Galactic dynamos

Anvar Shukurov

School of Mathematics and Statistics, Newcastle University, U.K.
Outline

1. Introduction: spiral galaxies
2. Necessity of dynamo action
3. Mean-field disc dynamos
4. The “no-z” approximation
5. Nonlinear dynamo action and magnetic helicity
6. The fluctuation dynamo and random magnetic fields
7. Elliptical galaxies
1. Introduction: spiral galaxies

**Spiral galaxies:**
thin rotating discs of $\approx 10^{11}$ stars (90% of the visible mass) 
and interstellar gas (10%) 
+ dark matter

**Interstellar gas:** 
$\langle n \rangle \approx 1 \text{ cm}^{-3}$, $\langle T \rangle \approx 10^4 \text{ K}$, 
$10^{-3} < n < 10^3 \text{ cm}^{-3}$, $10 < T < 10^6 \text{ K}$
Differential rotation: angular velocity varying with position.

Flat rotation curves at large radii:

\[ V = r\Omega \approx 200 \text{ km/s} \approx \text{const}; \quad \Omega \propto V_0/r, \quad V_0 \approx \text{const}. \]

Rotational shear rate: \[ G = r \frac{d\Omega}{dr} \approx -\Omega. \]

Rotation curve and shear: Milky Way (solid) and a generic galaxy (dashed)
Interstellar turbulence

Correlation scale:

\[ l_0 = 50\text{–}100 \text{ pc} \approx \text{SN shell radius @ pressure balance} \]

Turbulent velocity:

\[ v_0 \approx 10 \text{ km/s} \approx c_s \text{ at } z = 0, \]
Interstellar medium in spiral galaxies:

- rotating,
- stratified,
- electrically conducting fluid (plasma),
- randomly stirred by SNe & stellar winds

→ perfect environment for turbulence & various dynamos
2. Necessity of dynamo action

- Can the magnetic fields observed be primordial?

- Do they need to be maintained by ongoing dynamo action?

- Dynamo action: conversion of kinetic energy into magnetic energy *with no electric currents at infinity*
2.1. Magnetic fields in a highly conducting turbulent medium

“If $R_m \gg 1$, magnetic field decays only slowly and so does not necessarily need to be continuously maintained.”

Wrong, if the system is turbulent:
energy is transferred along the spectrum and then dissipates in a time of order $l_0/v_0$, and this time is much shorter than the Ohmic decay time $l_0^2/\eta$ if $R_m = l_0v_0/\eta \gg 1$.

**Conclusion:** any (3D, MHD) magnetised, turbulent system must host a dynamo (unless the magnetic field is driven by external currents or decays).

Even without turbulence, random magnetic fields drive random motions, subject to viscous dissipation.
2.2. Magnetic field in a differentially rotating, turbulent disc

(A) **The decay problem**  
(Parker 1979)

The Ohmic decay of a large-scale magnetic field is very slow:

\[ \eta = 10^7 \left( \frac{T}{10^4} \right)^{-3/2} \text{cm}^2/\text{s}, \quad v_0 = 10 \text{ km/s}, \quad h = 500 \text{ pc} \]

\[ \Rightarrow R_m \approx 10^{20} \text{ (!?)}, \quad \tau_{\text{decay}} = \frac{h^2}{\eta} \approx 10^{27} \text{ yr} >> \text{Hubble time.} \]

However, turbulent diffusion destroys the *large-scale* magnetic field much faster:

\[ \beta = \frac{1}{3} l_0 v_0 \approx 10^{26} \text{ cm}^2/\text{s} \quad \Rightarrow \quad \tau_{\text{decay}} = \frac{h^2}{\beta} \approx 5 \times 10^8 \text{ yr} = \frac{\text{galactic lifetime}}{20} \]

Without dynamo action, *turbulent magnetic diffusion* destroys a *large-scale* magnetic field in a fraction of the galactic lifetime.
(B) **The wrap-up problem** (Parker 1979)

Expulsion of magnetic field from a region with closed streamlines

(Moffatt, *Magnetic Field Generation in Electrically Conducting Fluids*, CUP, 1978, §3.7)
The action of differential rotation on magnetic field: flux expulsion from a region with closed streamlines

\[ z = e^{-r^2}, \quad \mathbf{B}\big|_{t=0} = (0, 1, 0). \]
$G = |r \, d\theta / dr|$: rotational shear rate,

$C_\omega = G R_0^2 / \beta$: turbulent Reynolds number

$\beta$: turbulent magnetic diffusivity.

Initial growth:

$$\Delta r \simeq \frac{R_0}{G t}, \quad p = \arctan \left( \frac{B_r}{B_\phi} \right) \simeq -\frac{1}{G t} \quad \text{ (magnetic pitch angle)}.$$ 

End of the growth phase, $t = t_0$: amplification time = diffusion time,

$$\frac{1}{G} = \left[ \Delta r(t_0) \right]^2 \\frac{\beta}{\beta} \quad \Rightarrow \quad t_0 \simeq \frac{C_\omega^{1/2}}{G}, \quad p(t_0) \simeq -C_\omega^{-1/2}.$$ 

$$B_{\max} \simeq B_0 G t_0 \simeq B_0 C_\omega^{1/2}, \quad \Delta r(t_0) \simeq \frac{R_0}{C_\omega^{1/2}}.$$
Galactic discs:

\[ G = \frac{V_0}{R_0}, \quad \beta = \frac{1}{3} l_0 v_0 \quad \Rightarrow \quad C_\omega = 3 \frac{V_0}{v_0} \frac{R_0}{l_0} \simeq 6000, \]

\[ p \simeq -C_\omega^{-1/2} \simeq -1^\circ, \quad \Delta r \simeq R_0 C_\omega^{-1/2} \simeq 100 \text{ pc}, \]

\[ B_{\text{max}} = B_0 C_\omega^{1/2} \simeq 0.1 \mu \text{G} \quad \text{for} \quad B_0 = 10^{-9} \text{ G}. \]

**Conclusion:** to avoid twisting by differential rotation, the *large-scale* galactic magnetic field has to be replenished (by a dynamo action).
**Conclusion:** to avoid twisting by differential rotation, the *large-scale* galactic magnetic field has to be replenished (by a dynamo action).
3. Mean-field disc dynamos
The physical picture of the galactic dynamo

(A) Helicity of interstellar turbulence: consequence of angular momentum conservation in a rotating, stratified layer

\[ \alpha \approx \frac{l_0^2}{h} \]  

(F. Krause, 1967)
(B) Differential rotation:

\[ B_\phi \text{ produced from } B_r \]

(C) Helical turbulence:

\[ B_r \text{ produced from } B_\phi \]
3.1. Basic equations

$\mathbf{B} =$ large-scale magnetic field

$$ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\alpha \mathbf{B} + \mathbf{V} \times \mathbf{B}) + \beta \nabla^2 \mathbf{B} $$

Cylindrical coordinates $(r, \phi, z)$,  $\mathbf{e}_z \parallel \tilde{\Omega}$,

$\mathbf{V} = (0, r \Omega, 0)$,  $\Omega = \Omega(r)$.

Thin disc:  $|z| \leq h$,  $r \leq R$,  $R \gg h$,

$\partial / \partial z \gg \partial / \partial r$,  $\partial / r \partial \phi \Rightarrow \nabla^2 \mathbf{B} = \frac{\partial^2 \mathbf{B}}{\partial z^2}$, ...

Axial symmetry:  $\partial / \partial \phi = 0$.

$\mathbf{B}(t, r) = \tilde{\mathbf{B}}(t, r, z) e^{im\phi}$,  $\epsilon = \frac{h_0}{R_0} \ll 1$. 
\[
\left( \frac{\partial}{\partial t} + imR_\omega \Omega + \frac{\epsilon^2 m^2}{r^2} \right) \tilde{B}_r = -R_\alpha \frac{\partial}{\partial z} (\alpha \tilde{B}_\phi) + \frac{\partial^2 \tilde{B}_r}{\partial z^2} + \epsilon^2 \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r \tilde{B}_r) \right] + im\epsilon R_\alpha \frac{\alpha}{r} \tilde{B}_z - \frac{2i \epsilon \epsilon^2}{r^2} \tilde{B}_\phi ,
\]

\[
\left( \frac{\partial}{\partial t} + imR_\omega \Omega + \frac{\epsilon^2 m^2}{r^2} \right) \tilde{B}_\phi = R_\omega G \tilde{B}_r + R_\alpha \frac{\partial}{\partial z} (\alpha \tilde{B}_r) + \frac{\partial^2 \tilde{B}_\phi}{\partial z^2} + \epsilon^2 \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r \tilde{B}_\phi) \right] - \epsilon R_\alpha \frac{\partial}{\partial r} (\alpha \tilde{B}_z) + \frac{2i \epsilon \epsilon^2}{r^2} \tilde{B}_r ,
\]

\[
\left( \frac{\partial}{\partial t} + imR_\omega \Omega + \frac{\epsilon^2 m^2}{r^2} \right) \tilde{B}_z = \frac{\partial^2 \tilde{B}_z}{\partial z^2} + R_\alpha \frac{\epsilon}{r} \frac{\partial}{\partial r} (r \alpha \tilde{B}_\phi) - im\epsilon R_\alpha \frac{\alpha}{r} \tilde{B}_r + \epsilon^2 \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r \tilde{B}_z) \right] + \frac{\epsilon^2}{r^2} \tilde{B}_z ,
\]

\[
\mathbf{B}(t, r) = \tilde{\mathbf{B}}(t, r, z)e^{im\phi} , \quad \epsilon = \frac{\hbar_0}{R_0} \ll 1 , \quad G = r \frac{d-}{dr} .
\]
Thin disc, axisymmetric solutions:

\[
\frac{\partial B_r}{\partial t} = -\frac{\partial}{\partial z}(\alpha B_\phi) + \beta \frac{\partial^2 B_r}{\partial z^2},
\]

\[
\frac{\partial B_\phi}{\partial t} = G B_r + \frac{\partial}{\partial z}(\alpha B_r) + \beta \frac{\partial^2 B_\phi}{\partial z^2},
\]

\[
\frac{\partial B_z}{\partial t} = \beta \frac{\partial^2 B_z}{\partial z^2}.
\]

\[G = r \, d\Omega/dr\]

Equation for \(B_z\) splits from the system.

\(B_z\) is supported through \(B_r\) and \(B_\phi\) via \(\partial/\partial r\)
Dimensionless variables

\[ \tilde{z} = \frac{z}{h} \Rightarrow \frac{\partial}{\partial z} = \frac{1}{h} \frac{\partial}{\partial \tilde{z}}, \quad \tilde{t} = \frac{t}{h^2/\beta} \Rightarrow \frac{\partial}{\partial t} = \frac{\beta}{h^2} \frac{\partial}{\partial \tilde{t}}, \]

\[ \tilde{\alpha} = \frac{\alpha(z)}{\alpha_0}. \]

\[ \frac{\partial B_r}{\partial \tilde{t}} = -R_\alpha \frac{\partial}{\partial \tilde{z}}(\tilde{\alpha}B_\phi) + \frac{\partial^2 B_r}{\partial \tilde{z}^2}, \quad R_\alpha = \frac{\alpha_0 h}{\beta} \]

\[ \frac{\partial B_\phi}{\partial \tilde{t}} = R_\omega B_r + R_\alpha \frac{\partial}{\partial \tilde{z}}(\tilde{\alpha}B_r) + \frac{\partial^2 B_\phi}{\partial \tilde{z}^2}, \quad R_\omega = \frac{Gh^2}{\beta}. \]
Drop~at dimensionless variables:

\[
\frac{\partial B_r}{\partial t} = -R_\alpha \frac{\partial}{\partial z} (\alpha B_\phi) + \frac{\partial^2 B_r}{\partial z^2}, \quad \text{via } \alpha\text{-effect}
\]

\[
\frac{\partial B_\phi}{\partial t} = \frac{R_\omega B_r}{B_r \rightarrow B_\phi \text{ via differential rotation}} + R_\alpha \frac{\partial}{\partial z} (\alpha B_r) + \frac{\partial^2 B_\phi}{\partial z^2}.
\]

\[
R_\alpha = \frac{\alpha_0 h}{\beta}
\]

\[
R_\omega = \frac{h^2 r d- / dr}{\beta}
\]
\( \alpha \omega \)-Dynamo: \( |R_\omega| \gg R_\alpha \)

\[
\begin{align*}
\frac{\partial B_r}{\partial t} &= -R_\alpha \frac{\partial}{\partial z}(\alpha B_\phi) + \frac{\partial^2 B_r}{\partial z^2}, \\
\frac{\partial B_\phi}{\partial t} &= R_\omega B_r + \frac{\partial^2 B_\phi}{\partial z^2}.
\end{align*}
\]

Introduce new variable \( B'_r = R_\alpha B'_r \) and drop the dash:

\[
\begin{align*}
\frac{\partial B_r}{\partial t} &= -\frac{\partial}{\partial z}(\alpha B_\phi) + \frac{\partial^2 B_r}{\partial z^2}, \\
\frac{\partial B_\phi}{\partial t} &= D B_r + \frac{\partial^2 B_\phi}{\partial z^2},
\end{align*}
\]

where \( D = R_\alpha R_\omega \) is the dynamo number.
Boundary conditions

\[ B_r \big|_{z=1} = B_\phi \big|_{z=1} = 0 \]  (vacuum boundary conditions)

\[ \frac{\partial B_r}{\partial z} \big|_{z=0} = \frac{\partial B_\phi}{\partial z} \big|_{z=0} = 0 \]  (quadrupole)

\[ B_r \big|_{z=0} = B_\phi \big|_{z=0} = 0 \]  (dipole)
3.2. Dynamo control parameters

NB! The Solar neighbourhood of the Milky Way, where these estimates apply, is not a typical galactic location.

Rotation \( \mathcal{R} = \frac{V_0}{r} \),

\[ V_0 \approx 200 \text{ km/s}, \quad r \approx 10 \text{ kpc}. \]

Ionised gas scale height

\[ h \approx 0.5 \text{ kpc}, \]

Turbulent velocity \( v_0 \approx 10 \text{ km/s} \).

Turbulent scale \( l_0 \approx 0.1 \text{ kpc} \).

- \( \alpha_0 \approx \frac{l_0^2}{h} \approx 0.4 \text{ km/s} \),
- \( \beta \approx \frac{1}{3} l_0 v_0 \approx 10^{26} \text{ cm}^2/\text{s} \),
- \( R_\alpha = \frac{\alpha_0 h}{\beta} \approx 0.6 \),
- \( R_\omega = \frac{(r \frac{d}{dr}) h^2}{\beta} \approx -15 \),
- \( D = R_\alpha R_\omega \approx - \left( \frac{3 - h}{v_0} \right)^2 \approx -10 \).
4. The “no-$z$” approximation (Subramanian & Mestel, 1993)

Thin disc, dimensional $\alpha \omega$-dynamo equations:

$$\frac{\partial B_r}{\partial t} = -\frac{\partial}{\partial z}(\alpha B_\phi) + \beta \frac{\partial^2 B_r}{\partial z^2},$$

$$\frac{\partial B_\phi}{\partial t} = GB_r + \beta \frac{\partial^2 B_\phi}{\partial z^2}.$$ 

Solutions have a simple form, e.g., $B_{r,\phi} \propto \cos z/h$:

$$\frac{\partial}{\partial z} \approx \frac{1}{h}, \quad \frac{\partial^2}{\partial z^2} \approx -\frac{1}{h^2}.$$ 

Kinematic solutions: $\vec{B} = \vec{B}_0 \exp(\gamma t)$. 

\[
\left( \gamma + \frac{\beta}{h^2} \right) B_{0r} + \frac{\alpha}{h} B_{0\phi} = 0, \\
-GB_{0r} + \left( \gamma + \frac{\beta}{h^2} \right) B_{0\phi} = 0.
\]

Nontrivial solutions exist if
\[
\begin{vmatrix}
\gamma + \beta/h^2 & \alpha/h \\
-G & \gamma + \beta/h^2
\end{vmatrix} = 0,
\]
i.e., $\gamma \simeq \frac{\beta}{h^2}(-1 + \sqrt{-D})$,

\[
\tan p = \frac{B_r}{B_\phi} \simeq -\sqrt{-\frac{\alpha}{-Gh}} = -\sqrt{\frac{R_\alpha}{|R_\omega|}}.
\]

Magnetic field grows if $D \lesssim -1$, with $p \simeq -\arctan \frac{1}{4} \simeq -15^\circ$. 
5. Nonlinear dynamo action and magnetic helicity

Magnetic helicity: $\chi = \langle \vec{A} \cdot \vec{B} \rangle$, $\vec{B} = \nabla \times \vec{A}$

(conserved in ideal MHD)

$t = 0 \Rightarrow \vec{B} \approx 0$ (weak seed field)

$\Rightarrow \chi \big|_{t=0} \approx 0 \Rightarrow \chi \big|_{\text{now}} \approx 0$

Introduce large- & small-scale magnetic fields and the corresponding helicities:

$\vec{B} = \vec{B} + \vec{b}$, $\vec{A} = \vec{A} + \vec{a}$,

$\chi = \chi_B + \chi_b$, $\chi_B = \vec{A} \cdot \vec{B}$, $\chi_b = \vec{a} \cdot \vec{b}$
\[ \chi = \chi_B + \chi_b, \quad \chi_B = \mathbf{A} \cdot \mathbf{B}, \quad \chi_b = \mathbf{a} \cdot \mathbf{b} \]

- \( \chi_B = \langle \mathbf{A} \cdot \mathbf{B} \rangle \simeq -LB_r B_\phi \simeq \frac{1}{4} LB^2 \),
  for \( B_r / B_\phi = -\sin p \), \( p = 15^\circ \); \( L \gtrsim 1 \) kpc.

- \( \chi_b = \langle \mathbf{a} \cdot \mathbf{b} \rangle \simeq -l_d b^2 \),
  \( l_d \lesssim l \simeq 100 \) pc \( (l_d = \text{scale of } \chi_b, \ l = \text{turbulent scale}) \)

- \( \chi = \chi_B + \chi_b = 0 \quad \Rightarrow \quad \frac{B^2}{b^2} \simeq \frac{4l_d}{L} \simeq 0.4, \text{ if } l_d \simeq l \)

**Catastrophic \( \alpha \)-quenching:**
\[ \frac{B^2}{b^2} \simeq R_m^{-1} \ll 1 \quad \text{if} \quad \frac{l_d}{L} \simeq R_m^{-1} \]
\( (R_m = \text{magnetic Reynolds number}) \)
A mechanical analogy of helicity conservation: twist & writhe of a hose pipe

Twist by $90^\circ$

Twist by $180^\circ$
Galactic discs are not closed systems:
galactic winds and fountains
⇒ the “unwanted” magnetic helicity can be removed from the disc
Galactic fountain/wind removes magnetic field from the disc

Hot gas outflow through the disc surface: $V_z = 150–200$ km/s

Outflow time scale: $H/V_z = 1$ kpc/150 km/s = $10^7$ yr

Surface filling factor of the hot gas: $f_s = 0.2–0.3$

Relative density of the hot gas: $\frac{\rho_h}{\langle \rho \rangle} \approx \frac{10^{-3} \text{ cm}^{-3}}{0.1 \text{ cm}^{-3}} = 10^{-2}$

Effective (mass-averaged) advection speed (for single-phase dynamo models):

$$U_z \approx f_s \frac{\rho_h}{\langle \rho \rangle} V_z \approx 10^{-2} V_z \approx 0.3–2 \text{ km/s}$$
Helicity balance

Random field $\vec{b}$ has finite correlation length $\Rightarrow$ define volume density of linkages of $\vec{b}$:

$$\chi \approx H_b \quad \text{for } \nabla \cdot \vec{a} = 0$$

Evolution equation:

$$\frac{\partial \chi}{\partial t} + \nabla \cdot \vec{F} = -2\vec{E} \cdot \vec{B} - 2\eta \vec{j} \cdot \vec{b}$$

$$\vec{j} = \nabla \times \vec{b}, \quad \vec{J} = \nabla \times \vec{B}, \quad \text{electric current densities}$$

$$\vec{E} \approx \alpha \vec{B} - \beta \nabla \times \vec{B}, \quad \text{mean electromotive force}$$

$$\vec{F} = \chi \vec{U}, \quad \text{advective flux.}$$

$$\alpha = \alpha_{\text{kinetic}} + \alpha_m, \quad \alpha_m \simeq \frac{1}{3} \tau k_0^2 \frac{\chi}{\rho}$$

$$\frac{\partial \alpha_m}{\partial t} = -2\beta k_0^2 \left( \frac{\vec{E} \cdot \vec{B}}{B_{eq}^2} + \frac{\alpha_m}{R_m} \right) - \nabla \cdot \left( \alpha_m \vec{U} \right)$$

+ mean-field dynamo equations for $B_r$ and $B_\phi$
Mean-field dynamo: 
\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{U} \times \vec{B} + \alpha \vec{B} - \beta \nabla \times \vec{B}) .
\]

Thin disc \(|z| \cdot h, \frac{\partial}{\partial z} \gg \frac{\partial}{\partial r}\), axial symmetry, \(\partial/\partial \phi = 0\),
\(\vec{U} = (0, r- (r), U_z)\), dimensionless equations:

\[
\begin{align*}
\frac{\partial B_r}{\partial t} &= - \frac{\partial}{\partial z} (R_u U_z B_r + R_\alpha \alpha B_\phi) + \frac{\partial^2 B_r}{\partial z^2}, \\
\frac{\partial B_\phi}{\partial t} &= R_\omega B_r - R_u \frac{\partial}{\partial z} (U_z B_\phi) + \frac{\partial^2 B_\phi}{\partial z^2}, \\
\frac{\partial \alpha_m}{\partial t} &= -C \left( \alpha B^2 - R_\alpha^{-1} \vec{J} \cdot \vec{B} + \frac{\alpha_m}{R_m} \right) - R_u \frac{\partial}{\partial z} (U_z \alpha_m),
\end{align*}
\]

\(\alpha = \alpha_K + \alpha_m, \quad \alpha_K \simeq \frac{l^2}{h}, \quad C = 2 \frac{h^2}{l^2}, \quad \vec{J} \cdot \vec{B} = B_\phi \frac{\partial B_r}{\partial z} - B_r \frac{\partial B_\phi}{\partial z}, \quad R_\alpha = \frac{\alpha_0 h}{\beta}, \quad R_\omega = \frac{(r \partial^2 / \partial r) h^2}{\beta} \quad R_u = \frac{U_z 0 h}{\beta}.
\]
Sur, Shukurov & Subramanian (MNRAS, 2007): Magnetic field evolution in a galactic disc with helicity advection by the galactic fountain/wind, "no-\(z\)" approximation: \(\partial / \partial z \rightarrow 1/h\), \(\partial^2 / \partial z^2 \rightarrow -1/h^2\)
1. Helicity balance: \( B^2 \approx 12\pi \rho v_0 \frac{l}{h} U_z \left( \frac{D}{D_{\text{crit}}} - 1 \right) \),

2. Balance of the Coriolis & Lorentz forces: \( B^2 \approx 4\pi \rho h v_0 - \frac{|R_\omega|}{\sqrt{|D_{\text{crit}}|}} \).

**Dependence of \( B^2 \) on galactic parameters**

1. Supernova (SN) frequency (per unit area): \( \nu_{\text{SN}} \propto \text{SFR} \)
   \[ \sigma_g = 2\rho h, \text{ gas surface density} \]
   Energy injection rate by SNe: \( \dot{E}_{\text{SN}} = E_{\text{SN}} \nu_{\text{SN}}, \)
   Outflow speed: \( \frac{1}{2} \frac{\sigma_h V_z^2}{\tau} \approx q \dot{E}_{\text{SN}}, \quad \tau = H/V_z, \)
   \( \sigma_h = 2\rho_h H, \text{ hot gas surface density}, \)
   \( q \approx 0.1, \text{ SN energy fraction converted into kinetic energy}, \)

\[ B^2 \propto (\rho_h \rho)^{1/3} v_0 \text{SFR}^{1/3} \left( \frac{D}{D_{\text{crit}}} - 1 \right). \]

2. \( B^2 \propto \rho h^3 - r \left| \frac{d}{dr} \right| \)

Distinct dependencies on observable parameters.
Galactic outflows & dynamos: magnetic pitch angle

Nonlinear, steady state: $\partial / \partial t = 0$,

$\alpha$-effect suppressed to its marginal level, $D(\alpha_K + \alpha_m) = D_{\text{crit}}$.

No-$z$ approximation, dynamo with an outflow, $R_u \neq 0$:

$$0 = -\frac{2}{\pi} R_\alpha \frac{D}{D_{\text{crit}}} B_\phi - \left( R_u + \frac{\pi^2}{4} \right) B_r,$$

$$0 = R_\omega B_r - \left( R_u + \frac{\pi^2}{4} \right) B_\phi.$$

$$\frac{B_r}{B_\phi} = \frac{R_u + \pi^2/4}{R_\omega},$$

$$R_U = 0 \quad \Rightarrow \quad \tan p = \frac{\pi^2}{4R_\omega}, \quad p \simeq -10^\circ$$
Magnetic pitch angle in M31

Dynamo model with helicity balance (Smith et al., 2011)

Observations (Fletcher et al., 2004)
6. The fluctuation dynamo and random magnetic fields

- Interstellar magnetic field ≠ a quasi-homogeneous Gaussian random vector field.
- Interstellar shocks, multi-phase structure, ...
- A quasi-homogeneous, weaker magnetic background from the tangling of the large-scale magnetic field by turbulence.

Further details in:
The fluctuation dynamo: exponential amplification and then maintenance of a random magnetic field in a random flow

- Growth due to random shearing (stretching) of magnetic fields.

- Only needs motions to be strong enough, $R_m > R_{m,cr}$, $R_{m,cr} = 30–100$ (depending on the flow type).

- Vortical motions are good dynamos; compressibility hinders the dynamo action.

- Very rapid amplification: growth time of rms magnetic field = eddy turnover time, $l_0/v_0 \approx 10^7\,\text{yr}$

- Conceptual model: Zeldovich’s stretch–twist–fold dynamo:
\[ B \propto 2^n \quad \Rightarrow \quad B \propto e^{\gamma t} \]
Fluctuation dynamo produces intermittent magnetic fields even in a homogeneous medium, magnetic filaments (+ ribbons & sheets?):

- \( B_{\text{max}} \approx B_{\text{eq}} = (4\pi\rho)^{1/2}v_0 \approx 5 \ \mu\text{G}. \)
- Length \( \approx l \approx 50\text{–}100 \ \text{pc}. \)
- Low volume filling factor, \( \langle B^2 \rangle \approx 0.1 B_{\text{max}}^2. \)
- Kinematic stage: magnetic energy max at \( l_\eta = l R_m^{-1/2}. \)
- Nonlinear, statistically steady state: controversial
  - folds at \( l_\eta = l R_m^{-1/2} \approx 10^{-7} \ \text{pc} \) (???) (Schekochihin et al. 2004) or
  - thicker structures \( l_{\eta,\text{cr}} = l R_{m,\text{cr}}^{-1/2} \approx 10 \ \text{pc} \) (Subramanian 1999).
Simulations of the fluctuation dynamo: magnetic isosurfaces, $B^2 = \text{const}$
Implications

- Power spectrum & structure/correlation function are not suitable tools to describe intermittent magnetic fields (intense flux ropes separated by extended regions with relatively weak magnetic field).

- Magnetic field estimates from synchrotron intensity can be strongly affected (underestimated random magnetic field).

- Cosmic ray propagation can be strongly affected by magnetic intermittency. No models available of cosmic ray propagation in such magnetic fields.

- Locally anisotropic magnetic fields are less efficient in cosmic ray scattering ($>10^2-10^3$ GeV) (Chandran 2000).
7. Elliptical galaxies
(10% of all galaxies)

- M87 (E1)
- M89 (E0)
- M59 (E5)
- M84 (E3)
- M49 (E4)

Defining the class of elliptical galaxy:

\[ n = 10 \times (1 - \frac{b}{a}) \]
Structureless, triaxial ellipsoids of old stars, no star formation, stellar density monotonously decreases outwards.

Wide range in size and mass:
- from giants (diameter a few Mpc, a few times $10^{12} M_\odot$, many have active nucleus, jet, are bright in radio, etc.)
- to dwarves (a kpc in diameter, $10^6 M_\odot$).

No overall rotation: randomly oriented stellar orbits, stellar velocity dispersion $\sigma_* \approx 300$ km/s.
M87: X-ray (gas) & optical (stars)
[Chandra X-ray satellite]
Interstellar gas and magnetic fields in ellipticals

- Hot, $T \approx 10^7$ K, detected in X-rays;
- tenuous, $n \approx 10^{-3}$ cm$^{-3}$ ($10^{-2}$ cm$^{-3}$ near the centre);
- total gas mass $10^9M_\odot \approx 1\%$ of the stellar mass;
- radius of the gas distribution $\approx 50$ kpc $\approx$ radius of the stellar distribution.
- No or weak spectral line emission $\Rightarrow$ little know about gas motions.
- Some observational evidence for random magnetic fields;