Numerical Resistivity, Towards SPH MRI studies

Ringberg – 20th July 2011

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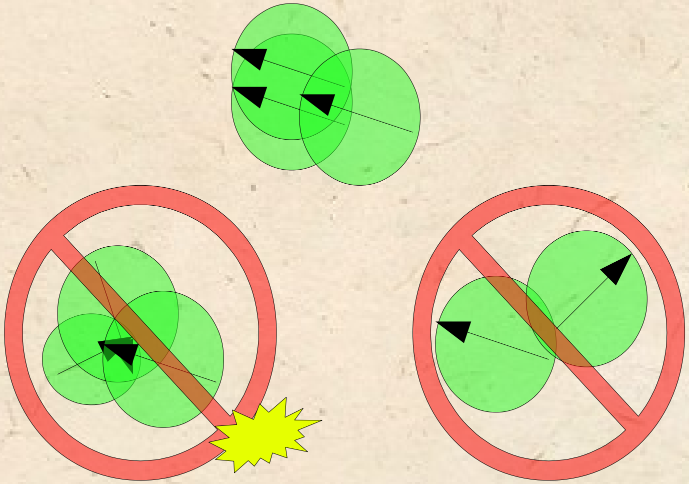
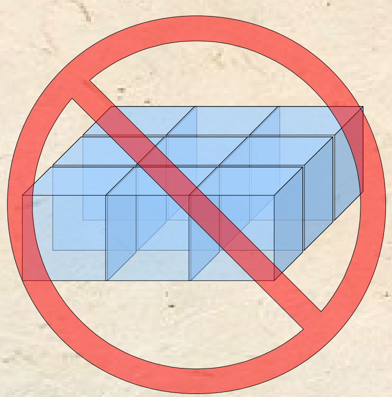
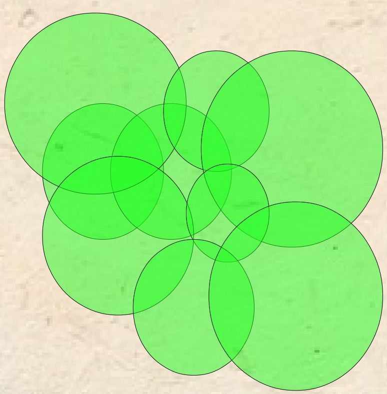
Numerical MHD

- Smoothed Particles Hydrodinamics:
  - Natural Adaptativity and Huge Dynamical Range
  - Easy Gravity Calculation
  - Small Mixing
  - Galilean Invariance
  - ...







Approaches

- Suppresion instabilities by
  - Cleaning Schemes
  - Smoothing of the Field
  - Art. Dissipation
- Euler Potentials
- Vector Potential (?)

Physical Motivation.....

Induction Eq.

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B} + \nabla \times \eta \vec{B})$$

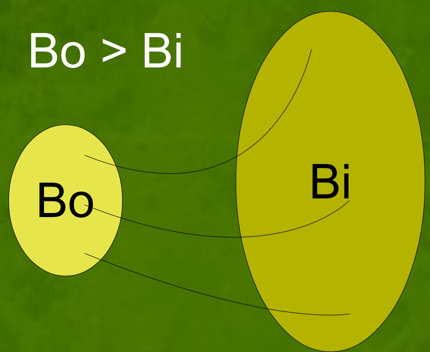
Ideal MHD

Non Ideal MHD

Not now

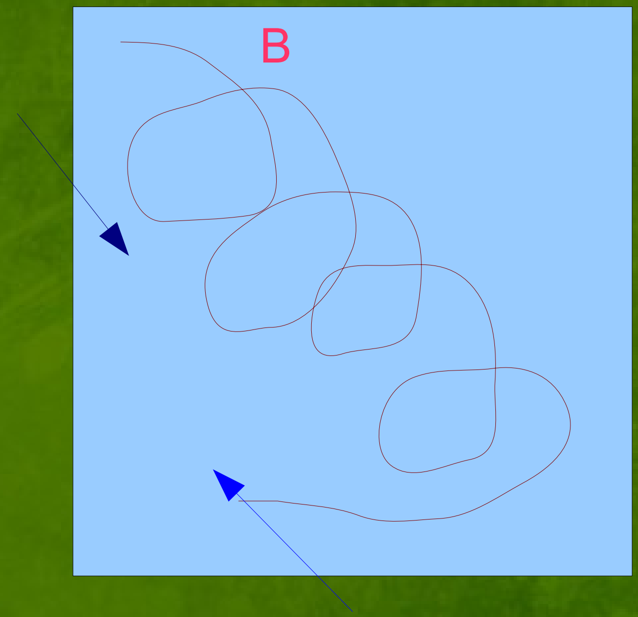
Ideal Induction: $\nabla \times (\vec{V} \times \vec{B})$

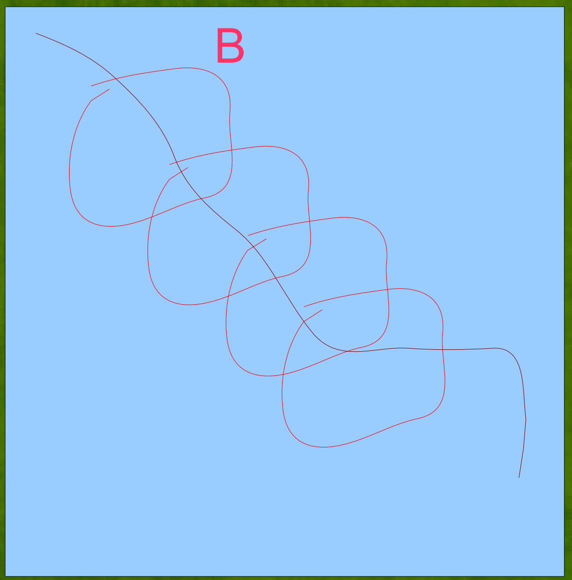
Bo > Bi



- Flux conservation
- No reconeccion!

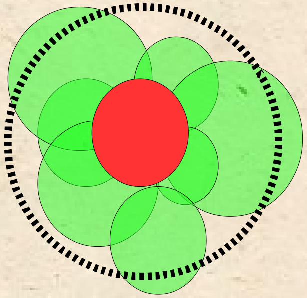
$$\vec{B} \propto \rho^{2/3}$$



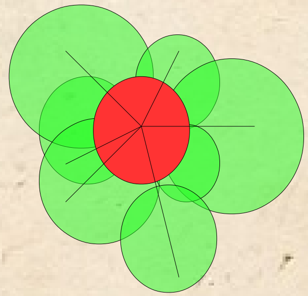


Orszag-Tang Vortex

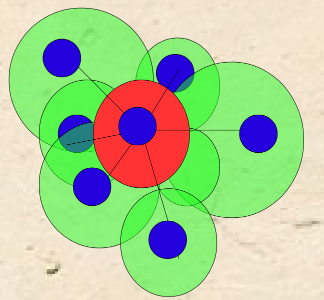
Smoothing:



Art. Dissipation:

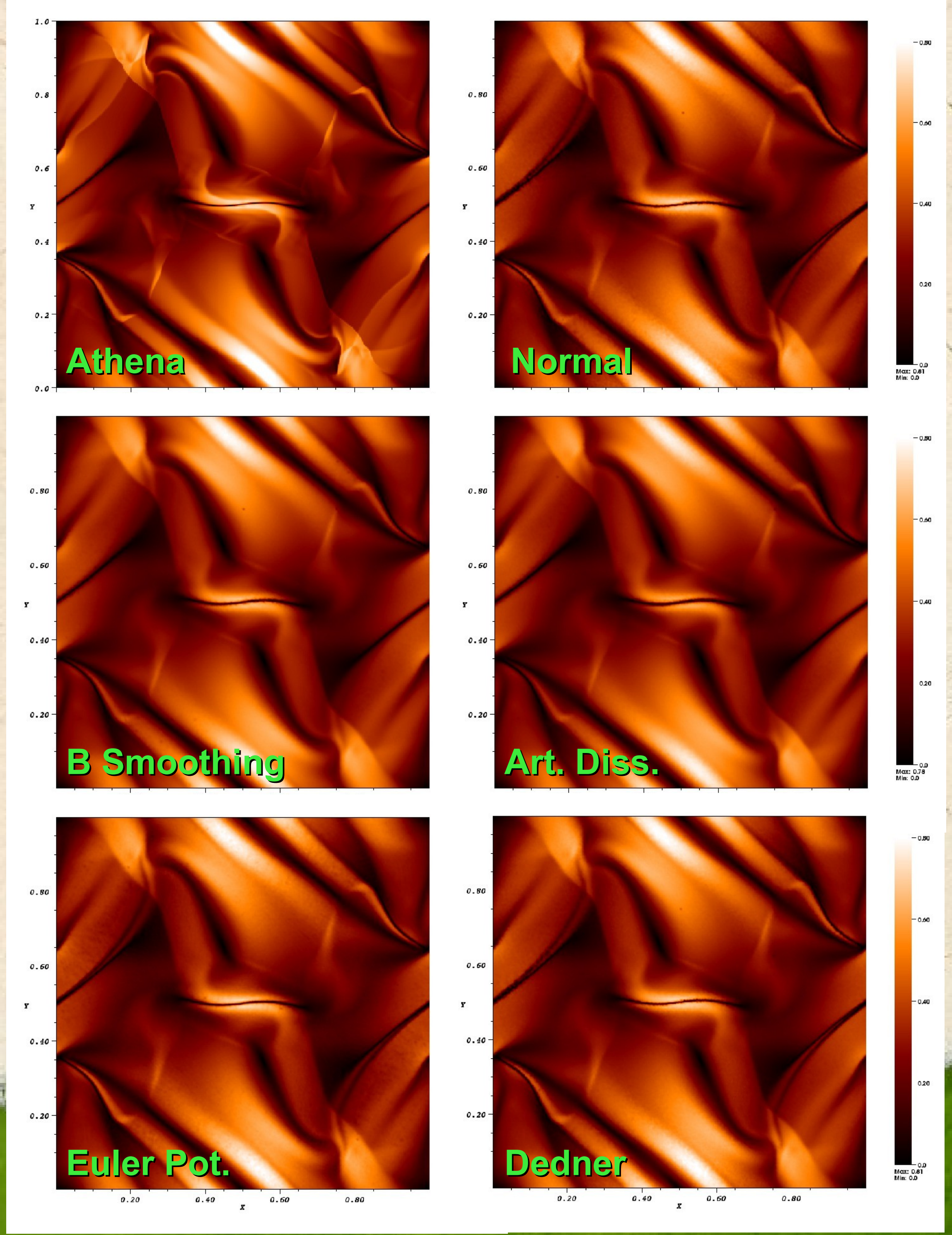


Cleaning Scheme (Dedner):

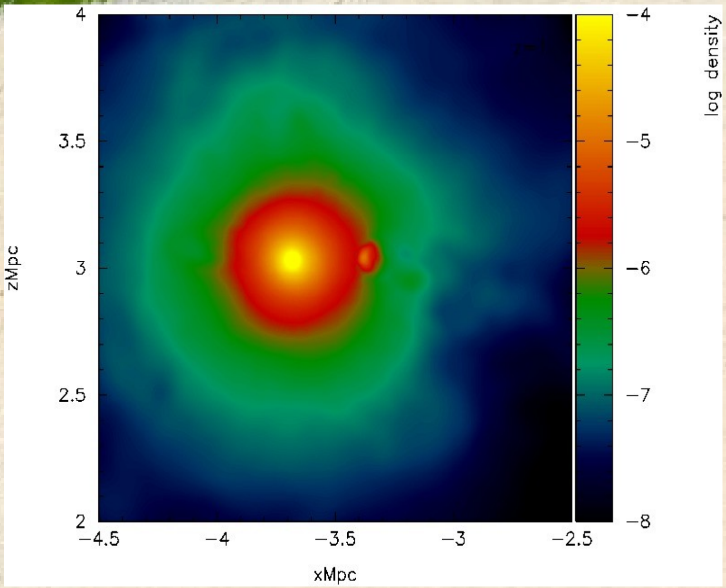


Euler Potencials: $\vec{B} = \vec{\nabla}\alpha \times \vec{\nabla}\beta$

To a code or scheme be reliable has to complete resolution convergence and pass the full 1D/2D/3D test suite.

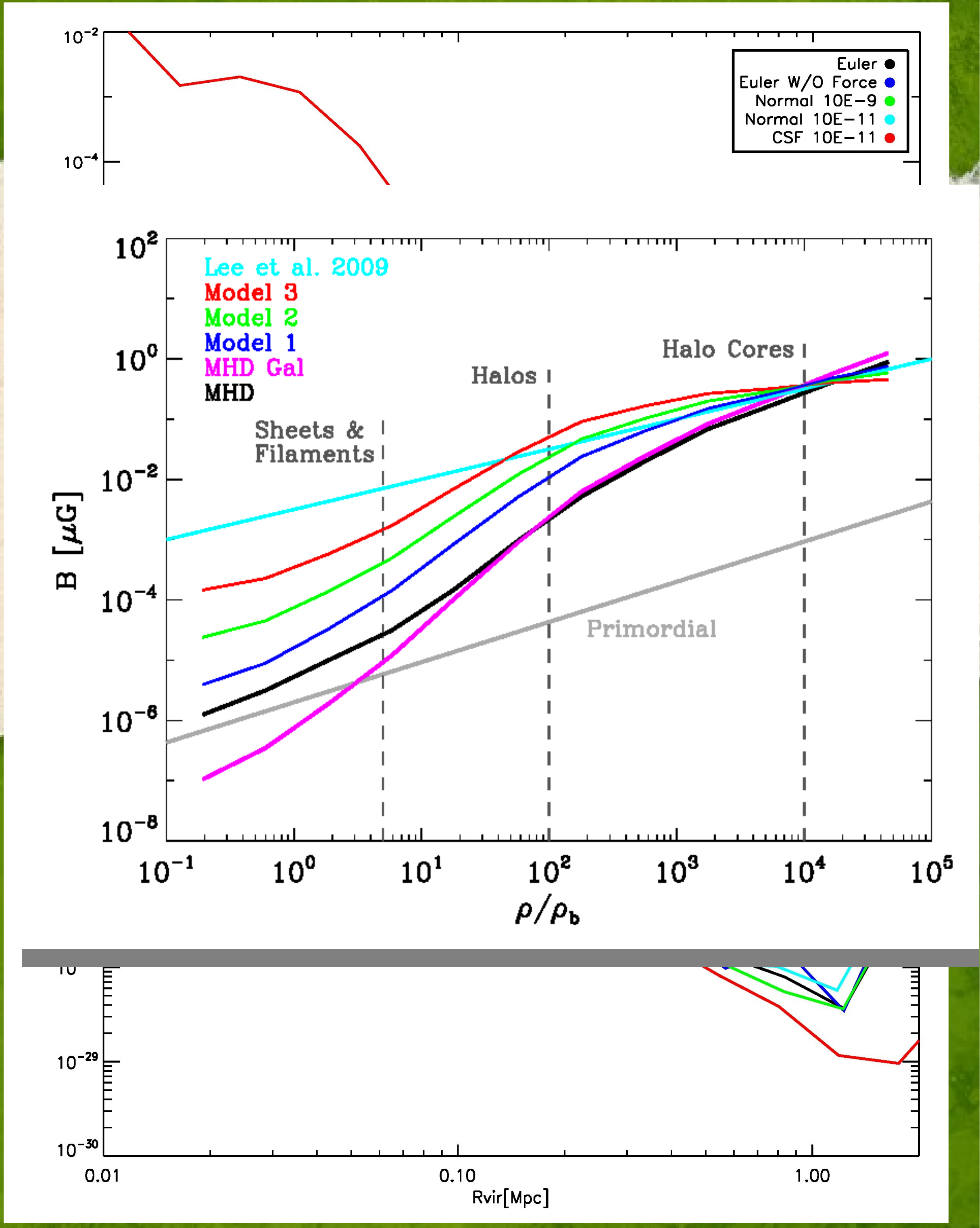


Galaxy Clusters

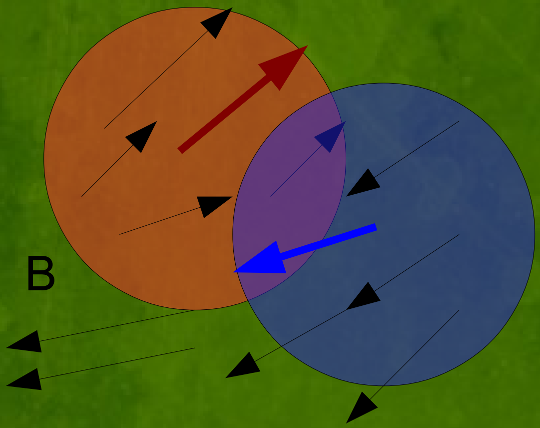


Everything is Nice, but....
Ideal MHD does not allow
Recconection, therefore:
$\vec{B} \propto \rho^{2/3}$

There is some recconection!

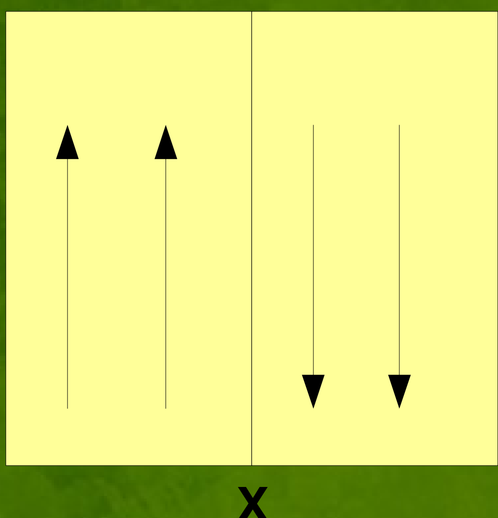


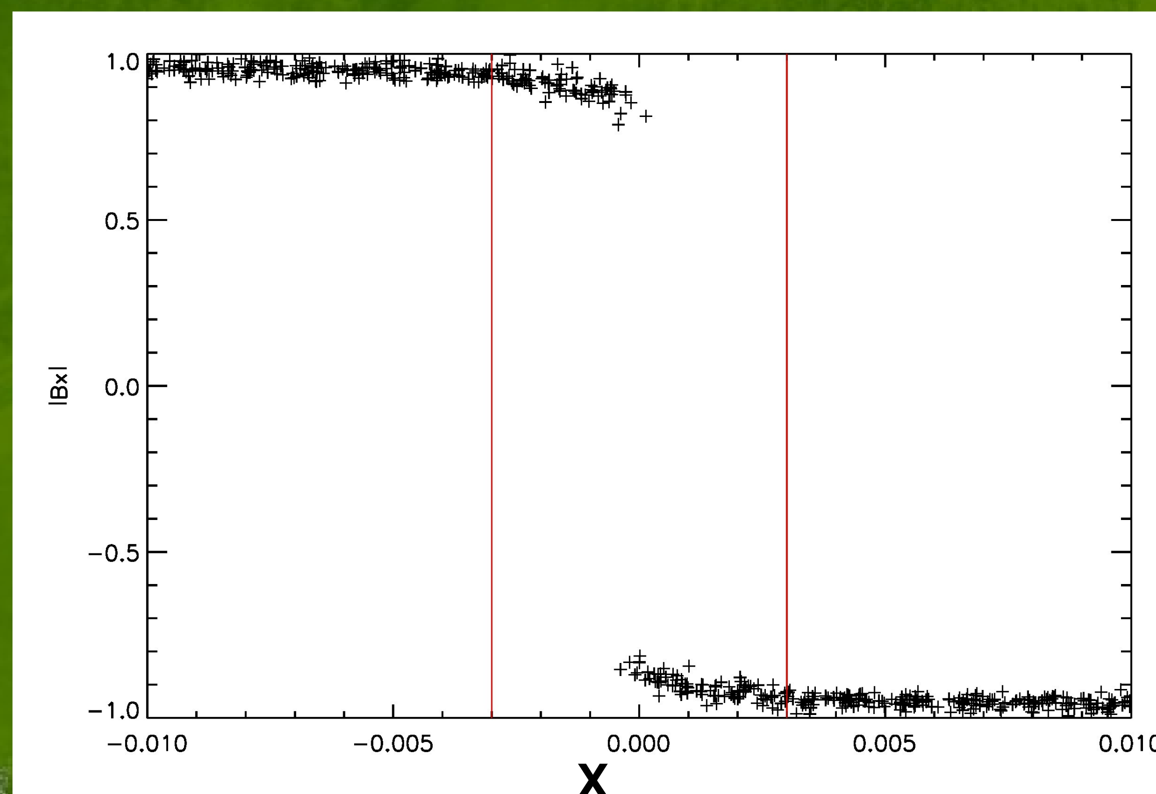
Numerical Dissipation - Shock



The SPH feature of deriving the quantities from the neighbours inside the SPH kernel (HSML) allows some “dissipation” to happen

Let's try to measure it





Easy case, we
can solve it analitically:

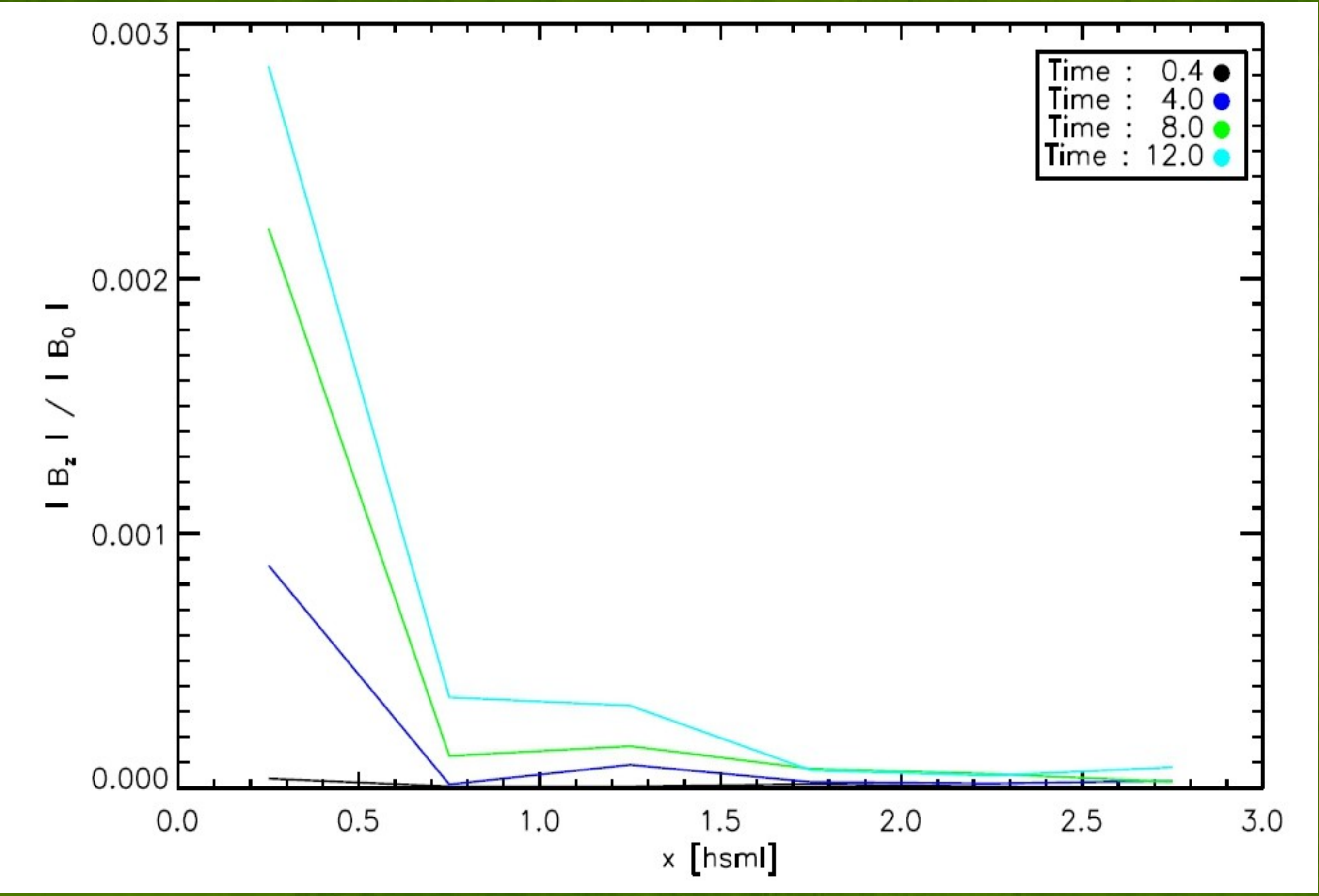
$$\frac{\Delta B}{\Delta T} = 2\,\eta_m \frac{B_0}{\Delta X^2}$$

Numerical Dissipation - Shock

Now we can measure as function of h and Vch:

$$\eta_m = \frac{1}{2}\frac{\Delta B}{B_0} \cdot h \cdot v_{ch}$$

Test	Hsml	v_a	η_m	Re_m
Standard	0.50	$2.8e-1$	$1.8e-6$	$7.8e+4$
Dedner	0.50	$2.8e-1$	$4.8e-6$	$2.9e+4$
B-Smooth #15	0.46	$1.2e-2$	$1.7e-5$	$3.2e+2$
B-Smooth #30	0.47	$1.3e-2$	$1.5e-5$	$4.1e+2$
Art. Diss. $\alpha_B = 0.1$	0.49	$3.2e-2$	$6.3e-5$	$2.5e+2$
Art. Diss. $\alpha_B = 0.5$	0.51	$1.3e-2$	$2.0e-5$	$3.3e+2$

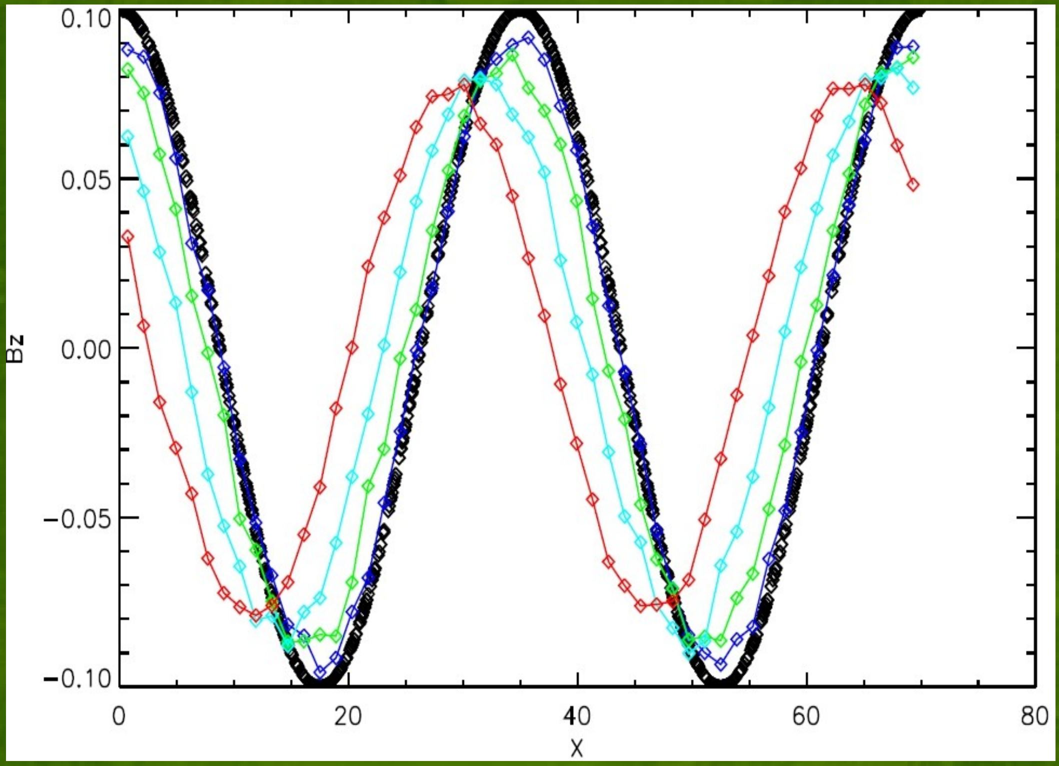


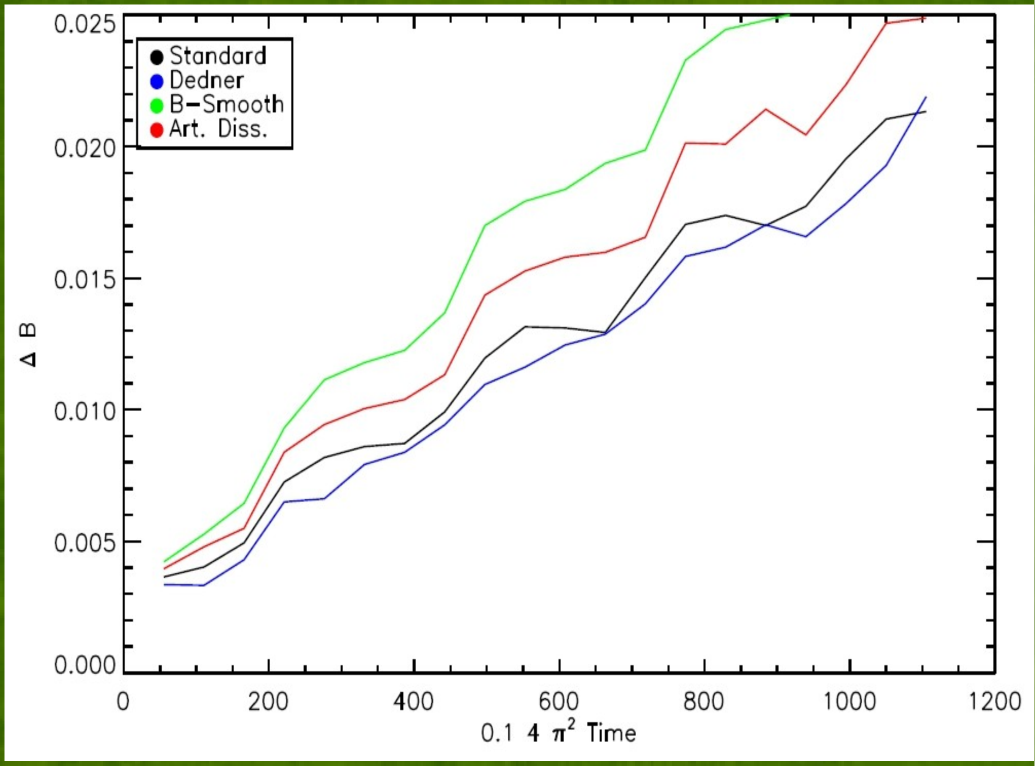
CAUTION:

•This effects only acts inside the kernel smoothing length, it does not dissipate at larger scales

•It is time independent and function of h

Numerical Dissipation – CPAW



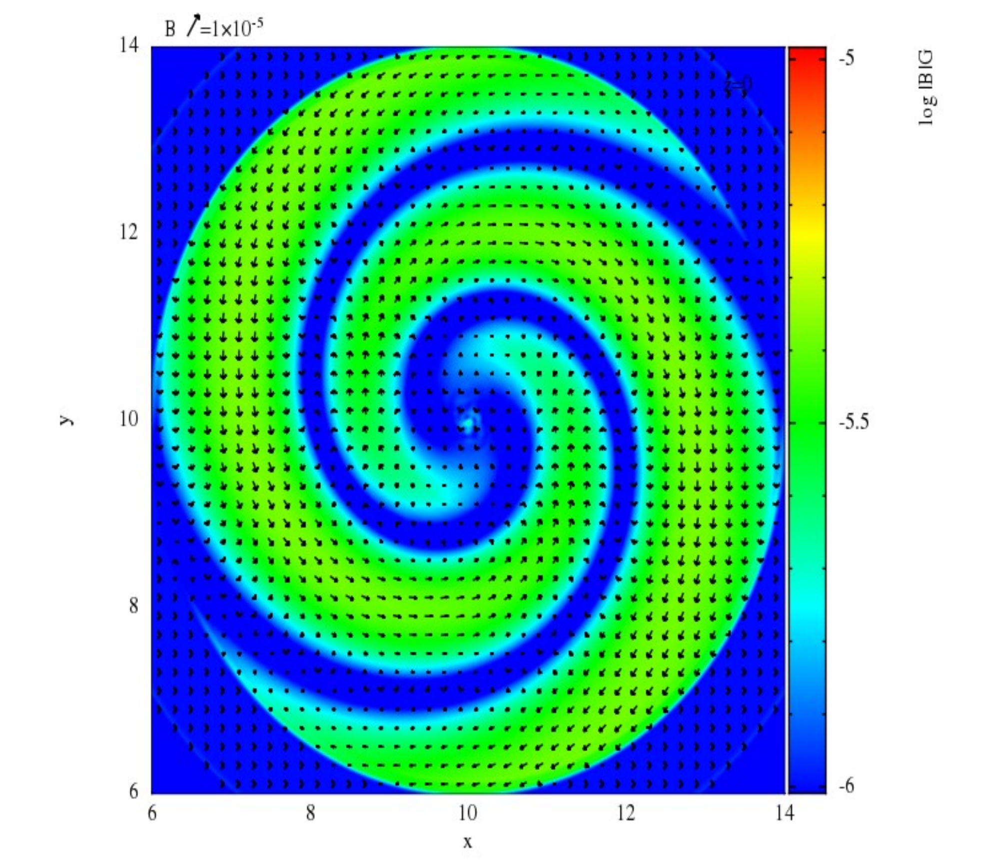


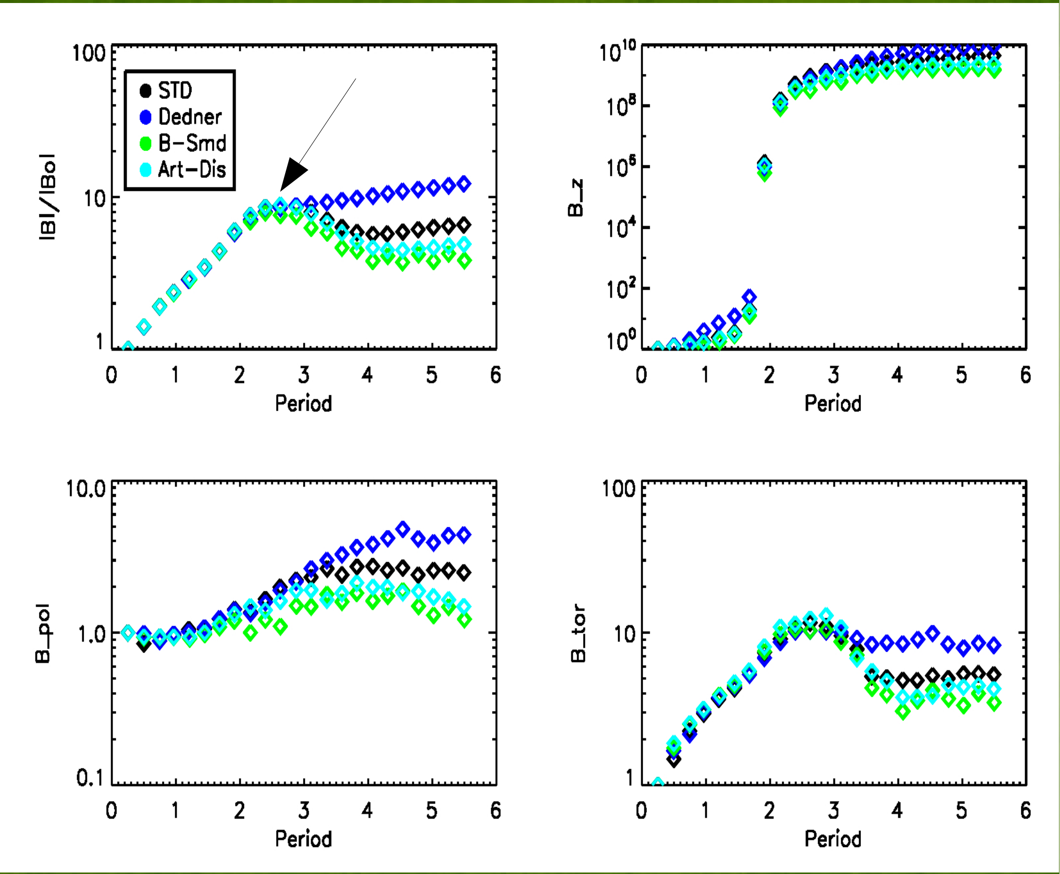
We can measure the change in each period:

$$\Delta B_Z = -0.1\,\eta_m 4\pi \Delta T$$

Test	η_m	Re_m
Standard	$1.69e-5$	$7.1e+3$
Dedner	$1.67e-5$	$7.2e+3$
B-Smooth #15	$2.42e-5$	$4.9e+3$
Art. Diss. $\alpha_B = 0.1$	$2.00e-5$	$6.0e+3$

Numerical Dissipation – Winding Up





We can measure the change in each period:

$$\eta_0 = \frac{G}{\tau_0^2}$$

Summary

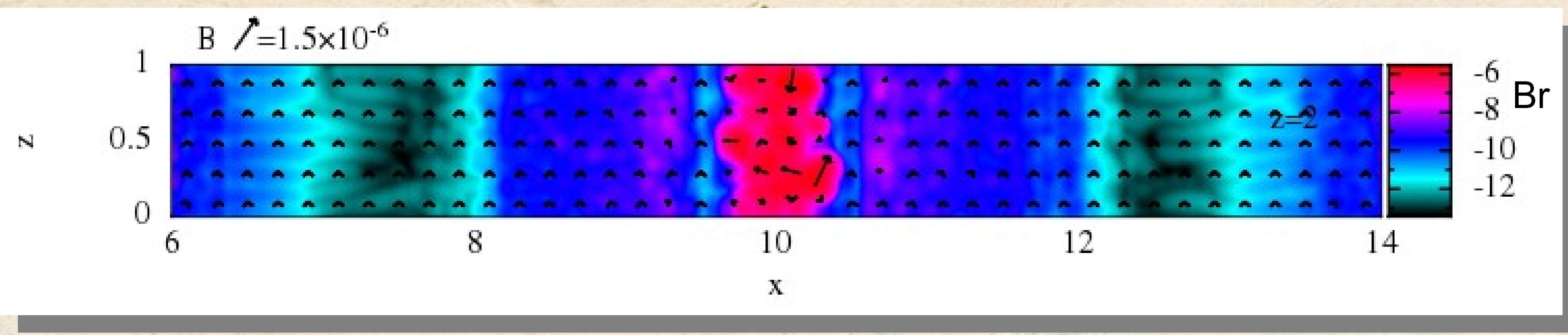
-Ideal MHD simulations have a small numerical dissipation.

-This feature, allow us to find simulation results that are in good agreemnt with observations, however we don't have a complete control. Need resolution studies.

- Help to explain some numerical instbilities.

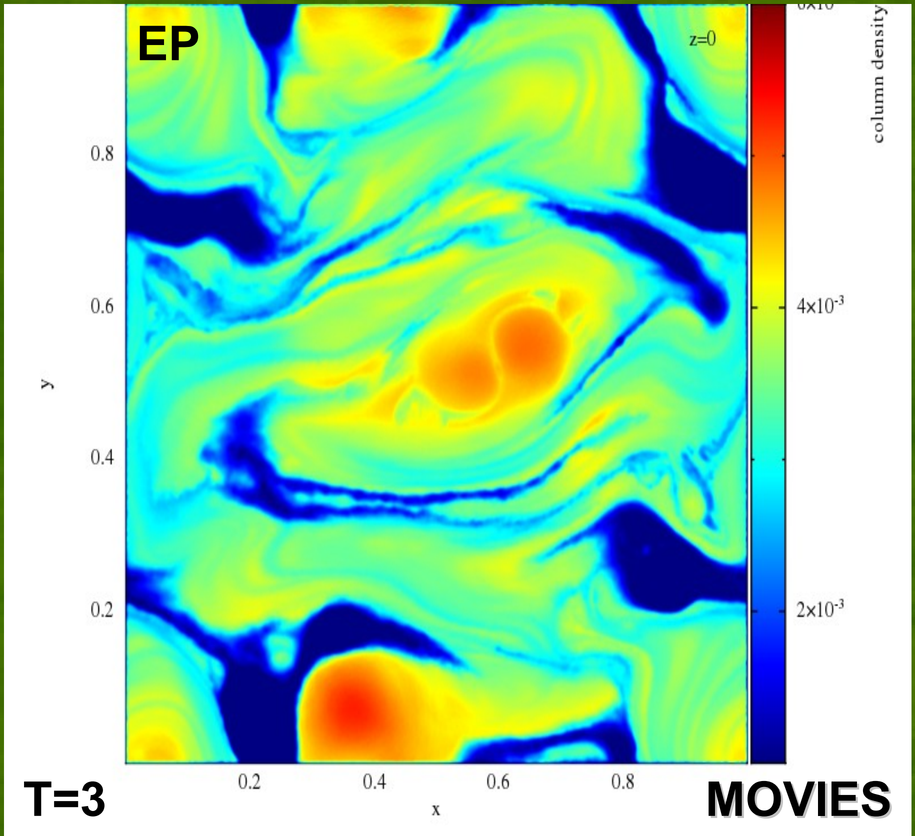
-The new physics implemented (or to be) takes profit of the Low numerical dissipation, therefore favoring regularization schemes that preserves the “idealness” of the MHD Implementation.

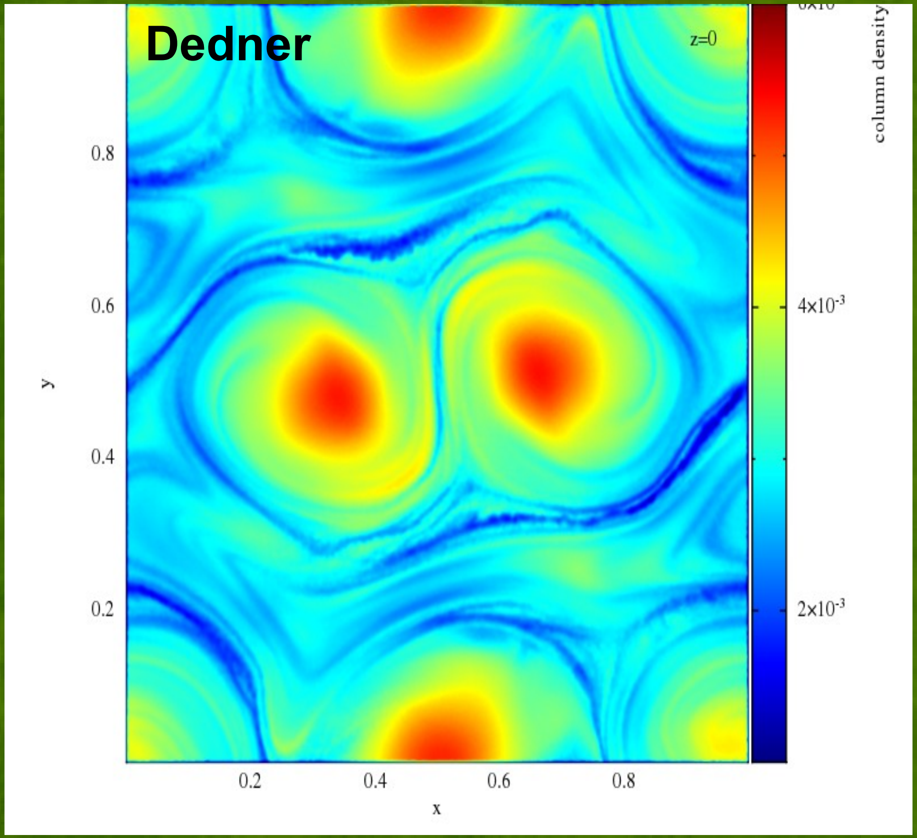
MRI



How this affects my life?

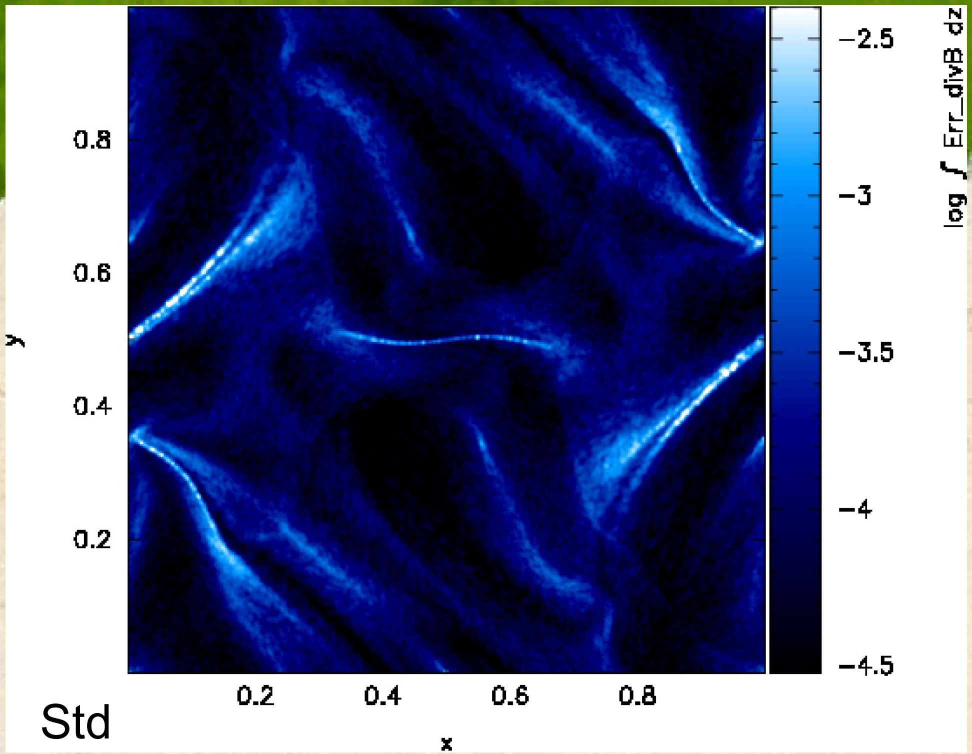
-Euler Potencial case

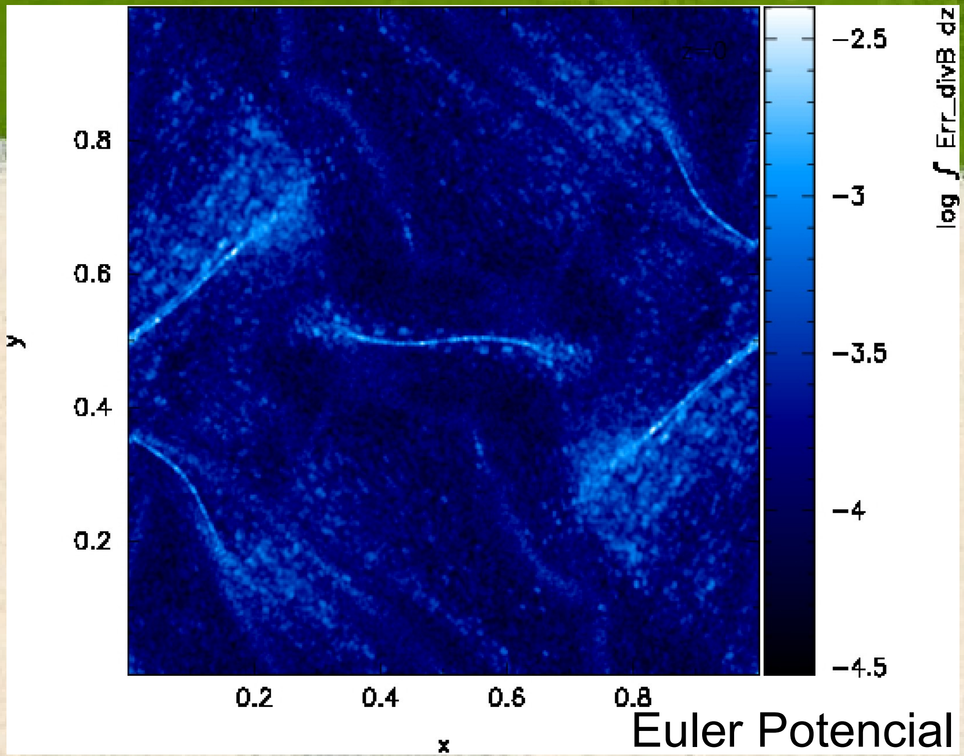


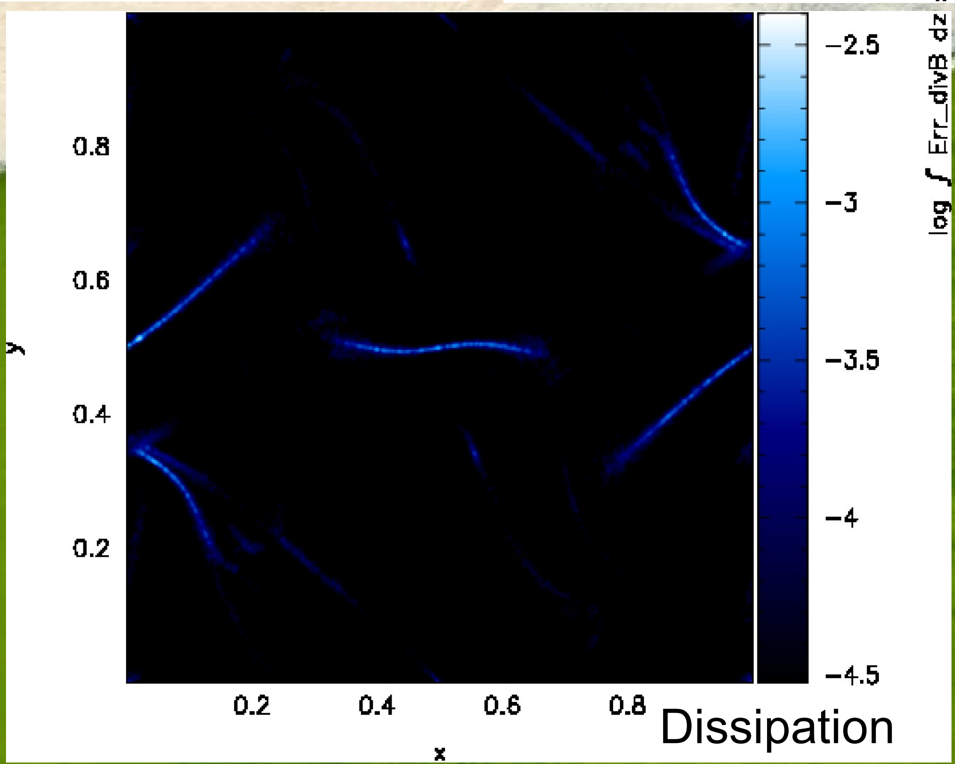


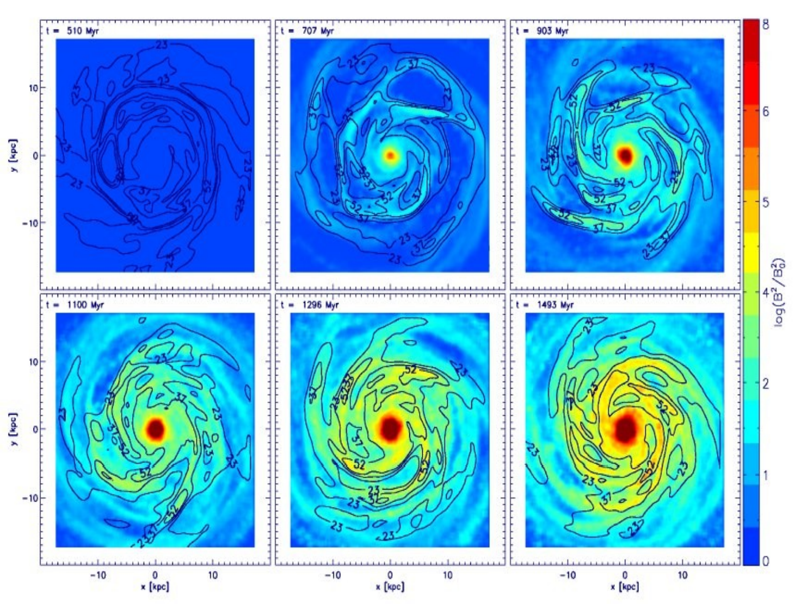
$$\vec{B} = \vec{\nabla}\alpha \times \vec{\nabla}\beta$$

EP are ideal by definition, but as implemented in SPH they can't get rid of this dissipation and the schemes Fails. Magnetic helicity not zero.

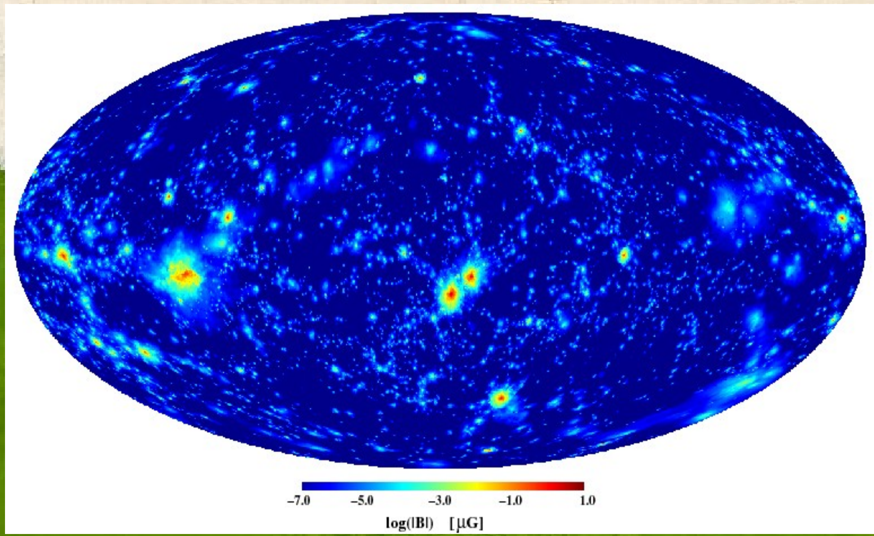

Std
x
y
log ∫ Err_divB dz
0.2
0.4
0.6
0.8
−2.5
−3
−3.5
−4
−4.5


Euler Potencial
x
y
log ∫ Err_divB dz
0.2
0.4
0.6
0.8
−2.5
−3
−3.5
−4
−4.5


Dissipation
x
y
log ∫ Err_divB dz
0.2
0.4
0.6
0.8
−2.5
−3
−3.5
−4
−4.5

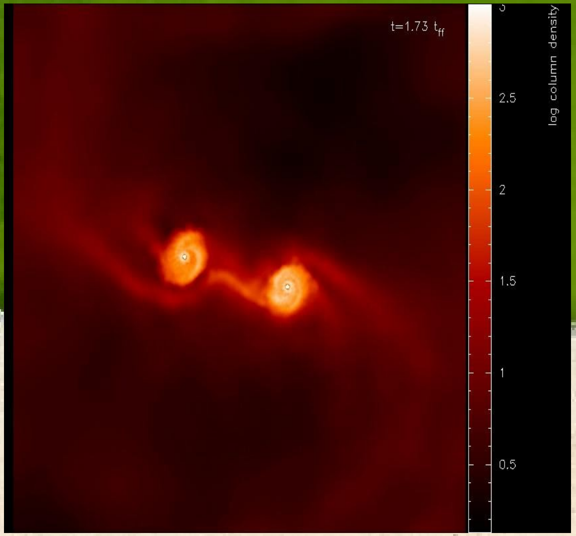


Hanna Kotarba – USM
Harald Lesch - USM
Detlef Elsner – AIP

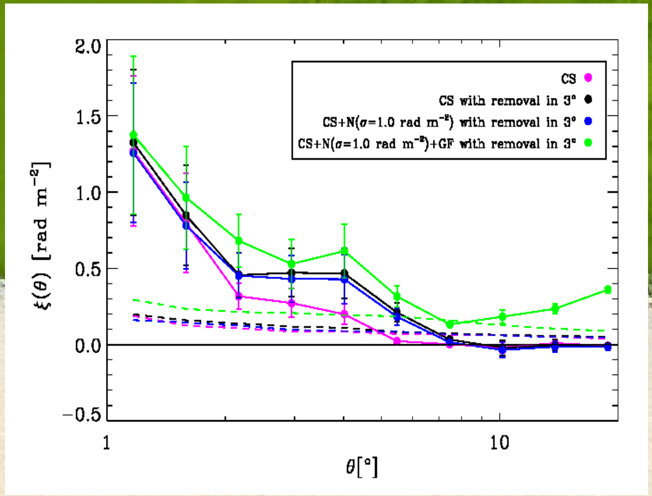


Julius Donnert - MPA

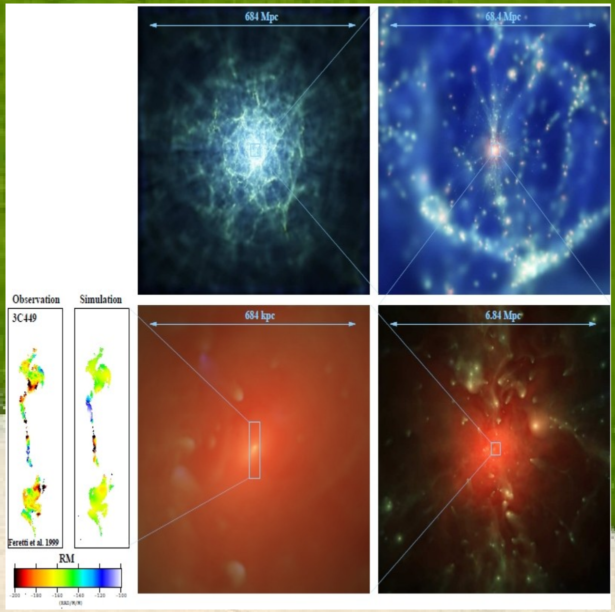
Florian Bruenzl – Uni. Konstanz

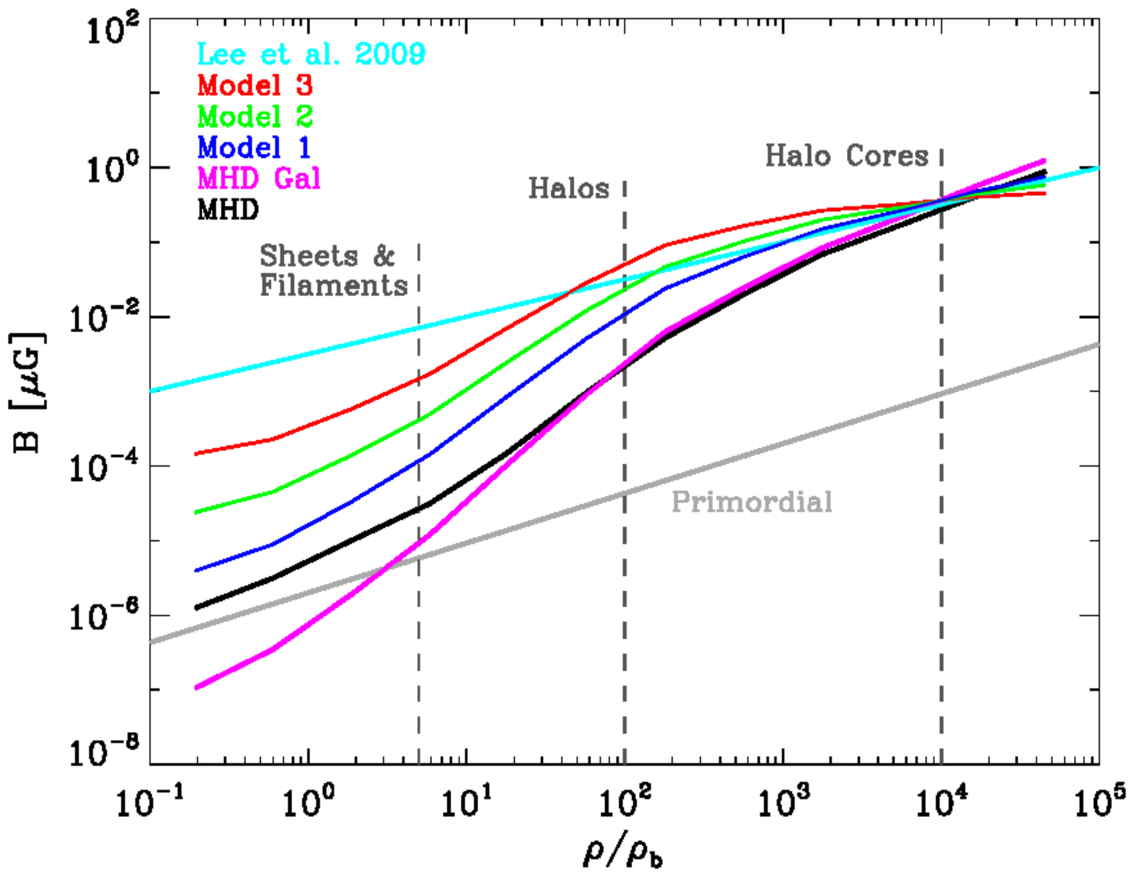


Sebastian Nuza – AIP



Klaus Dolag - USM





Need Non-Ideal MHD

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B} + \alpha \vec{B}) + \eta \nabla^2 \vec{B}$$

Induction: $\nabla \times (\vec{V} \times \vec{B})$

Diffusion: $\eta \nabla^2 \vec{B}$

$$\eta = \frac{1}{\mu \sigma} = [\Omega\, m] = [\frac{m^2}{sec}]$$

Dynamo: $\nabla \times (\alpha \vec{B})$

$$\alpha = -1/3 \langle \vec{V}_t \cdot \nabla \times \vec{V}_t \rangle$$

Brief Cosmic Magnetic problems:
- Galaxies: the actual MF should be vanished at 10^8 years.
-Galaxy Clusters: Only Gravitational Collapse does not explain their fields
-Stars/Sun: explanations of Acivity Cycle and MF reversals

continue.....