Galactic dynamos

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Outline

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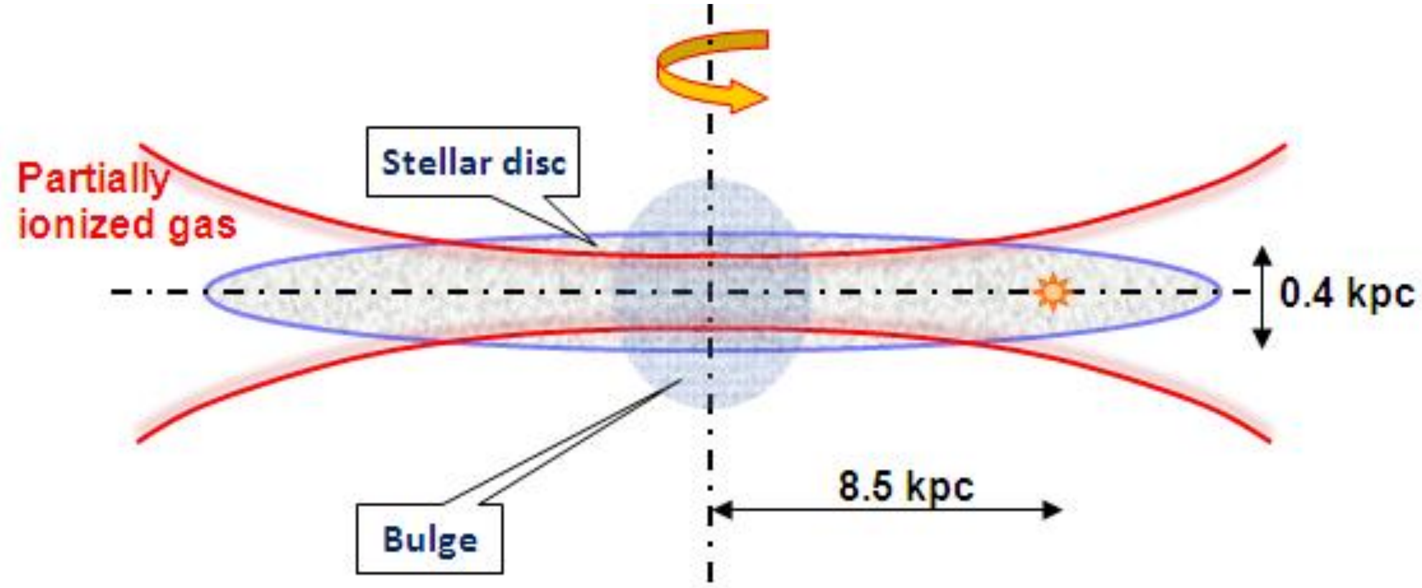
1. Introduction: spiral galaxies

Spiral galaxies:

thin rotating discs of $\simeq 10^{11}$ stars (90% of the visible mass)
and interstellar gas (10%)
+ dark matter



Interstellar gas: $\langle n \rangle \simeq 1\ \mathrm{cm}^{-3}$, $\langle T \rangle \simeq 10^4\ \mathrm{K}$,

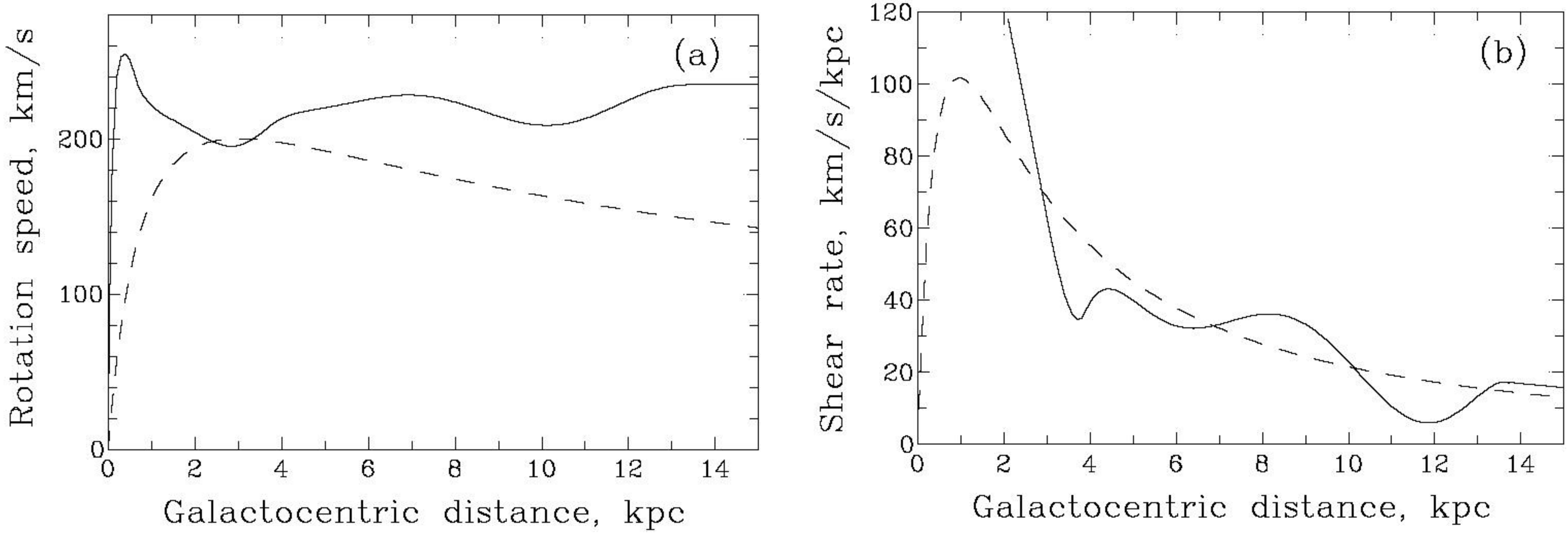
$10^{-3} < n < 10^3\ \mathrm{cm}^{-3}$, $10 < T < 10^6\ \mathrm{K}$

❑Differential rotation: angular velocity varying with position.

❑Flat rotation curves at large radii:

$V = r\Omega \simeq 200$ km/s $\simeq$ const; $\Omega \propto V_0/r$, $V_0 \simeq$ const.

Rotational shear rate: $G = r\, d\Omega/dr \simeq -\Omega$.



Rotation curve and shear: Milky Way (solid) and a generic galaxy (dashed)

Interstellar turbulence

Correlation scale:

l_0 = 50–100 pc $\cong$ SN shell radius @ pressure balance

Turbulent velocity:

$$v_0 \simeq 10\,\mathrm{km/s} \approx c_\mathrm{s} \text{ at } z = 0,$$

Interstellar medium in spiral galaxies:

- ❑ rotating,
- ❑ stratified,
- ❑ electrically conducting fluid (plasma),
- ❑ randomly stirred by SNe & stellar winds

➔ perfect environment for turbulence & various dynamos

2. Necessity of dynamo action

- Can the magnetic fields observed be primordial?
- Do they need to be maintained by ongoing dynamo action?
- Dynamo action: conversion of kinetic energy into magnetic energy *with no electric currents at infinity*

2.1. Magnetic fields in a highly conducting turbulent medium

"If $R_m >> 1$, magnetic field decays only slowly and so does not necessarily need to be continuously maintained."

Wrong, if the system is turbulent:
energy is transferred along the spectrum and then dissipates in a time of order l_0/v_0, and this time is much shorter than the Ohmic decay time l_0^2/η if $R_m = l_0 v_0/\eta \gg 1$.

Conclusion: any (3D, MHD) magnetised, turbulent system must host a dynamo (unless the magnetic field is driven by external currents or decays).

Even without turbulence, random magnetic fields drive random motions, subject to viscous dissipation.

2.2. Magnetic field in a differentially rotating, turbulent disc

(A) The decay problem (Parker 1979)

The Ohmic decay of a large-scale magnetic field is very slow:

$$\eta = 10^7 (T/10^4)^{-3/2}\ \mathrm{cm^2/s}, \qquad v_0 = 10\ \mathrm{km/s}, \qquad h = 500\ \mathrm{pc}$$

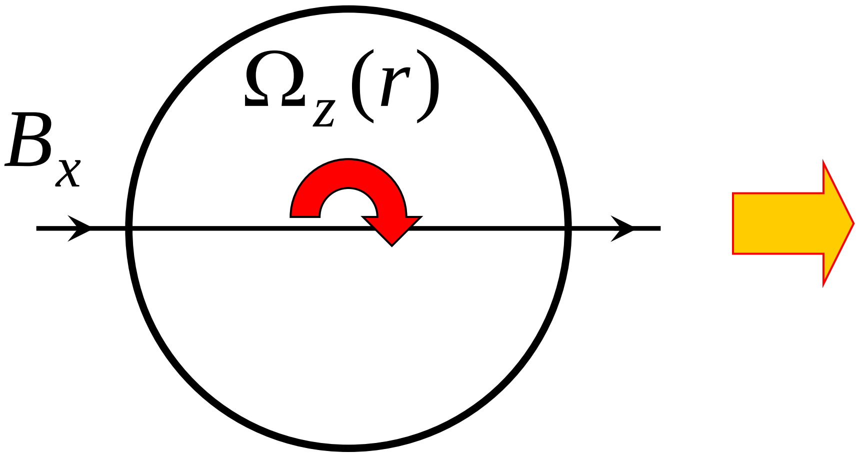
$$\Rightarrow \quad R_m \cong 10^{20}\ (!?), \qquad \tau_{decay} = h^2/\eta \cong 10^{27}\ \mathrm{yr} >> \text{Hubble time.}$$

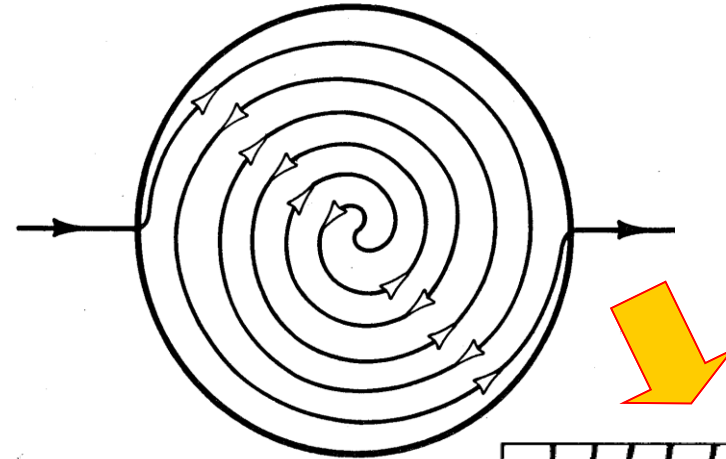
However, turbulent diffusion destroys the *large-scale* magnetic field much faster:

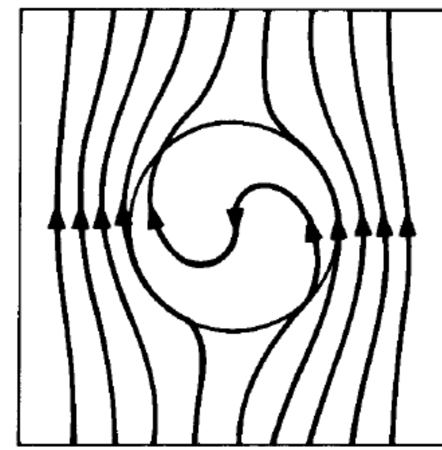
$$\beta = \tfrac{1}{3} l_0 v_0 \simeq 10^{26}\ \frac{\mathrm{cm}^2}{\mathrm{s}} \quad \Rightarrow \tau_{decay} = h^2/\beta \cong 5\times 10^8\ \mathrm{yr} = \frac{\text{galactic lifetime}}{20}$$

Without dynamo action, *turbulent magnetic diffusion* destroys a *large-scale* magnetic field in a fraction of the galactic lifetime.

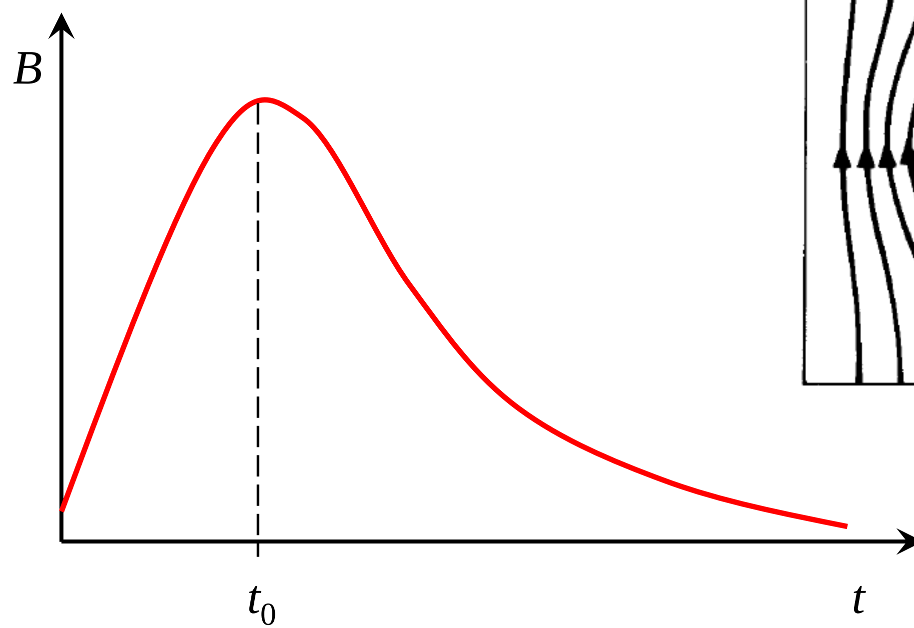
(B) The wrap-up problem (Parker 1979)







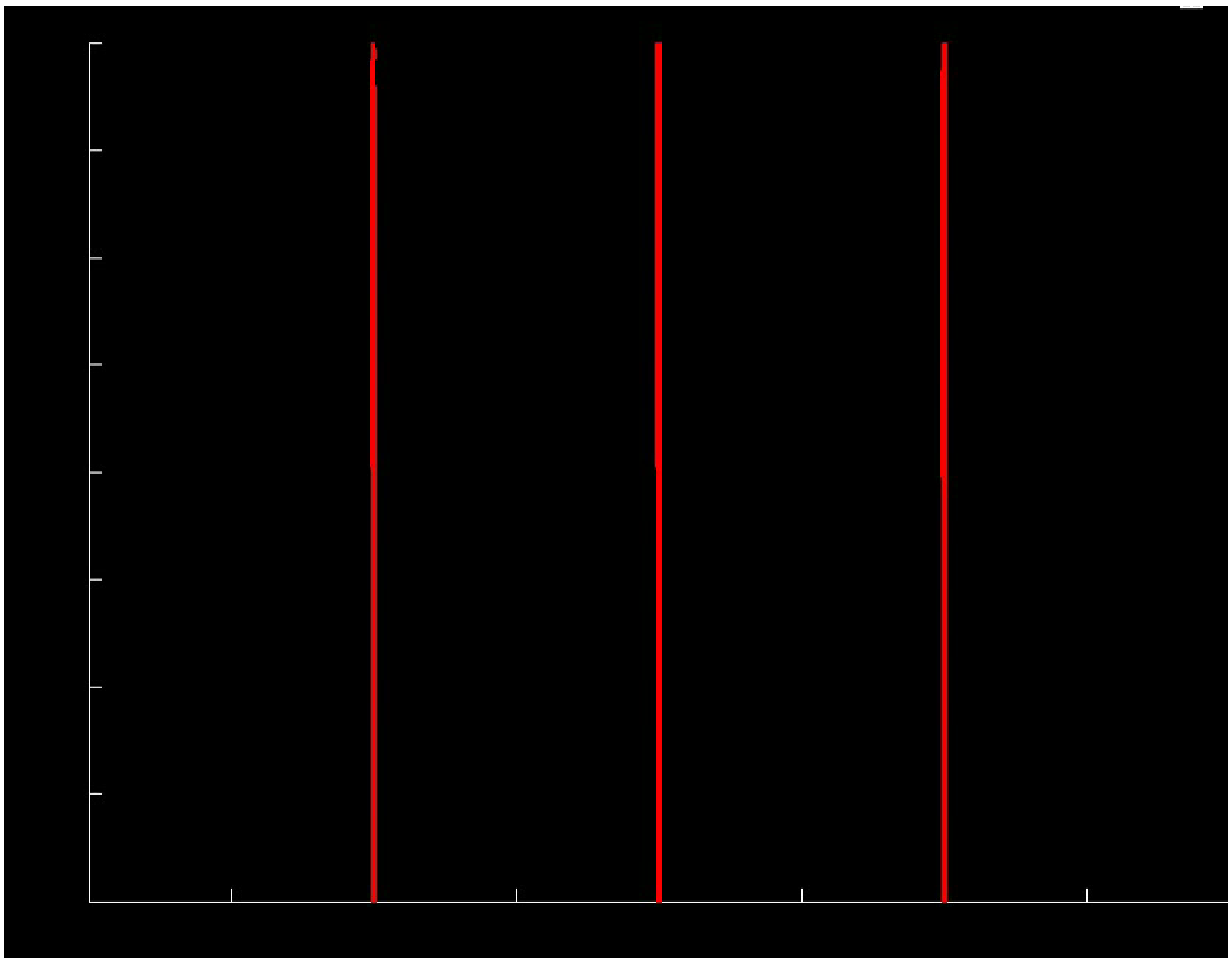
Expulsion of magnetic field from a region with closed streamlines



(Moffatt, *Magnetic Field Generation in Electrically Conducting Fluids*, CUP, 1978, §3.7)

The action of differential rotation on magnetic field: flux expulsion from a region with closed streamlines

$$-_{z} = e^{-r^2}, \qquad \mathbf{B}|_{t=0} = (0,1,0)\,.$$

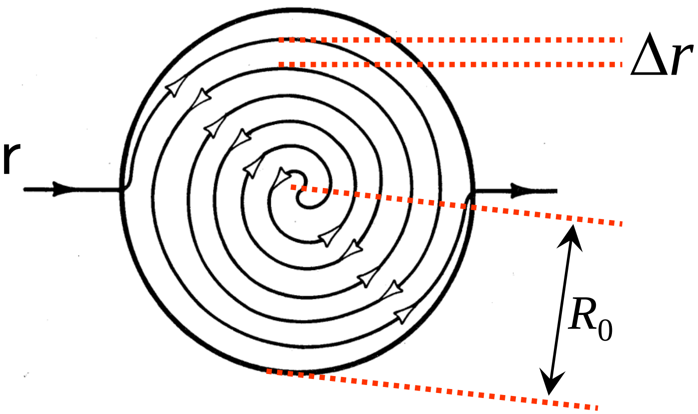


A. Baggaley, Newcastle

$G = |r\, d\text{-}\ /dr|$: rotational shear rate,

$C_\omega = G R_0^2/\beta$: turbulent Reynolds number

β: turbulent magnetic diffusivity.



Initial growth:

$$\Delta r \simeq \frac{R_0}{Gt}, \quad p = \arctan \frac{B_r}{B_\phi} \simeq -\frac{1}{Gt}$$ (magnetic pitch angle).

End of the growth phase, $t = t_0$: *amplification time = diffusion time*,

$$\frac{1}{G} = \frac{[\Delta r(t_0)]^2}{\beta} \quad \Rightarrow \quad t_0 \simeq \frac{C_\omega^{1/2}}{G}, \quad p(t_0) \simeq -C_\omega^{-1/2}.$$

$$B_{\max} \simeq B_0 G t_0 \simeq B_0 C_\omega^{1/2}, \quad \Delta r(t_0) \simeq \frac{R_0}{C_\omega^{1/2}}.$$

Galactic discs:

$$G = V_0/R_0, \quad \beta = \tfrac{1}{3} l_0 v_0 \quad \Rightarrow \quad C_\omega = 3\frac{V_0}{v_0}\frac{R_0}{l_0} \simeq 6000,$$

$$p \simeq -C_\omega^{-1/2} \simeq -1^\circ, \qquad \Delta r \simeq R_0 C_\omega^{-1/2} \simeq 100\,\text{pc},$$

$$B_{\max} = B_0 C_\omega^{1/2} \simeq 0.1\,\mu\text{G} \quad \text{for } B_0 = 10^{-9}\,\text{G}.$$

$p \simeq -1^\circ$, $\Delta r \simeq 100\,\text{pc}$, $B \simeq 0.1\,\mu\text{G}$:
a configuration very different from that observed,
$p \simeq -15^\circ$, $\Delta r > 1\,\text{kpc}$, $B \simeq 3\,\mu\text{G}$.

Conclusion: to avoid twisting by differential rotation, the *large-scale* galactic magnetic field has to be replenished (by a dynamo action).

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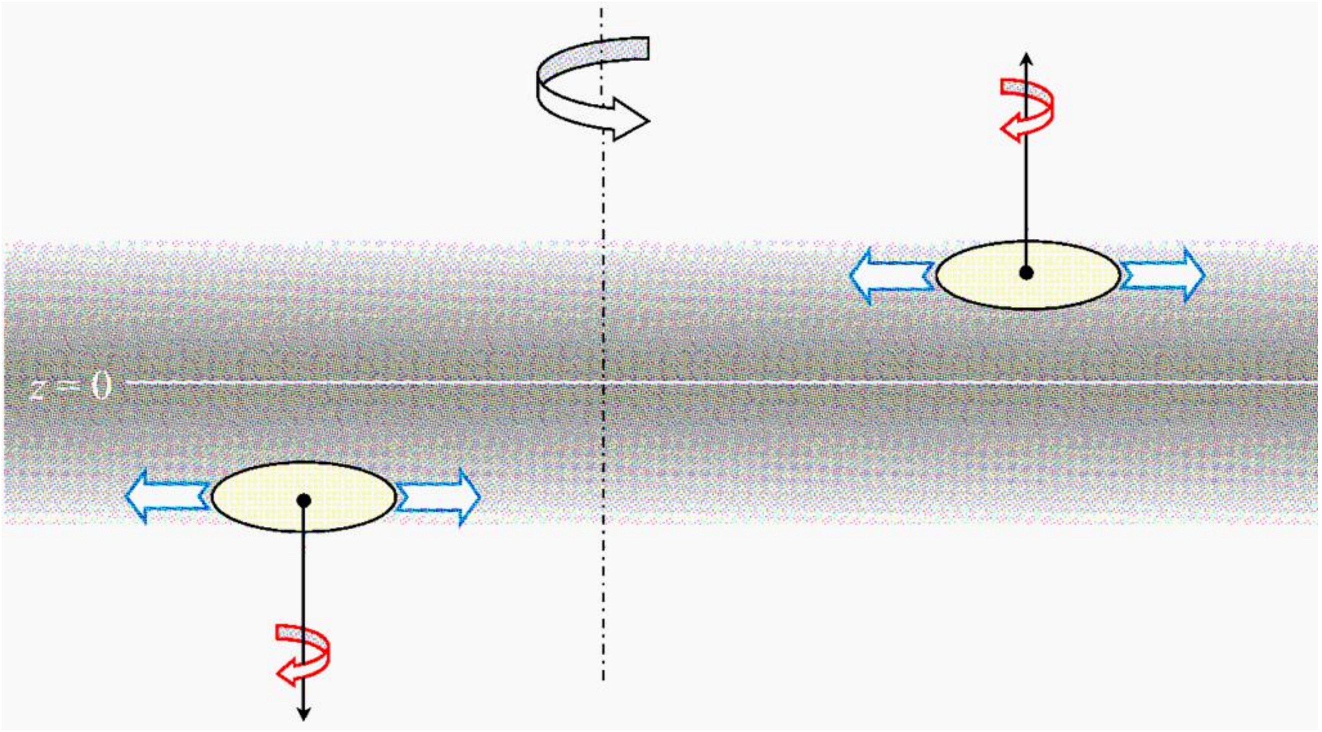
the *large-scale* galactic magnetic field
has to be replenished (by a dynamo action).

3. Mean-field disc dynamos

The physical picture of the galactic dynamo

(A) Helicity of interstellar turbulence:

consequence of angular momentum conservation in a rotating, stratified layer

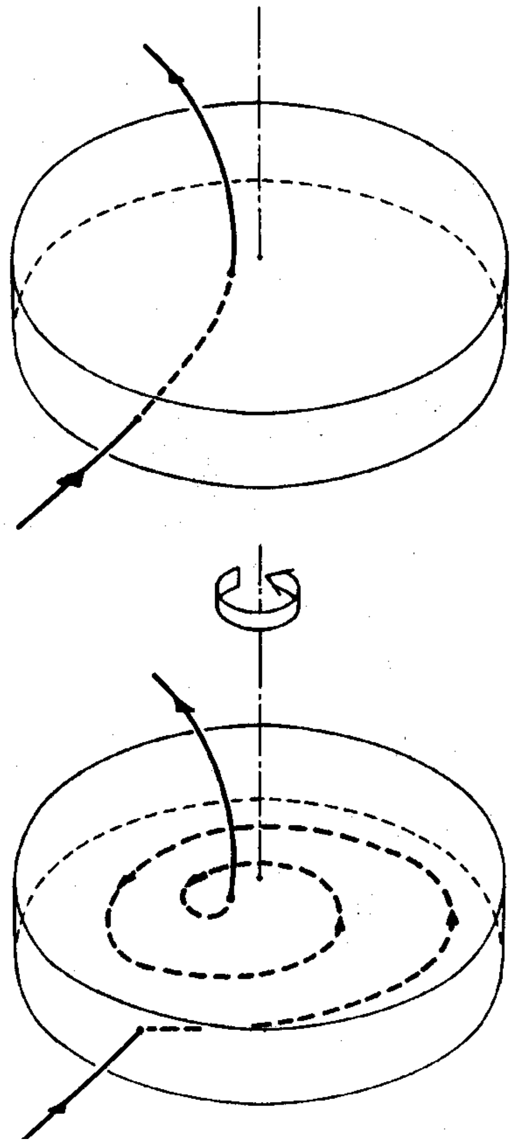


$$\alpha \simeq \frac{l_0^2 \Omega}{h}$$

(F. Krause, 1967)

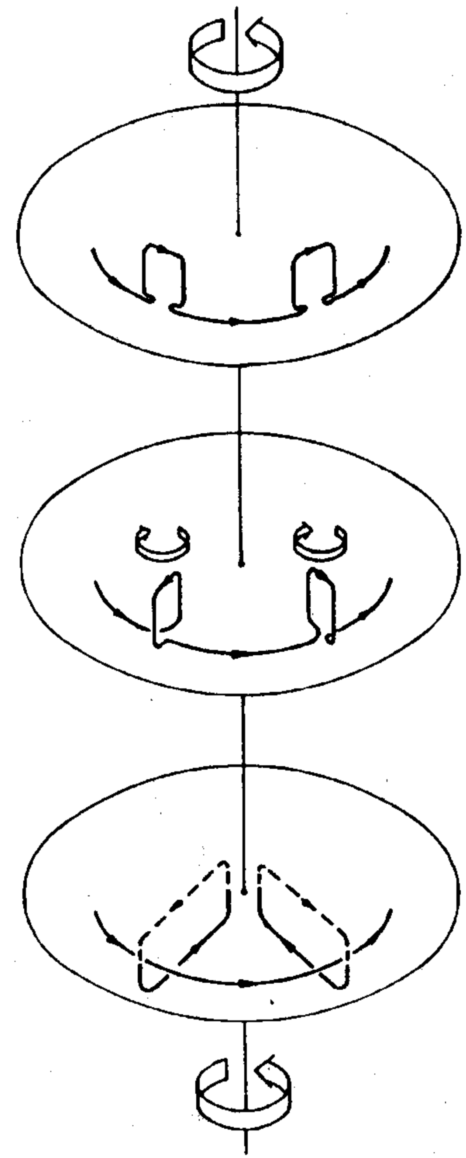
(B) Differential rotation:

B_ϕ produced from B_r



(C) Helical turbulence:

B_r produced from B_ϕ



3.1. Basic equations

$\vec{B}$ = large-scale magnetic field

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\alpha \vec{B} + \vec{V} \times \vec{B}) + \beta \nabla^2 \vec{B}$$

Cylindrical coordinates (r, ϕ, z), $\quad \vec{e}_z \parallel \vec{\Omega}$,

$\vec{V} = (0, r\Omega, 0), \quad \Omega = \Omega(r).$

Thin disc: $|z| \leq h, \quad r \leq R, \quad R >> h,$

$\partial/\partial z \gg \partial/\partial r,\ \partial/r\partial\phi \Rightarrow \nabla^2 \vec{B} = \dfrac{\partial^2 \vec{B}}{\partial z^2}, \cdots$

Axial symmetry: $\partial/\partial\phi = 0.$

$$\mathbf{B}(t, \mathbf{r}) = \tilde{\mathbf{B}}(t, r, z) e^{im\phi}\ , \qquad \epsilon = \frac{h_0}{R_0} \ll 1\ .$$

$$
\begin{aligned}
\left(\frac{\partial}{\partial t} + imR_\omega\Omega + \frac{\epsilon^2 m^2}{r^2}\right)\tilde{B}_r \;&= -R_\alpha \frac{\partial}{\partial z}(\alpha \tilde{B}_\phi) + \frac{\partial^2 \tilde{B}_r}{\partial z^2} + \epsilon^2 \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_r)\right] \\
&\quad + im\epsilon R_\alpha \frac{\alpha}{r}\tilde{B}_z - \frac{2im\epsilon^2}{r^2}\tilde{B}_\phi \,, \\
\left(\frac{\partial}{\partial t} + imR_\omega\Omega + \frac{\epsilon^2 m^2}{r^2}\right)\tilde{B}_\phi \;&= R_\omega G \tilde{B}_r + R_\alpha \frac{\partial}{\partial z}(\alpha \tilde{B}_r) + \frac{\partial^2 \tilde{B}_\phi}{\partial z^2} \\
&\quad + \epsilon^2 \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_\phi)\right] - \epsilon R_\alpha \frac{\partial}{\partial r}(\alpha \tilde{B}_z) \\
&\quad + \frac{2im\epsilon^2}{r^2}\tilde{B}_r \,, \\
\left(\frac{\partial}{\partial t} + imR_\omega\Omega + \frac{\epsilon^2 m^2}{r^2}\right)\tilde{B}_z \;&= \frac{\partial^2 \tilde{B}_z}{\partial z^2} + R_\alpha \frac{\epsilon}{r}\frac{\partial}{\partial r}(r\alpha \tilde{B}_\phi) - im\epsilon R_\alpha \frac{\alpha}{r}\tilde{B}_r \\
&\quad + \epsilon^2 \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tilde{B}_z)\right] + \frac{\epsilon^2}{r^2}\tilde{B}_z \,,
\end{aligned}
$$

$$
\mathbf{B}(t,\mathbf{r}) = \tilde{\mathbf{B}}(t,r,z)e^{im\phi} \,, \qquad \epsilon = \frac{h_0}{R_0} \ll 1 \,, \qquad G = r\frac{d-}{dr} \,.
$$

Thin disc, axisymmetric solutions:

$$\frac{\partial B_r}{\partial t} = -\frac{\partial}{\partial z}(\alpha B_\phi) + \beta\frac{\partial^2 B_r}{\partial z^2}\ ,$$
$$\frac{\partial B_\phi}{\partial t} = G B_r + \frac{\partial}{\partial z}(\alpha B_r) + \beta\frac{\partial^2 B_\phi}{\partial z^2}\ ,$$
$$\frac{\partial B_z}{\partial t} = \beta\frac{\partial^2 B_z}{\partial z^2}\ .$$

$$G = r\, d\Omega/dr$$

Equation for B_z splits from the system.

B_z is supported through B_r and B_ϕ via $\partial/\partial r$

Dimensionless variables

$$\tilde{z} = \frac{z}{h} \Rightarrow \frac{\partial}{\partial z} = \frac{1}{h}\,\frac{\partial}{\partial \tilde{z}}\,, \qquad \tilde{t} = \frac{t}{h^2/\beta} \Rightarrow \frac{\partial}{\partial t} = \frac{\beta}{h^2}\,\frac{\partial}{\partial \tilde{t}}\,,$$

$$\tilde{\alpha} = \frac{\alpha(z)}{\alpha_0}\,.$$

$$\begin{aligned}
\frac{\partial B_r}{\partial \tilde{t}} &= -R_\alpha \frac{\partial}{\partial \tilde{z}}(\tilde{\alpha} B_\phi) + \frac{\partial^2 B_r}{\partial \tilde{z}^2}\,, \qquad R_\alpha = \frac{\alpha_0 h}{\beta} \\
\frac{\partial B_\phi}{\partial \tilde{t}} &= R_\omega B_r + R_\alpha \frac{\partial}{\partial \tilde{z}}(\tilde{\alpha} B_r) + \frac{\partial^2 B_\phi}{\partial \tilde{z}^2}\,, \qquad R_\omega = \frac{G h^2}{\beta}\,.
\end{aligned}$$

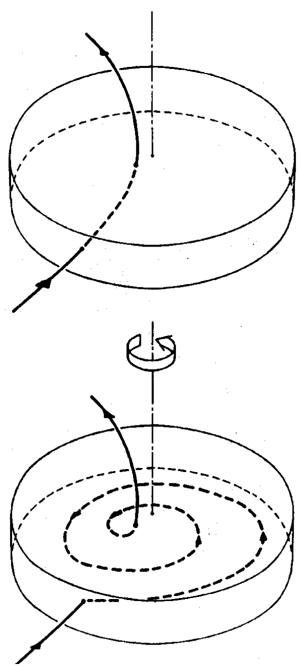
Drop˜at dimensionless variables:

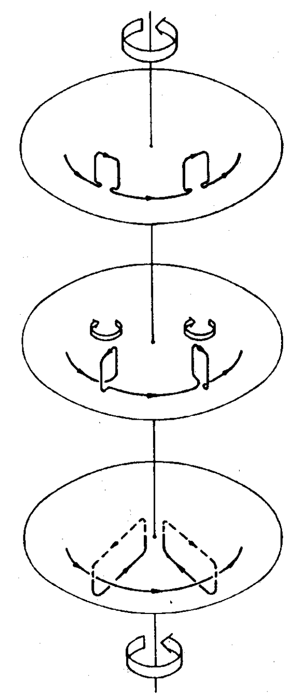
$$\frac{\partial B_r}{\partial t} = \underbrace{-R_\alpha \frac{\partial}{\partial z}(\alpha B_\phi)}_{B_\phi \to B_r \text{ via } \alpha\text{-effect}} + \frac{\partial^2 B_r}{\partial z^2} ,$$

$$\frac{\partial B_\phi}{\partial t} = \underbrace{R_\omega B_r}_{B_r \to B_\phi \text{ via differential rotation}} + \underbrace{R_\alpha \frac{\partial}{\partial z}(\alpha B_r)}_{B_r \to B_\phi \text{ via } \alpha\text{-effect}} + \frac{\partial^2 B_\phi}{\partial z^2} .$$

$$R_\alpha = \frac{\alpha_0 h}{\beta}$$

$$R_\omega = \frac{h^2\, r\, d\text{-}\ /dr}{\beta}$$





αω-Dynamo: $|R_\omega| >> R_\alpha$

$$\begin{aligned} \frac{\partial B_r}{\partial t} &= -R_\alpha \frac{\partial}{\partial z}(\alpha B_\phi) + \frac{\partial^2 B_r}{\partial z^2} , \\ \frac{\partial B_\phi}{\partial t} &= R_\omega B_r + \frac{\partial^2 B_\phi}{\partial z^2} . \end{aligned}$$

Introduce new variable $B_r = R_\alpha B_r'$ and drop the dash:

$$\begin{aligned} \frac{\partial B_r}{\partial t} &= -\frac{\partial}{\partial z}(\alpha B_\phi) + \frac{\partial^2 B_r}{\partial z^2} , \\ \frac{\partial B_\phi}{\partial t} &= D B_r + \frac{\partial^2 B_\phi}{\partial z^2} , \end{aligned}$$

where $D = R_\alpha R_\omega$ is the dynamo number.

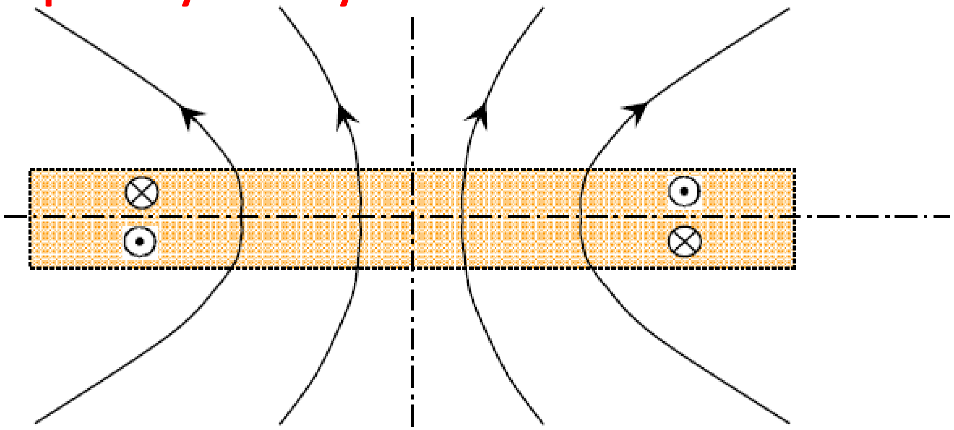
Boundary conditions

$$B_r|_{z=1} = B_\phi|_{z=1} = 0 \qquad \text{(vacuum boundary conditions)}$$

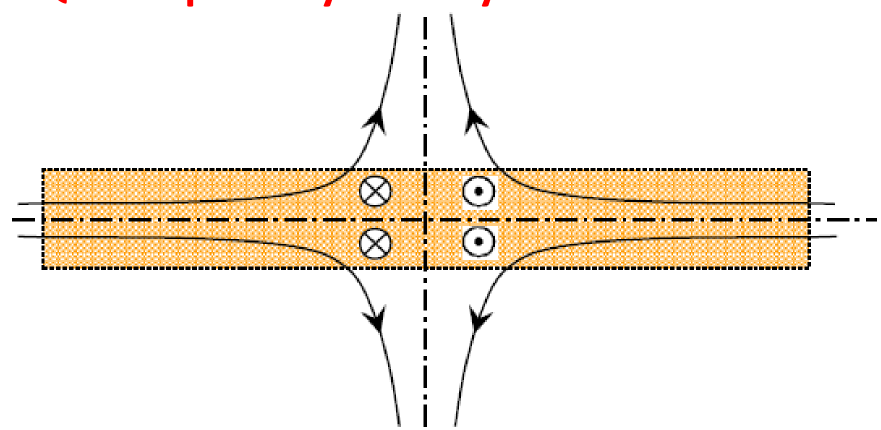
$$\left.\frac{\partial B_r}{\partial z}\right|_{z=0} = \left.\frac{\partial B_\phi}{\partial z}\right|_{z=0} = 0 \qquad \text{(quadrupole)}$$

$$B_r|_{z=0} = B_\phi|_{z=0} = 0 \qquad \text{(dipole)}$$

Dipolar symmetry



Quadrupolar symmetry



3.2. Dynamo control parameters

NB! The Solar neighbourhood of the Milky Way, where these estimates apply, is not a typical galactic location.

Rotation - $= \dfrac{V_0}{r}$,

$$V_0 \simeq 200\,\mathsf{km/s},\ \ r \simeq 10\,\mathsf{kpc}.$$

Ionised gas scale height
$h \simeq 0.5\,\mathsf{kpc}$,

Turbulent velocity $v_0 \simeq 10\,\mathsf{km/s}$.

Turbulent scale $l_0 \simeq 0.1\,\mathsf{kpc}$.

$$\alpha_0 \simeq \frac{l_0^2 -}{h} \simeq 0.4\,\mathsf{km/s},$$

$$\beta \simeq \tfrac{1}{3} l_0 v_0 \simeq 10^{26}\,\mathsf{cm^2/s},$$

$$R_\alpha = \frac{\alpha_0 h}{\beta} \simeq 0.6$$

$$R_\omega = \frac{(r\,d- \ /dr)h^2}{\beta} \simeq -15$$

$$D = R_\alpha R_\omega \simeq -\left(\frac{3- \ h}{v_0}\right)^2 \simeq -10$$

4. The "no-z" approximation (Subramanian & Mestel, 1993)

Thin disc, dimensional αω-dynamo equations:

$$\frac{\partial B_r}{\partial t} = -\frac{\partial}{\partial z}(\alpha B_\phi) + \beta \frac{\partial^2 B_r}{\partial z^2},$$
$$\frac{\partial B_\phi}{\partial t} = G B_r + \beta \frac{\partial^2 B_\phi}{\partial z^2}.$$

Solutions have a simple form, e.g., $B_{r,\phi} \propto \cos z/h$:

$$\frac{\partial}{\partial z} \simeq \frac{1}{h}, \qquad \frac{\partial^2}{\partial z^2} \simeq -\frac{1}{h^2}.$$

Kinematic solutions: $\vec{B} = \vec{B}_0 \exp(\gamma t)$.

$$
\begin{aligned}
\left(\gamma + \frac{\beta}{h^2}\right) B_{0r} + \frac{\alpha}{h} B_{0\phi} &= 0, \\
-G B_{0r} + \left(\gamma + \frac{\beta}{h^2}\right) B_{0\phi} &= 0.
\end{aligned}
$$

Nontrivial solutions exist if

$$
\begin{vmatrix} \gamma + \beta/h^2 & \alpha/h \\ -G & \gamma + \beta/h^2 \end{vmatrix} = 0,
$$

$$
\text{i.e.,} \quad \gamma \simeq \frac{\beta}{h^2}(-1 + \sqrt{-D}),
$$

$$
\tan p = \frac{B_r}{B_\phi} \simeq -\sqrt{\frac{\alpha}{-Gh}} = -\sqrt{\frac{R_\alpha}{|R_\omega|}}.
$$

Magnetic field grows if $D \lesssim -1$, with $p \simeq -\arctan\frac{1}{4} \simeq -15^\circ$.

5. Nonlinear dynamo action and magnetic helicity

Magnetic helicity: $\chi = \langle \vec{\mathcal{A}} \cdot \vec{\mathcal{B}} \rangle, \quad \vec{\mathcal{B}} = \nabla \times \vec{\mathcal{A}}$

(conserved in ideal MHD)

$$t = 0 \Rightarrow \vec{\mathcal{B}} \approx 0 \text{ (weak seed field)}$$

$$\Rightarrow \chi\big|_{t=0} \approx 0 \Rightarrow \chi\big|_{\text{now}} \approx 0$$

Introduce large- & small-scale magnetic fields and the corresponding helicities:

$$\vec{\mathcal{B}} = \vec{B} + \vec{b}, \quad \vec{\mathcal{A}} = \vec{A} + \vec{a},$$

$$\chi = \chi_B + \chi_b, \quad \chi_B = \vec{A} \cdot \vec{B}, \quad \chi_b = \vec{a} \cdot \vec{b}$$

$$\chi = \chi_B + \chi_b, \quad \chi_B = \vec{A}\cdot\vec{B}, \quad \chi_b = \vec{a}\cdot\vec{b}$$

- $\chi_B = \langle \vec{A}\cdot\vec{B}\rangle \simeq -LB_rB_\phi \simeq \frac{1}{4}LB^2$,

  for $B_r/B_\phi = -\sin p$, $\quad p = 15^\circ; \quad L \gtrsim 1$ kpc.

- $\chi_b = \langle \vec{a}\cdot\vec{b}\rangle \simeq -l_d b^2$,

  $l_d \lesssim l \simeq 100$ pc $\qquad$ ($l_d =$ scale of χ_b, $l =$ turbulent scale)

- $\chi = \chi_B + \chi_b = 0 \quad \Rightarrow \quad \dfrac{B^2}{b^2} \simeq \dfrac{4l_d}{L} \simeq 0.4$, **if** $l_d \simeq l$

Catastrophic α-quenching: $\dfrac{B^2}{b^2} \simeq R_m^{-1} \ll 1$ **if** $\dfrac{l_d}{L} \simeq R_m^{-1}$

(R_m =magnetic Reynolds number)

A mechanical analogy of helicity conservation: twist & writhe of a hose pipe

Twist by 90°

Twist by 180°

Galactic discs are not closed systems:

galactic winds and fountains

$\Rightarrow$ the “unwanted” magnetic helicity can be removed from the disc

Galactic fountain/wind removes magnetic field from the disc

Hot gas outflow through the disc surface: $V_z = 150\text{–}200$ km/s

Outflow time scale: $H/V_z = 1$ kpc/150 km/s $= 10^7$ yr

Surface filling factor of the hot gas: $f_\mathrm{s} = 0.2\text{–}0.3$

Relative density of the hot gas: $$\frac{\rho_h}{\langle\rho\rangle} \simeq \frac{10^{-3}\,\mathrm{cm}^{-3}}{0.1\,\mathrm{cm}^{-3}} = 10^{-2}$$

Effective (mass-averaged) advection speed (for single-phase dynamo models):

$$U_z \simeq f_\mathrm{s}\frac{\rho_h}{\langle\rho\rangle}V_z \simeq 10^{-2}V_z \simeq 0.3\!-\!2\,\mathrm{km/s}$$

Helicity balance

(Shukurov et al. A&A 448, L33, 2006)

Random field $\vec{b}$ has finite correlation length $\Rightarrow$ define volume density of linkages of $\vec{b}$:

$\chi \approx H_b$ for $\nabla \cdot \vec{a} = 0$

Evolution equation:

$$\frac{\partial \chi}{\partial t} + \nabla \cdot \vec{F} = -2\vec{\mathcal{E}} \cdot \vec{B} - 2\eta\overline{\vec{j} \cdot \vec{b}}$$

$\vec{j} = \nabla \times \vec{b}$, $\vec{J} = \nabla \times \vec{B}$, electric current densities

$\vec{\mathcal{E}} \approx \alpha\vec{B} - \beta\nabla \times \vec{B}$, mean electromotive force

$\vec{F} = \chi\vec{U}$, advective flux.

$\alpha = \alpha_{\rm kinetic} + \alpha_{\rm m}$, $\alpha_{\rm m} \simeq \frac{1}{3}\tau k_0^2 \frac{\chi}{\rho}$

$$\frac{\partial \alpha_{\rm m}}{\partial t} = -2\beta k_0^2 \left(\frac{\vec{\mathcal{E}} \cdot \vec{B}}{B_{\rm eq}^2} + \frac{\alpha_{\rm m}}{R_{\rm m}} \right) - \nabla \cdot \left(\alpha_{\rm m}\vec{U} \right)$$

+ mean-field dynamo equations for B_r and B_ϕ

Mean-field dynamo: $\dfrac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{U} \times \vec{B} + \alpha \vec{B} - \beta \nabla \times \vec{B})\,.$

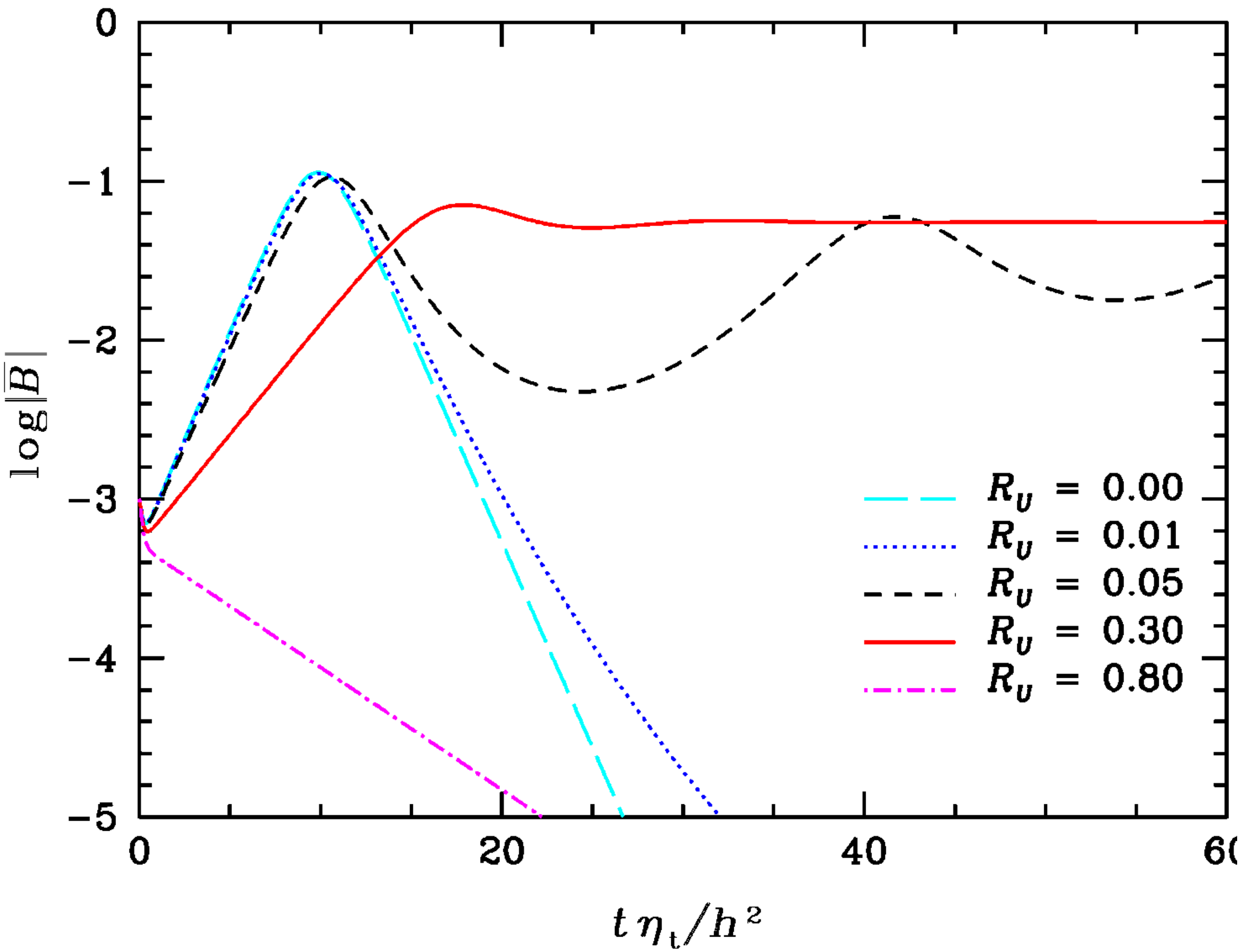
Thin disc $|z| \cdot\ h$, $\dfrac{\partial}{\partial z} \gg \dfrac{\partial}{\partial r}$, axial symmetry, $\partial / \partial \phi = 0$,
$\vec{U} = (0, r\text{-}\ (r), U_z)$, dimensionless equations:

$$
\begin{aligned}
\frac{\partial B_r}{\partial t} &= -\frac{\partial}{\partial z}(R_u U_z B_r + R_\alpha \alpha B_\phi) + \frac{\partial^2 B_r}{\partial z^2}\,, \\
\frac{\partial B_\phi}{\partial t} &= R_\omega B_r - R_u \frac{\partial}{\partial z}(U_z B_\phi) + \frac{\partial^2 B_\phi}{\partial z^2}\,, \\
\frac{\partial \alpha_m}{\partial t} &= -C\left(\alpha B^2 - R_\alpha^{-1} \vec{J} \cdot \vec{B} + \frac{\alpha_m}{R_m}\right) - R_u \frac{\partial}{\partial z}(U_z \alpha_m)\,,
\end{aligned}
$$

$$\alpha = \alpha_K + \alpha_m, \quad \alpha_K \simeq \frac{l^2\text{-}}{h}, \quad C = 2\frac{h^2}{l^2}, \quad \vec{J} \cdot \vec{B} = B_\phi \frac{\partial B_r}{\partial z} - B_r \frac{\partial B_\phi}{\partial z},$$

$$R_\alpha = \frac{\alpha_0 h}{\beta}\,, \quad R_\omega = \frac{(r\partial\text{-}\ /\partial r)_0 h^2}{\beta}\,, \quad R_u = \frac{U_{z0} h}{\beta}.$$

Sur, Shukurov & Subramanian (MNRAS, 2007): Magnetic field evolution in a galactic disc with helicity advection by the galactic fountain/wind, "no-z" approximation: $\partial/\partial z \to 1/h$, $\partial^2/\partial z^2 \to -1/h^2$



1. Helicity balance: $B^2 \simeq 12\pi\rho v_0 \frac{l}{h} U_z \left(\frac{D}{D_{\rm crit}} - 1\right)$,
2. Balance of the Coriolis & Lorentz forces: $B^2 \simeq 4\pi\rho h v_0\text{-}\ \frac{|R_\omega|}{\sqrt{|D_{\rm crit}|}}$.

Dependence of B^2 on galactic parameters

1. Suprenova (SN) frequency (per unit area): $\nu_{\rm SN} \propto \rm SFR$

   $\sigma_{\rm g} = 2\rho h$, gas surface density

   Energy injection rate by SNe: $\dot{E}_{\rm SN} = E_{\rm SN}\nu_{\rm SN}$,

   Outflow speed: $\frac{1}{2}\frac{\sigma_{\rm h} V_z^2}{\tau} \simeq q\dot{E}_{\rm SN}, \qquad \tau = H/V_z,$

   $\sigma_{\rm h} = 2\rho_{\rm h} H$, hot gas surface density,

   $q \simeq 0.1$, SN energy fraction converted into kinetic energy,

   $$B^2 \propto (\rho_{\rm h}\rho)^{1/3} v_0\, {\rm SFR}^{1/3} \left(\frac{D}{D_{\rm crit}} - 1\right).$$

2. $B^2 \propto \rho h^3\text{-}\ r \left|\frac{d\text{-}}{dr}\right|$

Distinct dependencies on observable parameters.

Galactic outflows & dynamos: magnetic pitch angle

Nonlinear, steady state: $\partial/\partial t = 0$,

α-effect suppressed to its marginal level, $D(\alpha_K + \alpha_m) = D_{\mathrm{crit}}$.

No-z approximation, dynamo with an outflow, $R_u \neq 0$:

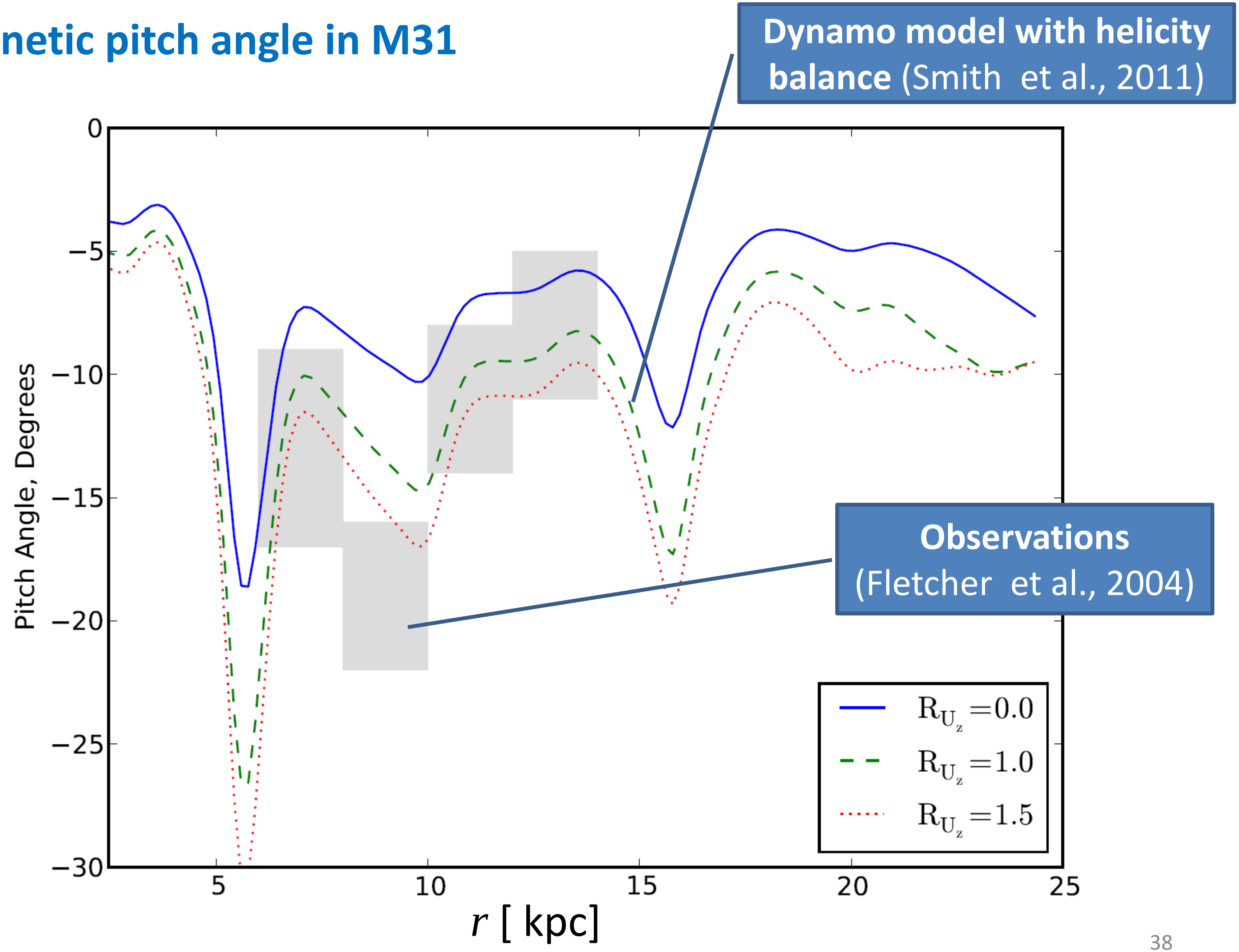
$$0 = -\frac{2}{\pi} R_\alpha \frac{D}{D_{\mathrm{crit}}} B_\phi - \left(R_u + \frac{\pi^2}{4} \right) B_r \,,$$

$$0 = R_\omega B_r - \left(R_u + \frac{\pi^2}{4} \right) B_\phi \,.$$

$$\frac{B_r}{B_\phi} = \frac{R_u + \pi^2/4}{R_\omega} \,,$$

$$R_U = 0 \quad \Rightarrow \quad \tan p = \frac{\pi^2}{4R_\omega}, \qquad p \simeq -10^\circ$$

Magnetic pitch angle in M31



6. The fluctuation dynamo and random magnetic fields

- Interstellar magnetic field ≠ a quasi-homogeneous Gaussian random vector field.

- Interstellar shocks, multi-phase structure, …

- A quasi-homogeneous, weaker magnetic background from the tangling of the large-scale magnetic field by turbulence.

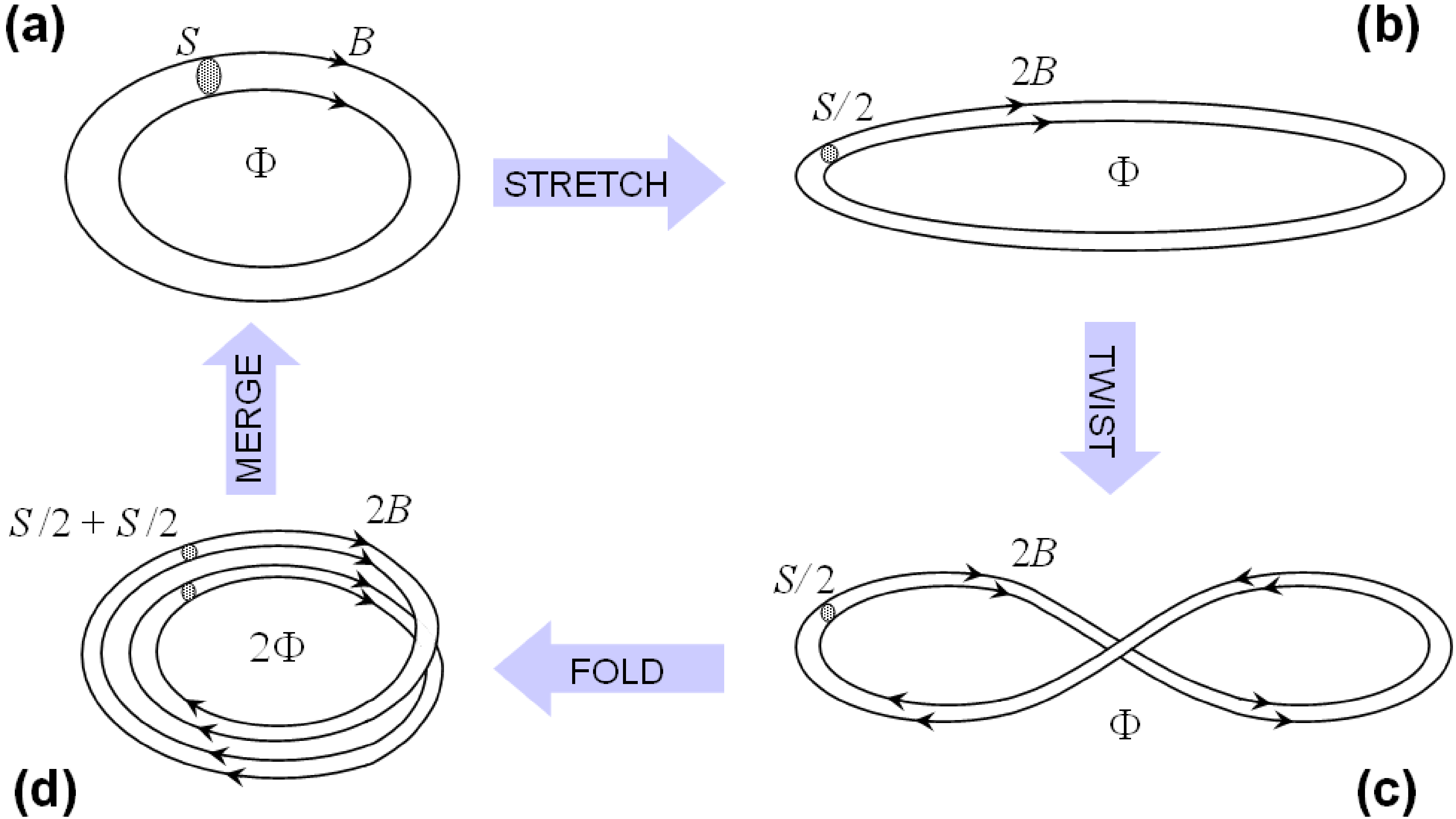
Further details in:

A. Shukurov & D. Sokoloff, Astrophysical dynamos. In: Ph. Cardin, L.F. Cugliandolo, editors, *Les Houches, Session LXXXVIII, 2007, Dynamos*. Elsevier, 2008, p. 251—299.

A. Shukurov, Introduction to galactic dynamos. In: *Mathematical Aspects of Natural Dynamos*, eds E. Dormy & A. M. Soward, Chapman & Hall/CRC, 2007, pp. 313--359 (astro-ph/0411739).

The fluctuation dynamo: exponential amplification and then maintenance of a random magnetic field in a random flow

- Growth due to random shearing (stretching) of magnetic fields.
- Only needs motions to be strong enough,
  $R_m > R_{m,cr}$, $R_{m,cr} = 30–100$ (depending on the flow type).
- Vortical motions are good dynamos; compressibility hinders the dynamo action.
- Very rapid amplification: growth time of rms magnetic field = eddy turnover time, $l_0/v_0 \simeq 10^7\,\mathrm{yr}$
- Conceptual model: Zeldovich's stretch–twist–fold dynamo:



$$B \propto 2^n \quad \Rightarrow \quad B \propto e^{\gamma t}$$

Fluctuation dynamo produces intermittent magnetic fields even in a homogeneous medium,

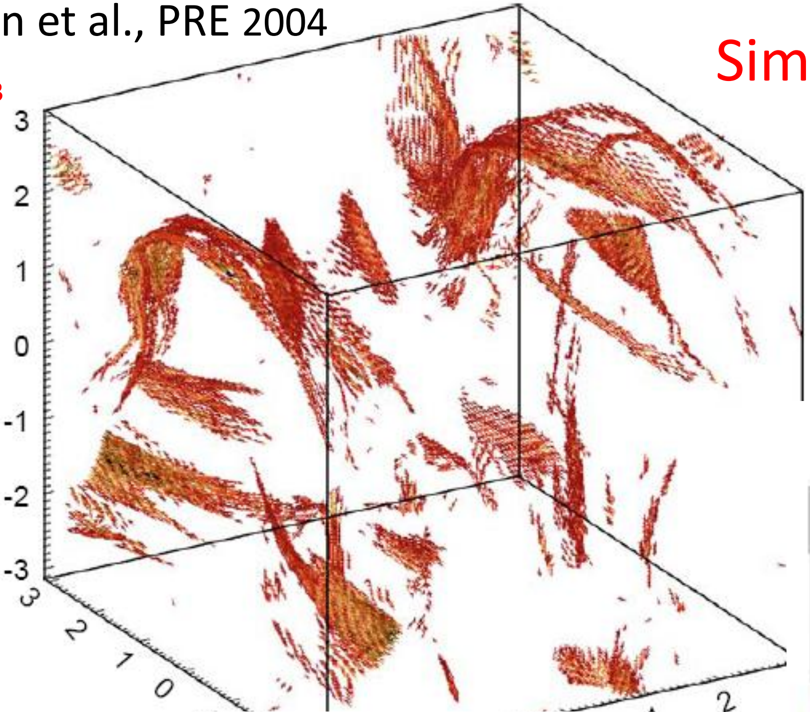
magnetic filaments (+ ribbons & sheets?):

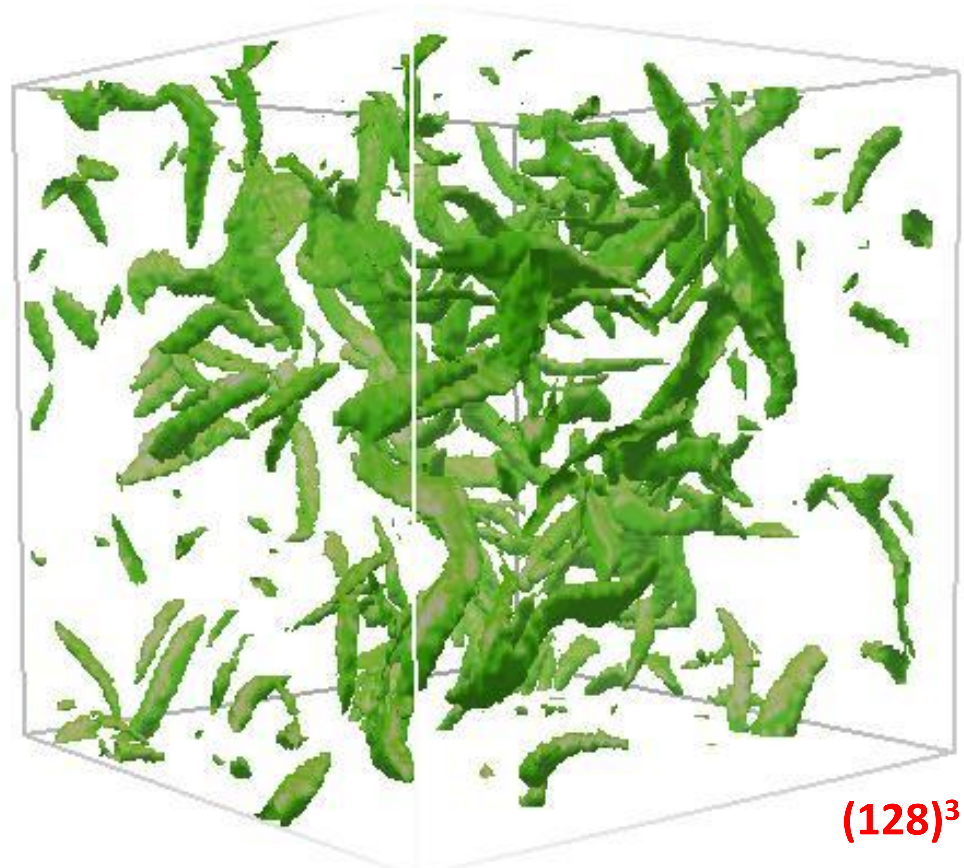
- $B_{\max} \simeq B_{\rm eq} = (4\pi\rho)^{1/2} v_0 \simeq 5\ \mu$G.
- Length $\simeq l \simeq$ 50–100 pc.
- Low volume filling factor, $\langle B^2 \rangle \simeq 0.1\, B^2_{\max}$.
- Kinematic stage: magnetic energy max at $l_\eta = l\, R_{\rm m}^{-1/2}$.
- Nonlinear, statistically steady state: controversial
  - folds at $l_\eta = l\, R_{\rm m}^{-1/2} \simeq 10^{-7}$ pc (???) (Schekochihin et al. 2004) or
  - thicker structures $l_{\eta,\rm cr} = l\, R_{\rm m,cr}^{-1/2} \simeq 10$ pc (Subramanian 1999).

Simulations of the fluctuation dynamo:
magnetic isosurfaces, B^2 = const

Haugen et al., PRE 2004

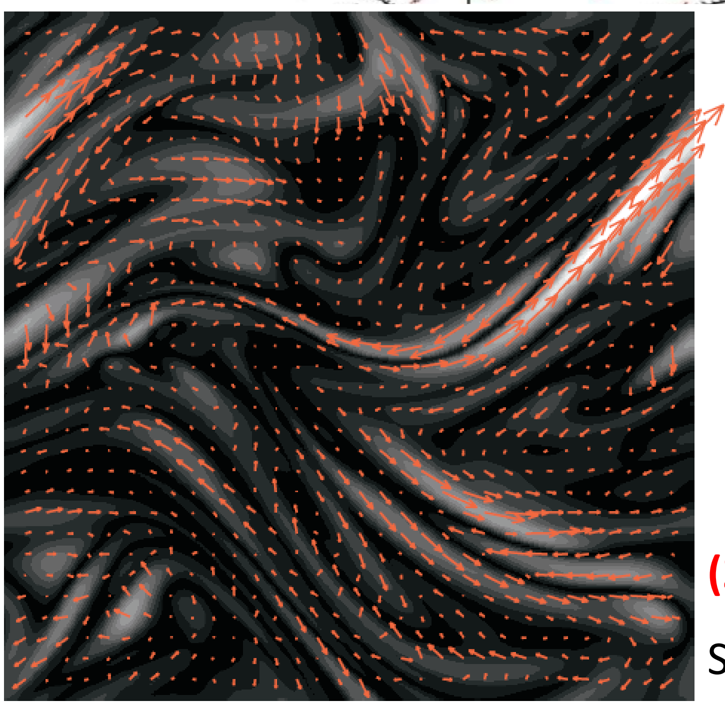
$(1024)^3$





$(128)^3$

Wilkin et al., PRL 2007



$(256)^3$

Schekochihin et al., ApJ 2004

Implications

- Power spectrum & structure/correlation function are not suitable tools to describe intermittent magnetic fields

  (intense flux ropes separated by extended regions with relatively weak magnetic field).

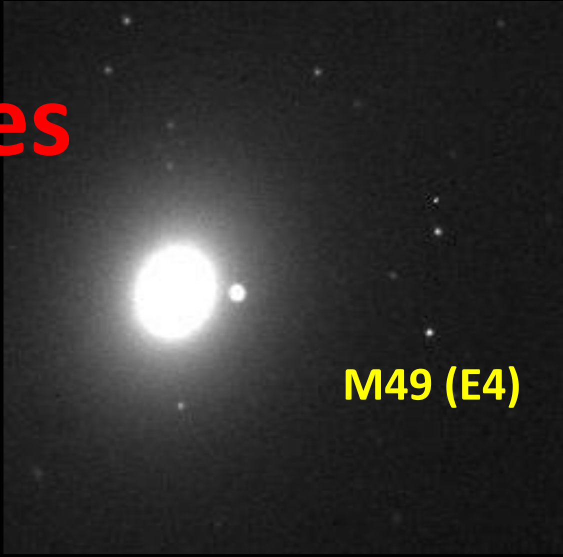
- Magnetic field estimates from synchrotron intensity can be strongly affected (underestimated random magnetic field).

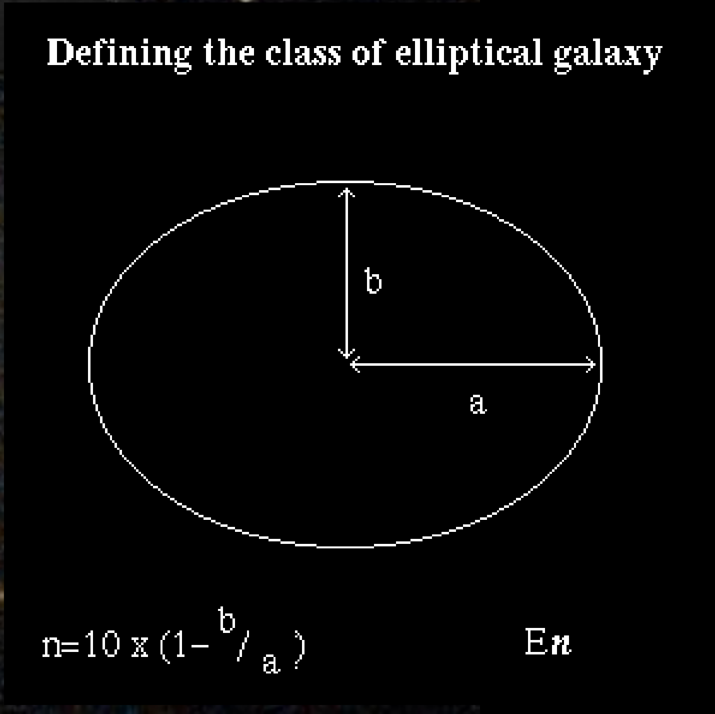
- Cosmic ray propagation can be strongly affected by magnetic intermittency. No models available of cosmic ray propagation in such magnetic fields.

- Locally anisotropic magnetic fields are less efficient in cosmic ray scattering (> 10^2–10^3 GeV) (Chandran 2000).

7. Elliptical galaxies

(10% of all galaxies)

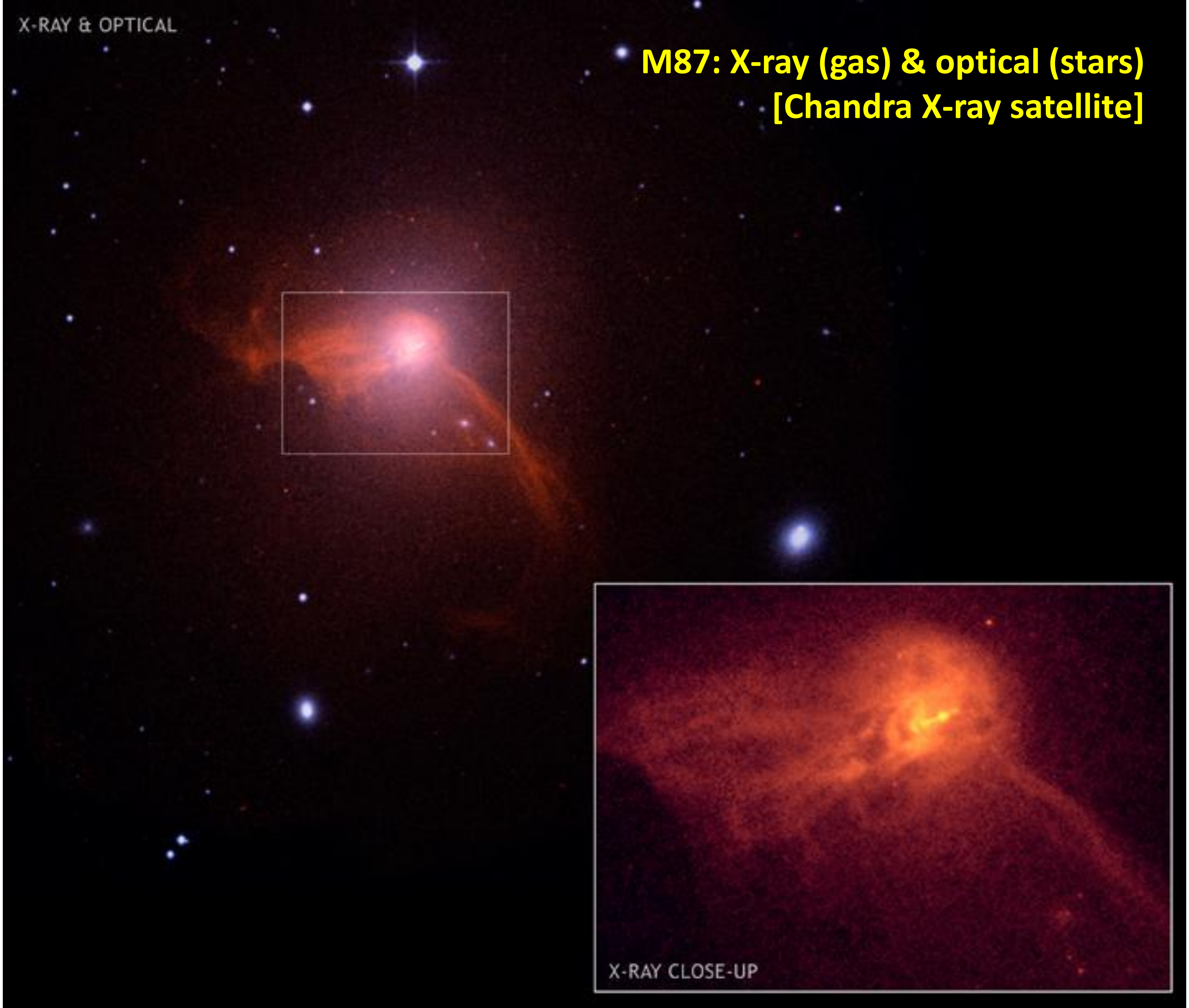


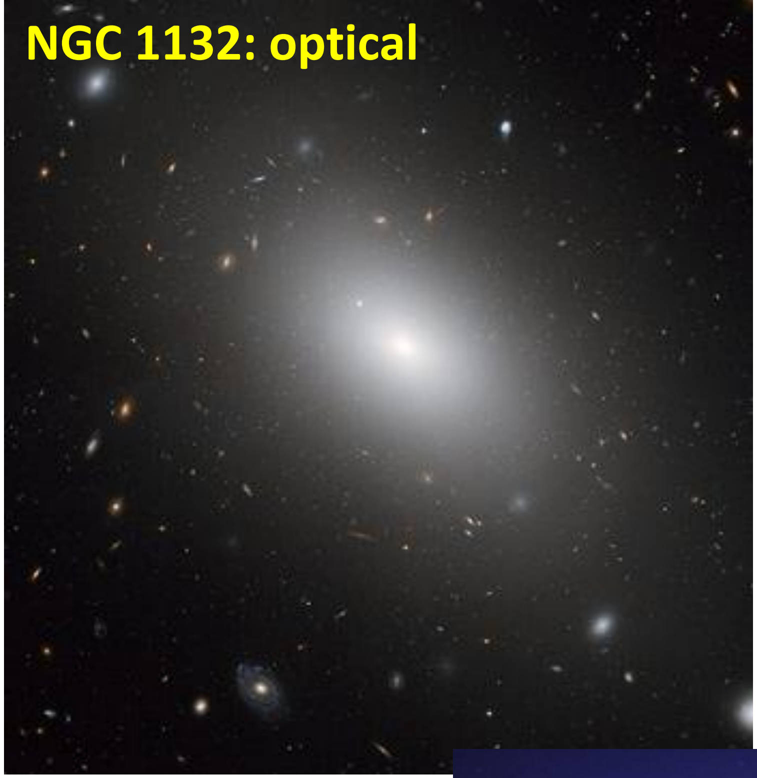


- Structureless, triaxial ellipsoids of old stars, no star formation, stellar density monotonously decreases outwards.

- Wide range in size and mass:
  - from giants (diameter a few Mpc, a few times $10^{12}\ M_{\odot}$, many have active nucleus, jet, are bright in radio, etc.)
  - to dwarves (a kpc in diameter, $10^{6}\ M_{\odot}$).

- No overall rotation: randomly oriented stellar orbits, stellar velocity dispersion $\sigma_{*} \simeq 300$ km/s.


X-RAY & OPTICAL
M87: X-ray (gas) & optical (stars)
[Chandra X-ray satellite]
X-RAY CLOSE-UP



NGC 1132: optical

NGC 1132: X-rays [Chandra]

NGC 4649: X-ray (purple), optical (blue) [Chandra]

Interstellar gas and magnetic fields in ellipticals

- Hot, $T \cong 10^7$ K, detected in X-rays;
- tenuous, $n \cong 10^{-3}\ \mathrm{cm}^{-3}$ ($10^{-2}\ \mathrm{cm}^{-3}$ near the centre);
- total gas mass $10^9 M_\odot \cong 1\%$ of the stellar mass;
- radius of the gas distribution $\cong 50$ kpc $\cong$ radius of the stellar distribution.
- No or weak spectral line emission $\Rightarrow$ little know about gas motions.
- Some observational evidence for random magnetic fields;
- possibility of the fluctuation dynamo action in elliptical galaxies (Moss & Shukurov, MNRAS 1996).