{2l+3}\frac{\partial f_{l+1}^m}{\partial x}\right]$$
$$+im\frac{ceB_x}{p}f_l^m+\frac{ceB_z}{2p}\beta_l^m$$
$$-p\frac{\partial u}{\partial x}\left[\frac{(l-m)(l-m-1)}{(2l-3)(2l-1)}\left(\frac{\partial f_{l-2}^m}{\partial p}-(l-2)\frac{f_{l-2}^m}{p}\right)\right.$$
$$+\frac{(l-m)(l+m)}{(2l-1)(2l+1)}\left(\frac{\partial f_l^m}{\partial p}+(l+1)\frac{f_l^m}{p}\right)$$
$$+\frac{(l-m+1)(l+m+1)}{(2l+1)(2l+3)}\left(\frac{\partial f_l^m}{\partial p}-l\frac{f_l^m}{p}\right)$$
$$\left.+\frac{(l+m+1)(l+m+2)}{(2l+3)(2l+5)}\left(\frac{\partial f_{l+2}^m}{\partial p}+(l+3)\frac{f_{l+2}^m}{p}\right)\right]$$
$$-pu\frac{\partial u}{\partial x}\left[\frac{l-m}{2l-1}\left(\frac{\partial f_{l-1}^m}{\partial p}-(l-1)\frac{f_{l-1}^m}{p}\right)\right.$$
$$\left.+\frac{l+m+1}{2l+3}\left(\frac{\partial f_{l+1}^m}{\partial p}+(l+2)\frac{f_{l+1}^m}{p}\right)\right]$$
$$=-\frac{l(l+1)}{2}\nu f_l^m$$

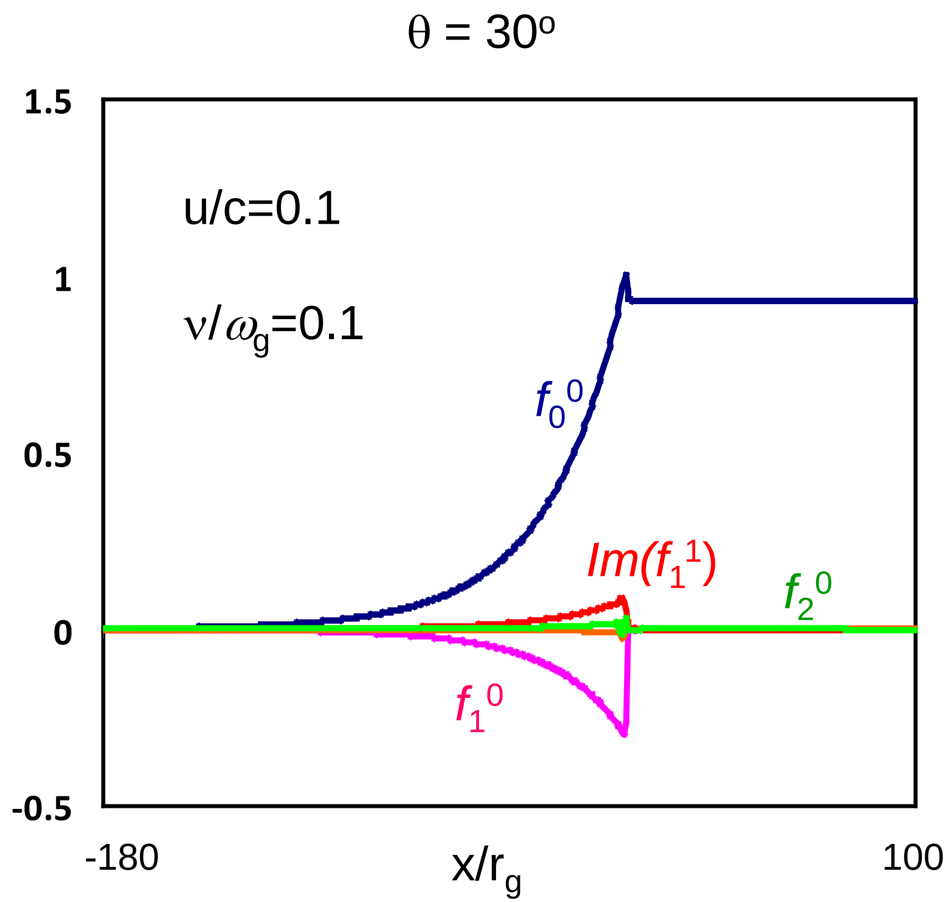
Solve numerically

where $\beta_l^m=(l-m)(l+m+1)f_l^{m+1}-f_l^{m-1}$ for $m>0$ and $\beta_l^0=2\Re(f_l^1)$.

Parallel shock



Oblique shock (nearly parallel)



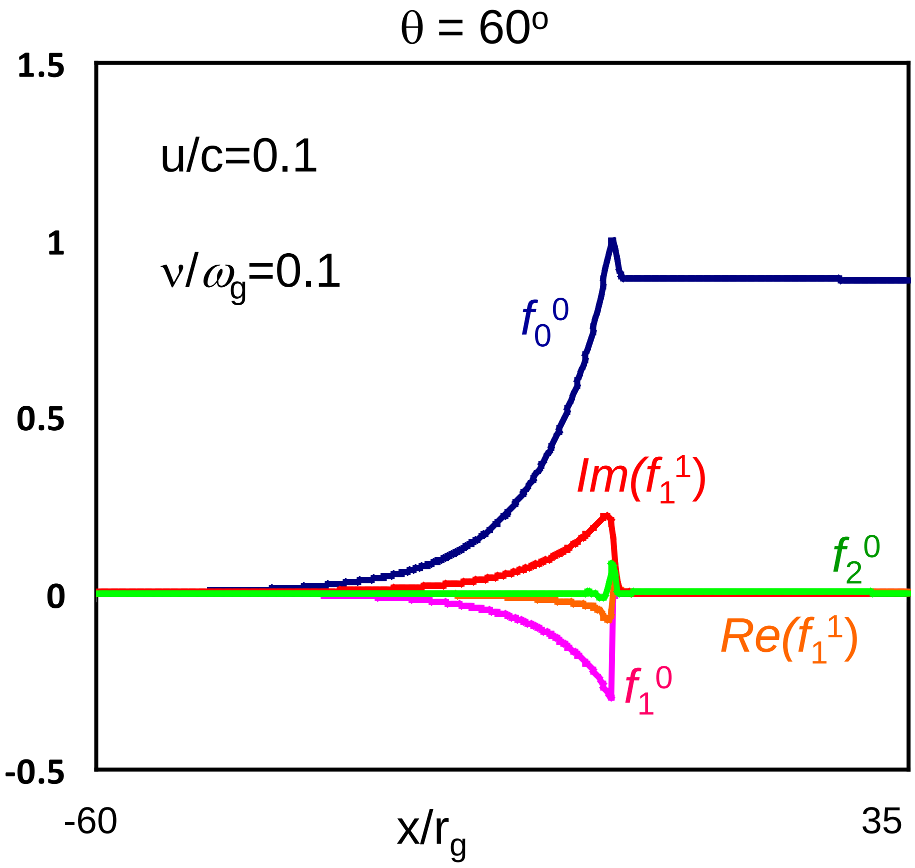
Density spike at shock as seen by

Ostrowski MNRAS **249** 551 (1991)

Ruffalo, ApJ **515** 787 (1999)

Gieseler, Kirk, Heavens & Achterberg A&A **345** 298 (1999)

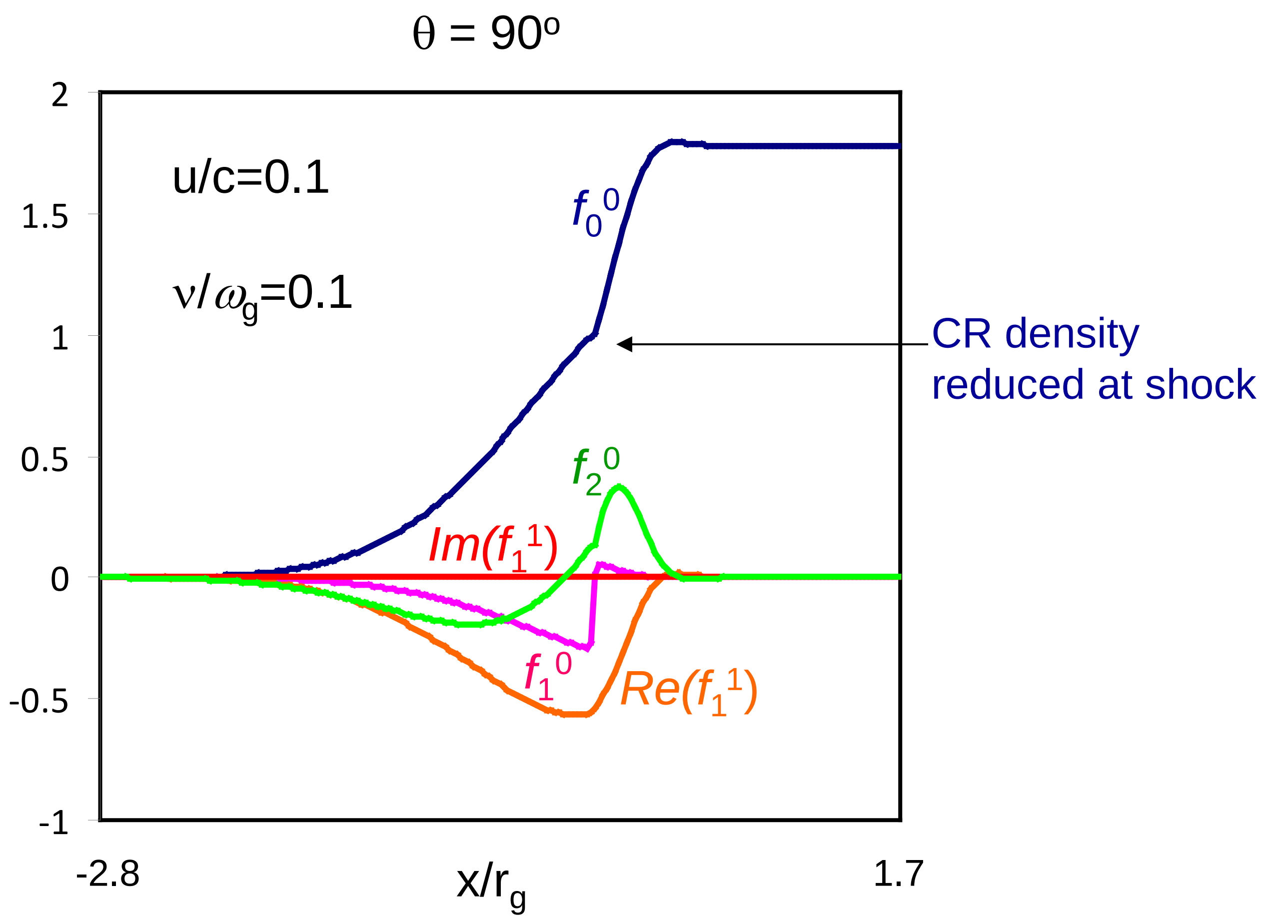
More perpendicular, less parallel



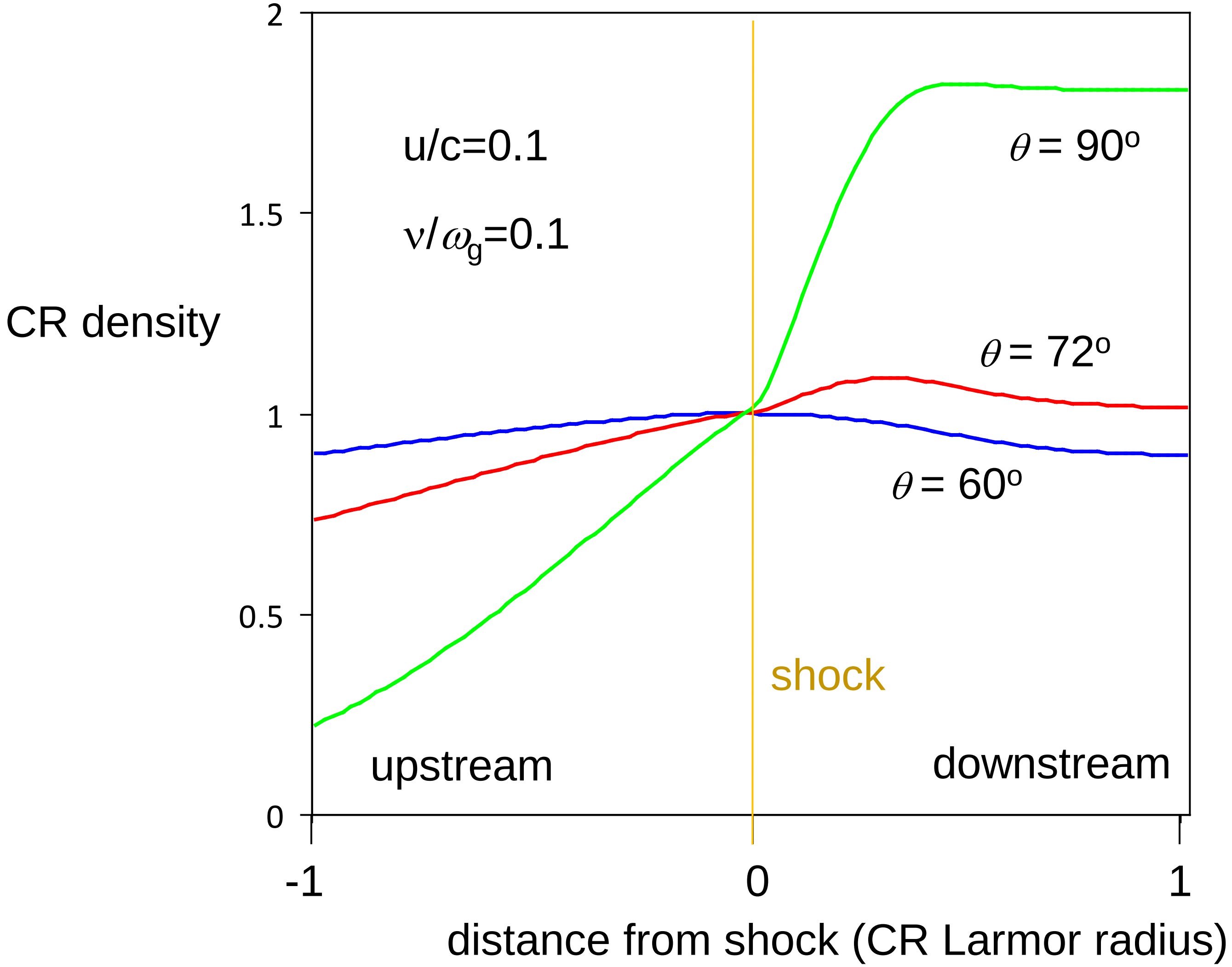
Re(f_1^1) represents cross-field drift

Im(f_1^1) represents drift along oblique field lines

Peperpendicular shock

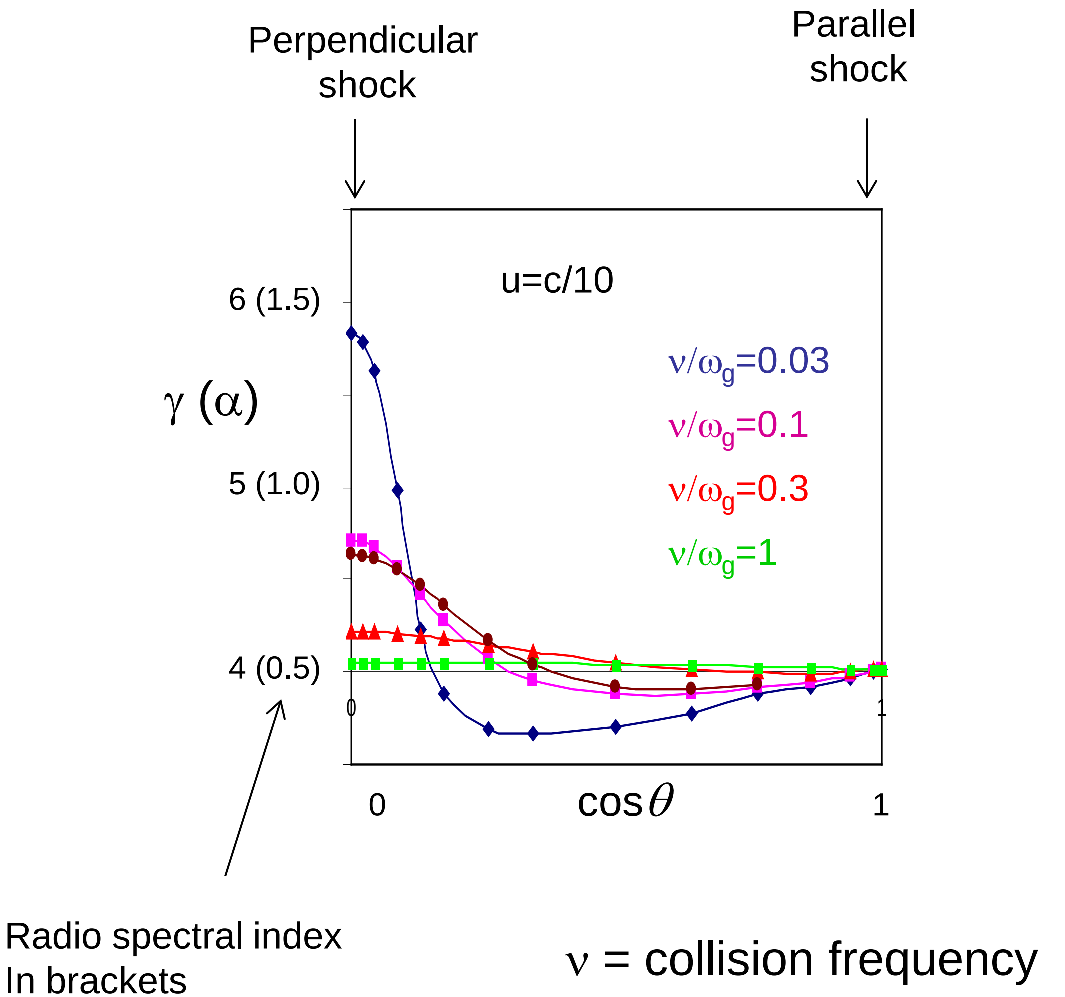


CR density profiles near shock



Spectral index plotted against shock obliquity

Shock compression x4 in all cases



ν = collision frequency

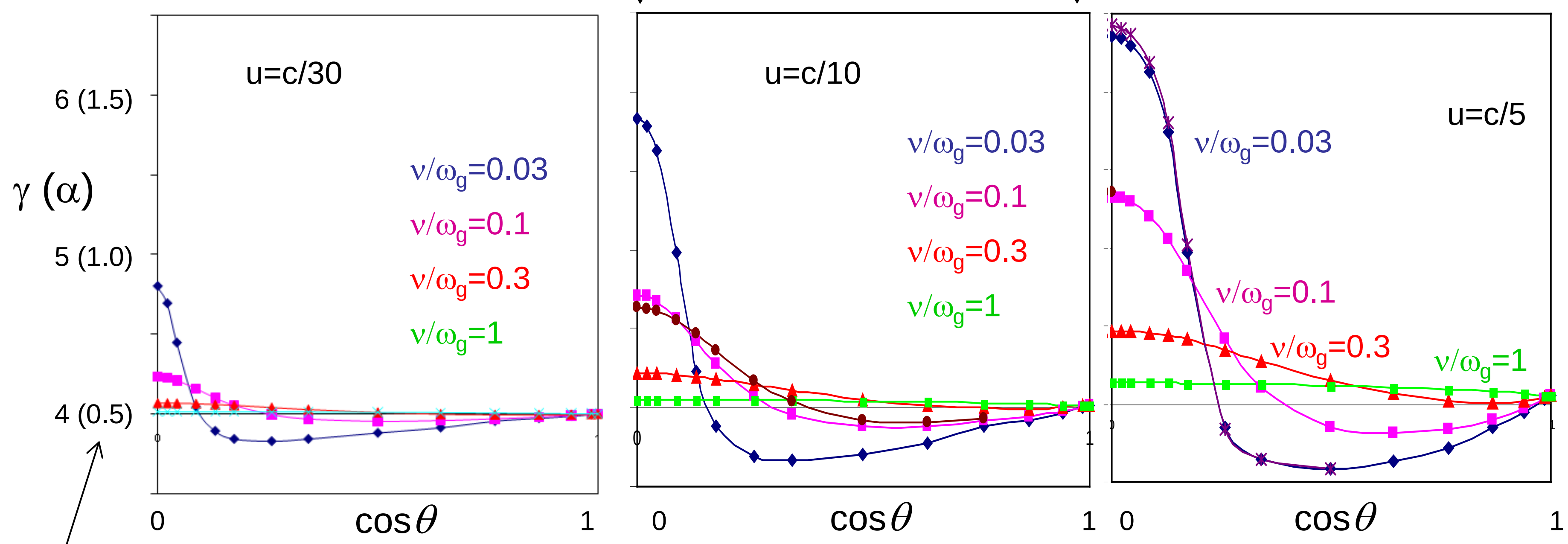
ω_g = Larmor frequency

Spectral index plotted against shock obliquity

Shock compression x4 in all cases

Perpendicular shock

Parallel shock



Radio spectral index
In brackets

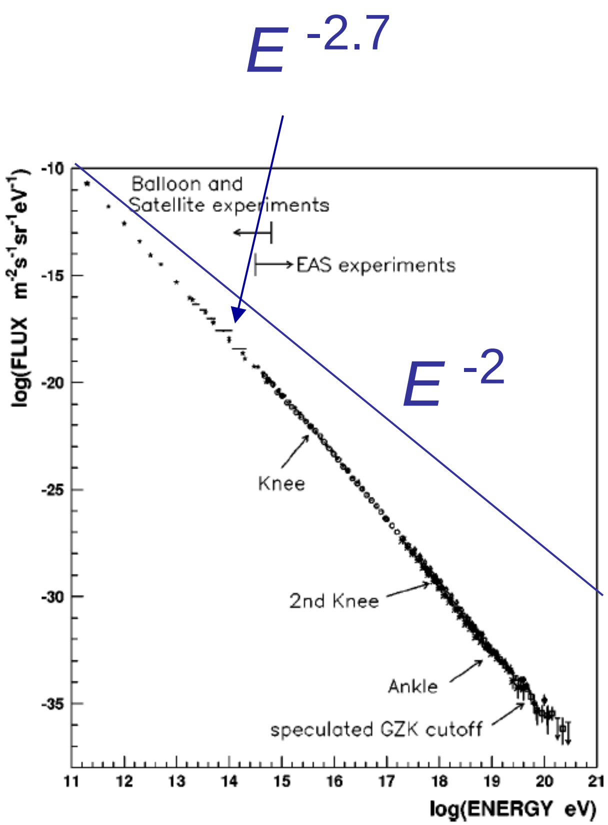
ν = collision frequency

ω_g = Larmor frequency

Observations

Cosmic Ray spectrum arriving at earth

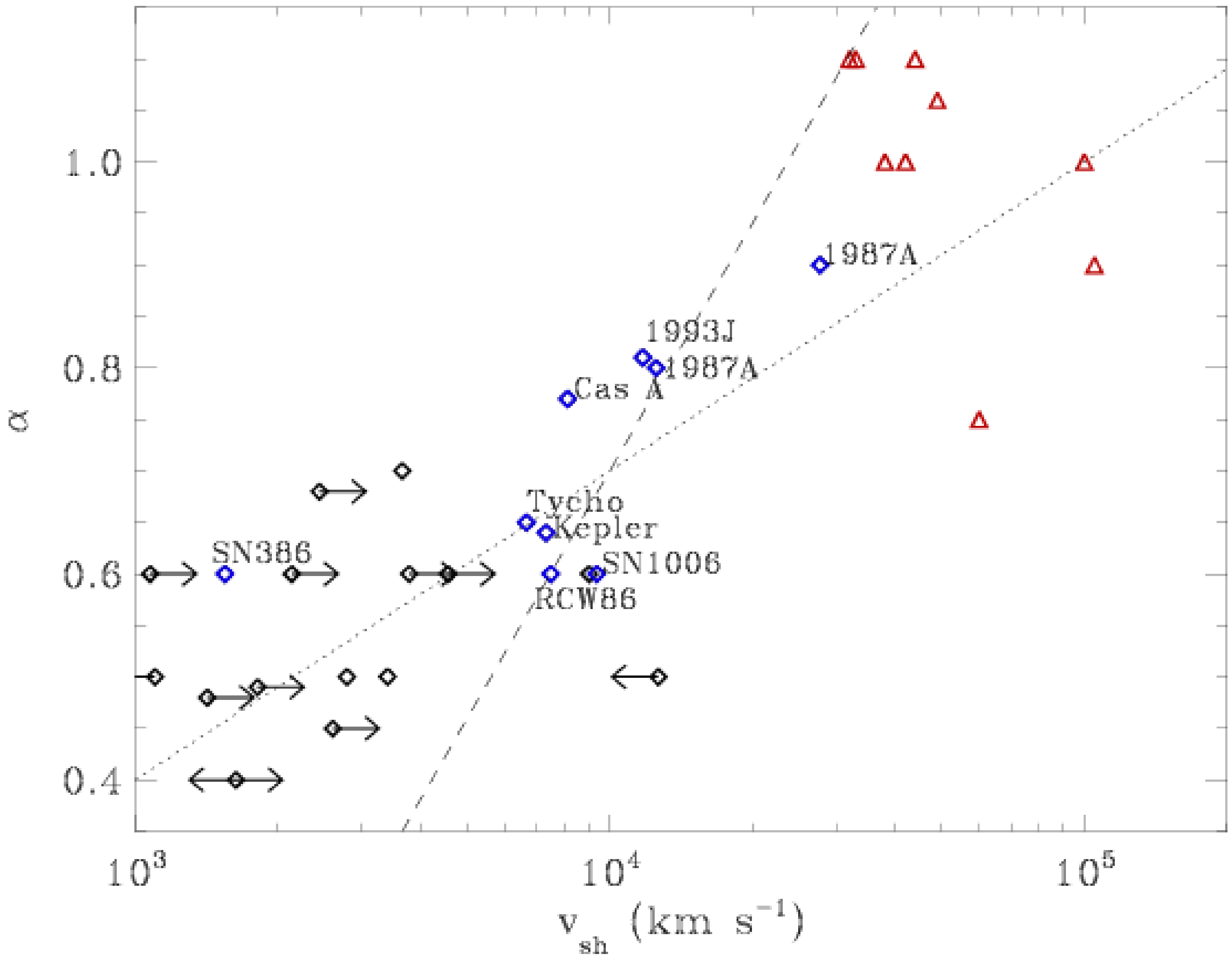
(Nagano & Watson 2000)



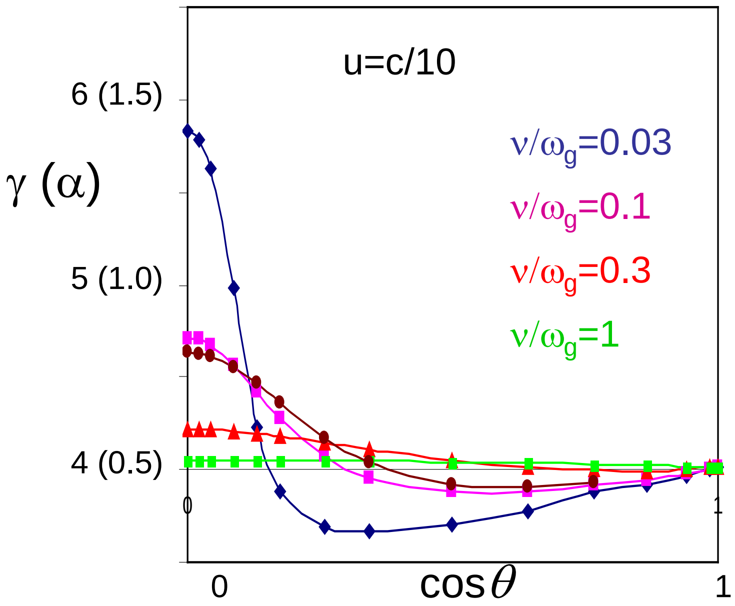
Leakage from galaxy accounts for some of difference (Hillas 2005)

Observed radio spectral index v. mean expansion velocity

(following Glushak 1985)



Spectral steepening suggests quasi-perpendicular shocks

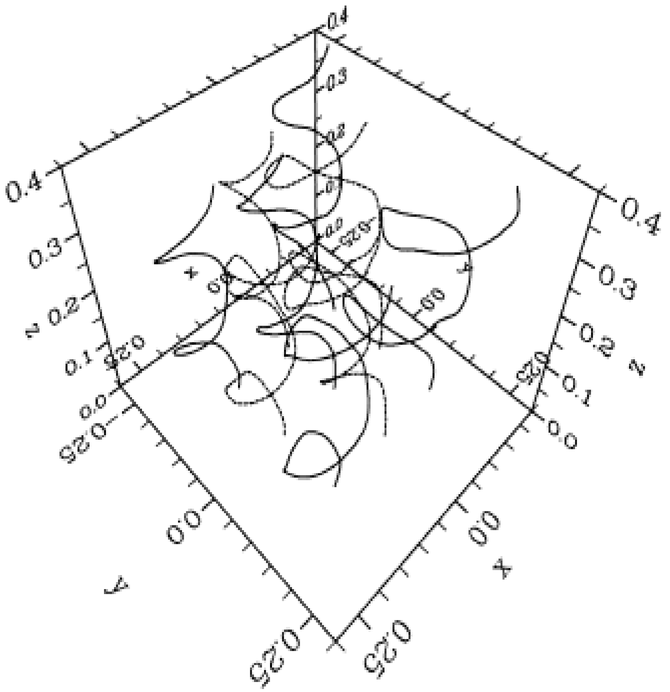


For random field orientation,
steepening & flattening nearly cancel out

Steepening at high velocity might be due to

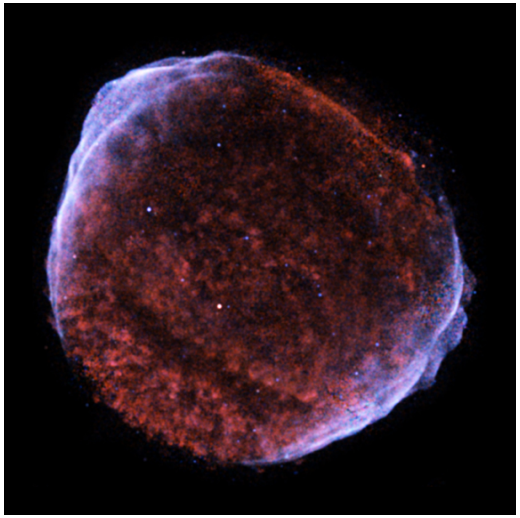
1) expansion into Parker spiral

2) magnetic field amplification
jxB stretches field perpendicular to shock normal



SNR morphology: spectral steepening/flattening

SN1006 (Chandra)
CR electrons (10-100TeV)



bi-polar emission (Rothenflug et al 2004)

Could be:

1) Quasi-perpendicular shocks accelerate fewer CR to high energy
2) Poor injection at quasi-perpendicular shocks

What we think we know, and what we know we don’t know

What we think we know

- Galactic CR are accelerated to 10^{15} eV by diffusive shock acceleration by SNR
- Streaming CR amplify the magnetic field which confines CR near shock
- CR spectra are often steeper than p^{-4} (E^{-2})

What we don’t know

- How CR reach 10^{16} -10^{17}eV
- When CR are accelerated to what energy at different stages of SNR evolution
- How CR escape SNR without losing energy adiabatically
- When & where non-linear effects are important
- Why typically the spectrum is flatter than p^{-4} in older SNR
- Why is the spectrum so straight?
- Whether second order Fermi acceleration contributes substantially
- Whether perpendicular shocks are good injectors of low energy CR
- How the above applies extra-galactically - the origin of 10^{20}eV CR