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Netherlands Institute for Radio Astronomy

RM Synthesis

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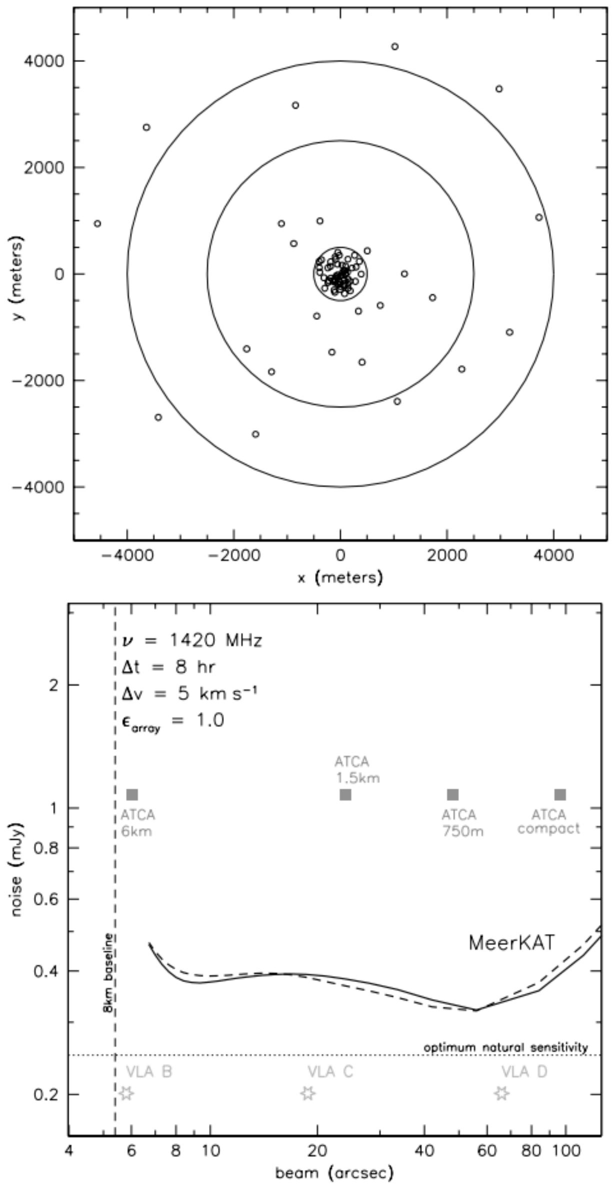
Sensitivity of interferometer image

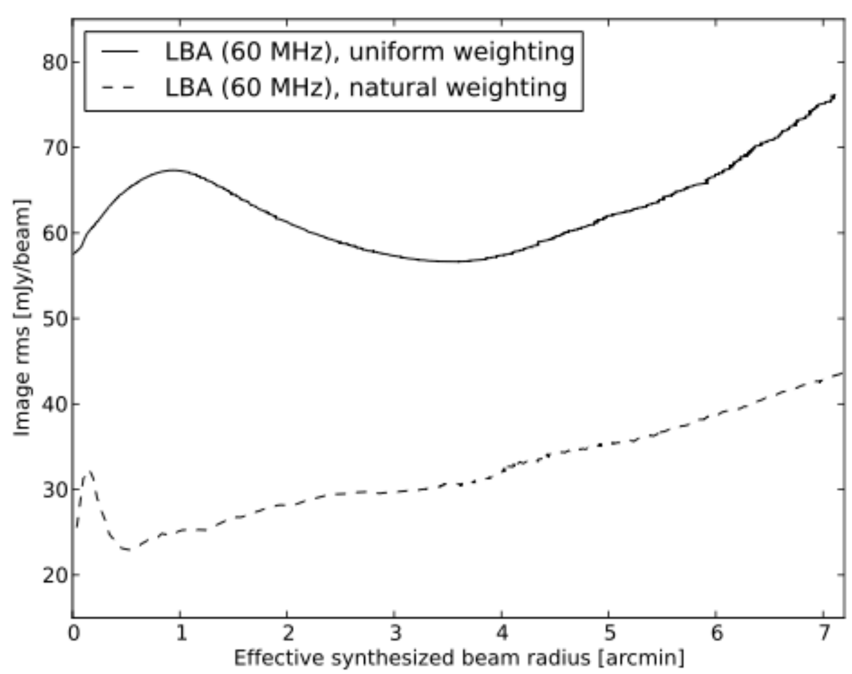
- See “Sensitivity” chapter by Wrobel & Walker in VLA white book:

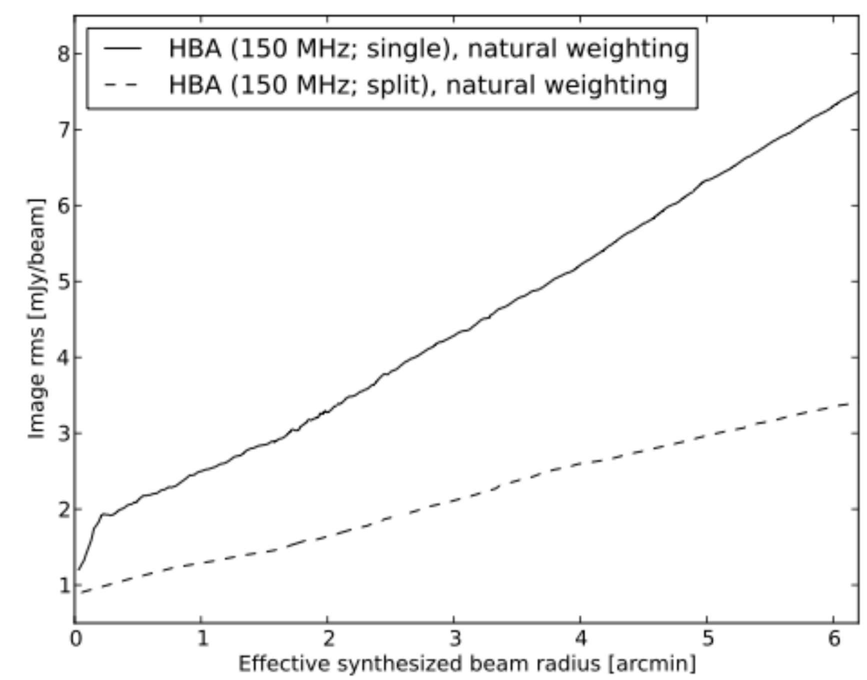
$$\Delta I_{\mathrm{m}} = \frac{SEFD}{\sqrt{N\,(N-1)\,\Delta\nu\, t_{\mathrm{int}}}}$$

- Examples, for MeerKAT (planned) and LOFAR (future array configuration):

de Blok et al. (SKA Forum 2009)







Heald et al. (2011)

Overview

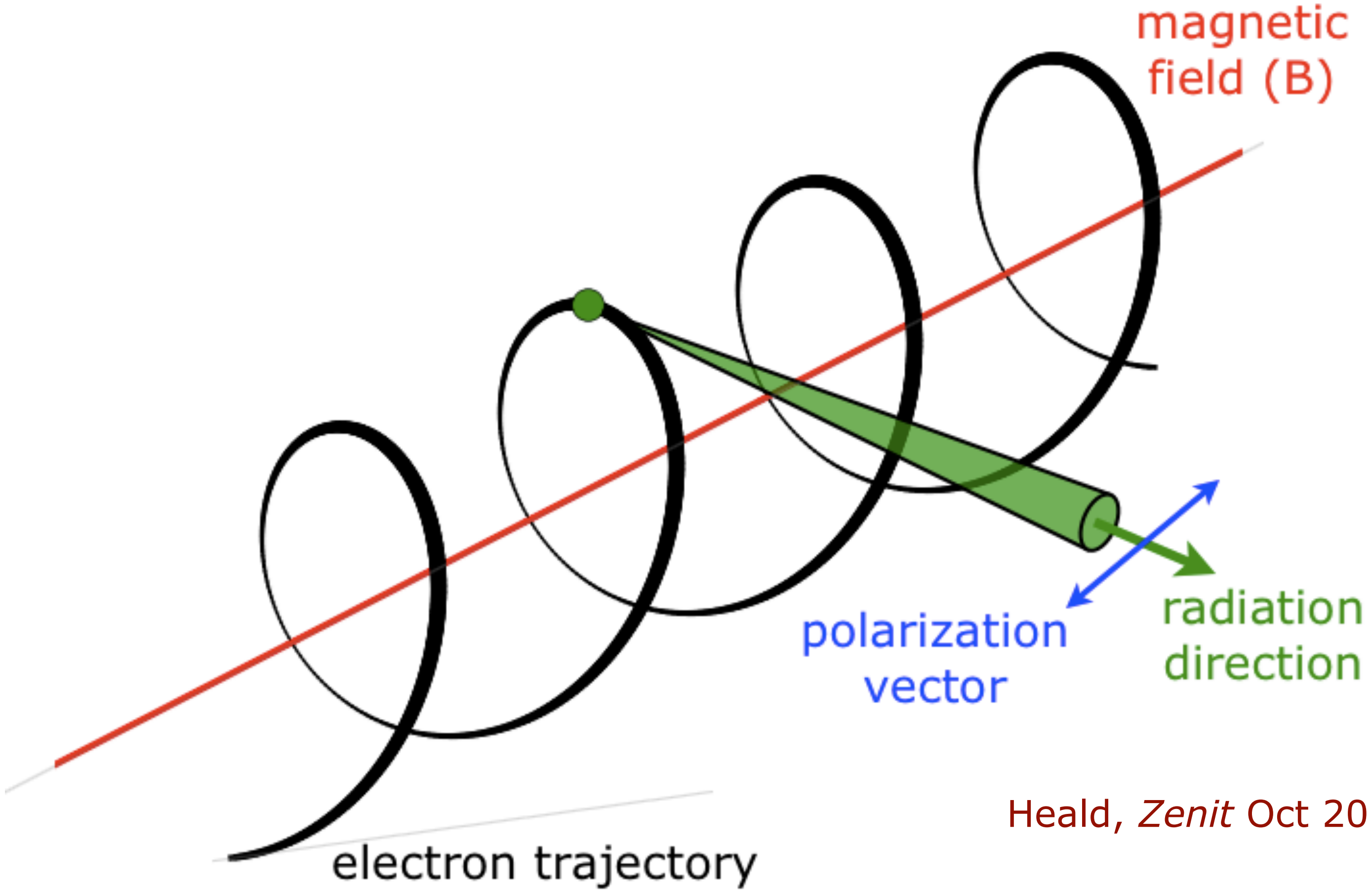
- Review of the RM Synthesis technique
- RM CLEAN
- Practical aspects
- Example: WSRT-SINGS
- New techniques ...

- See:
  Brentjens & de Bruyn 2005 (A&A, 441, 1217) [RM Synthesis]
  Heald 2009 (IAUS, 259, 591) [RM Synthesis Overview]
  Heald et al. 2009 (A&A, 503, 409) [RM Synthesis & RM-CLEAN]
  Frick et al. 2010 (MNRAS, 401, 24) [Wavelet-based RM Synthesis]
  Li et al. 2011 (A&A, 531, 126) [Compressive sensing]

Synchrotron radiation

- cosmic ray electrons spiraling around magnetic field lines
- beamed radiation in direction of electron motion
- linearly polarized perpendicular to magnetic field
- continuum radiation with power law index related to power law index of electron energy spectrum

Polarized synchrotron radiation traces $B_{\perp}$

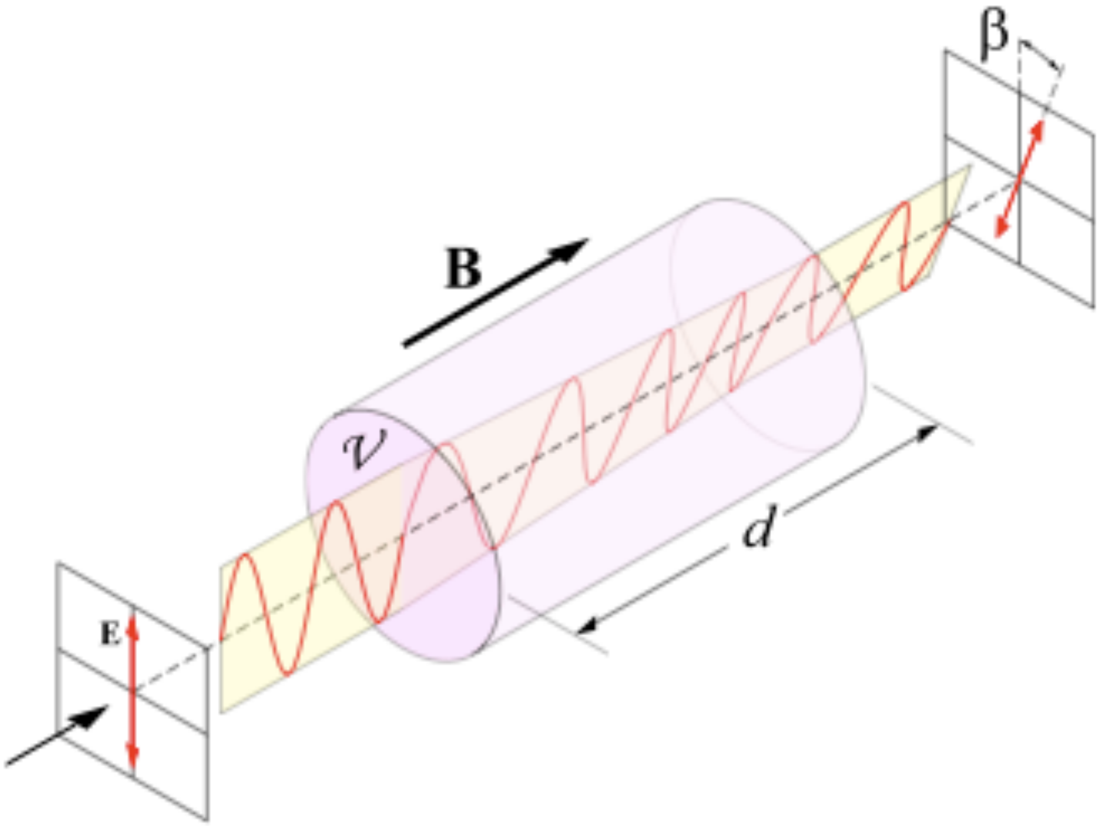


Heald, *Zenit* Oct 2010

Faraday rotation

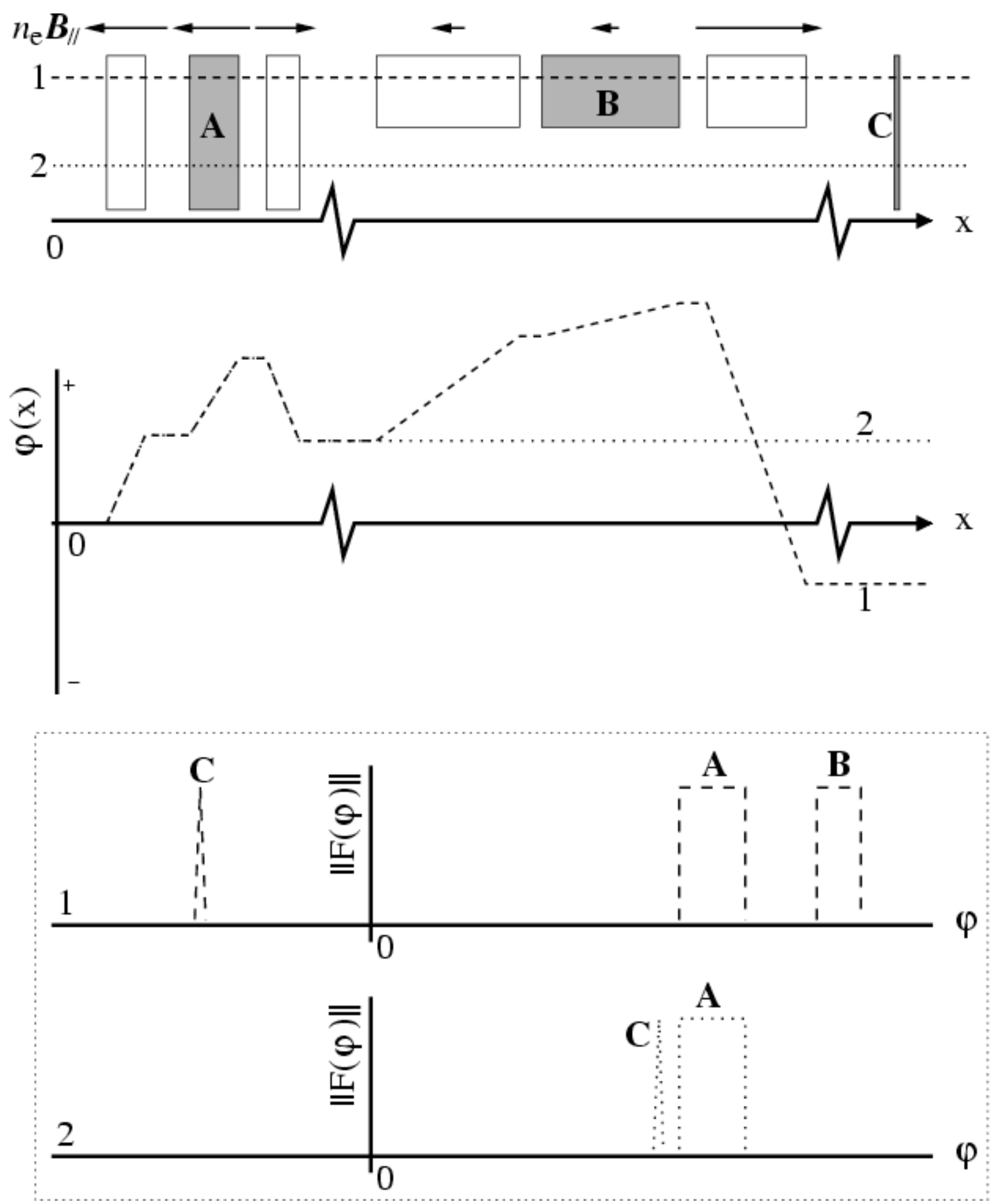
- Magnetized plasma is birefringent - causes Faraday rotation
- Speed of light different for right and left circular polarization
- Because RL phase difference is determined by
  - difference in c for R,L
  - wavelength of radiation
- and RL phase difference determines the polarization angle, we get a frequency-dependent change in linear polarization angle

Faraday rotation traces $B_{||}$



Faraday depth

- Not directly related to physical depth!



Brentjens & de Bruyn (2005)

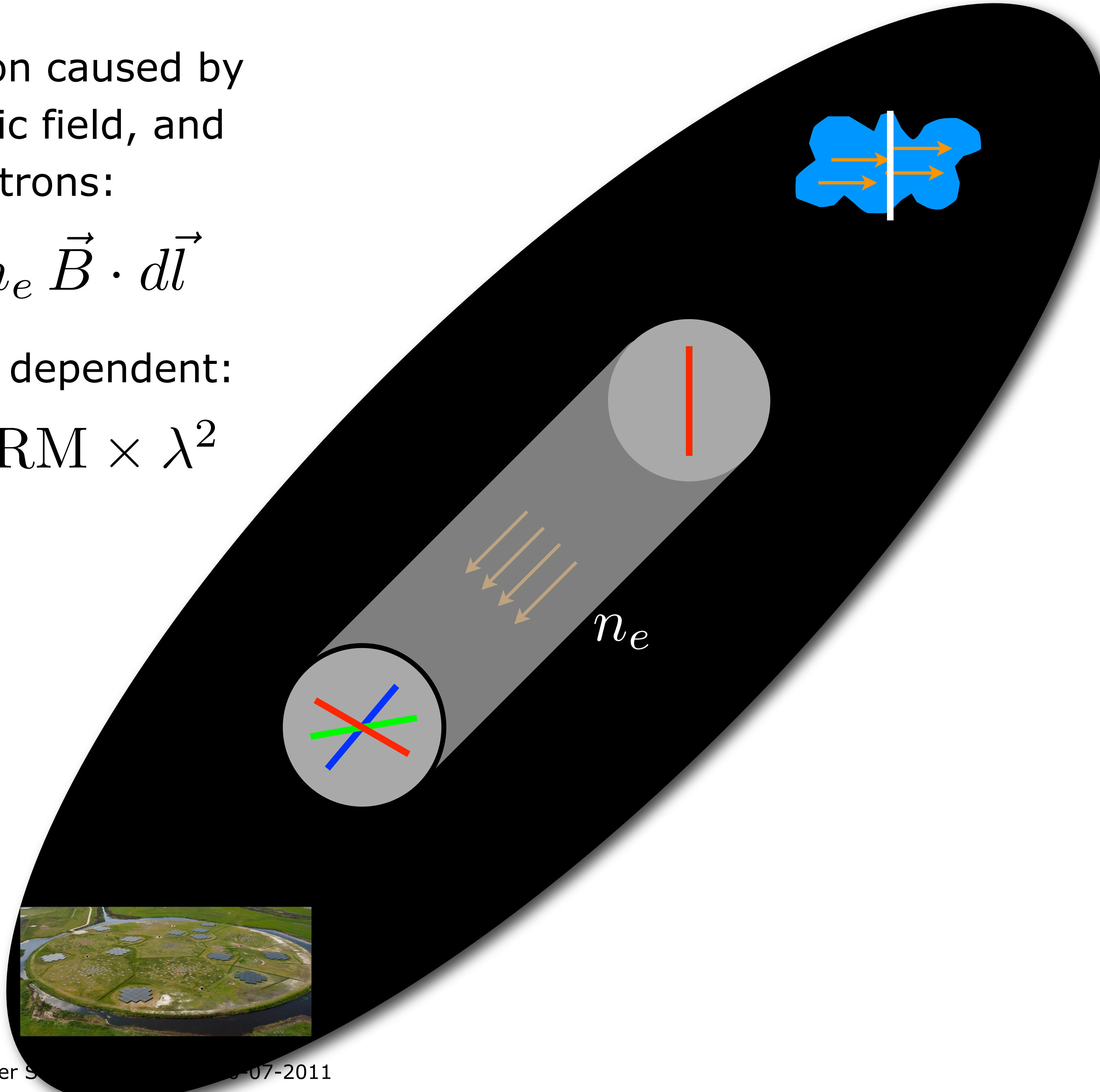
Faraday Rotation

- Faraday rotation caused by LOS magnetic field, and thermal electrons:

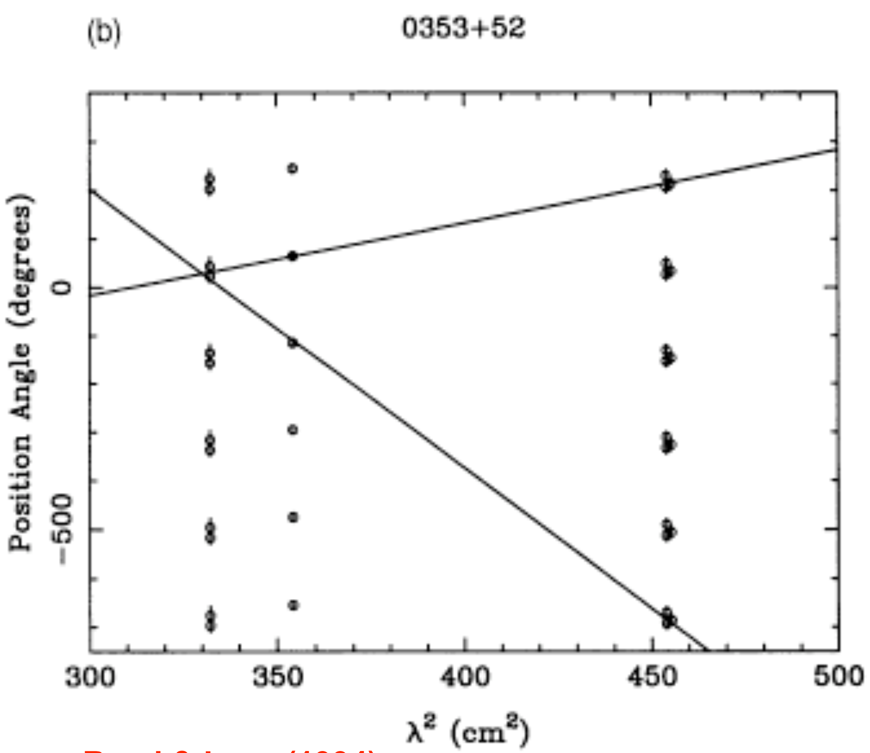
$$\mathrm{RM} \propto \int n_e \, \vec{B} \cdot d\vec{l}$$

- It is frequency dependent:

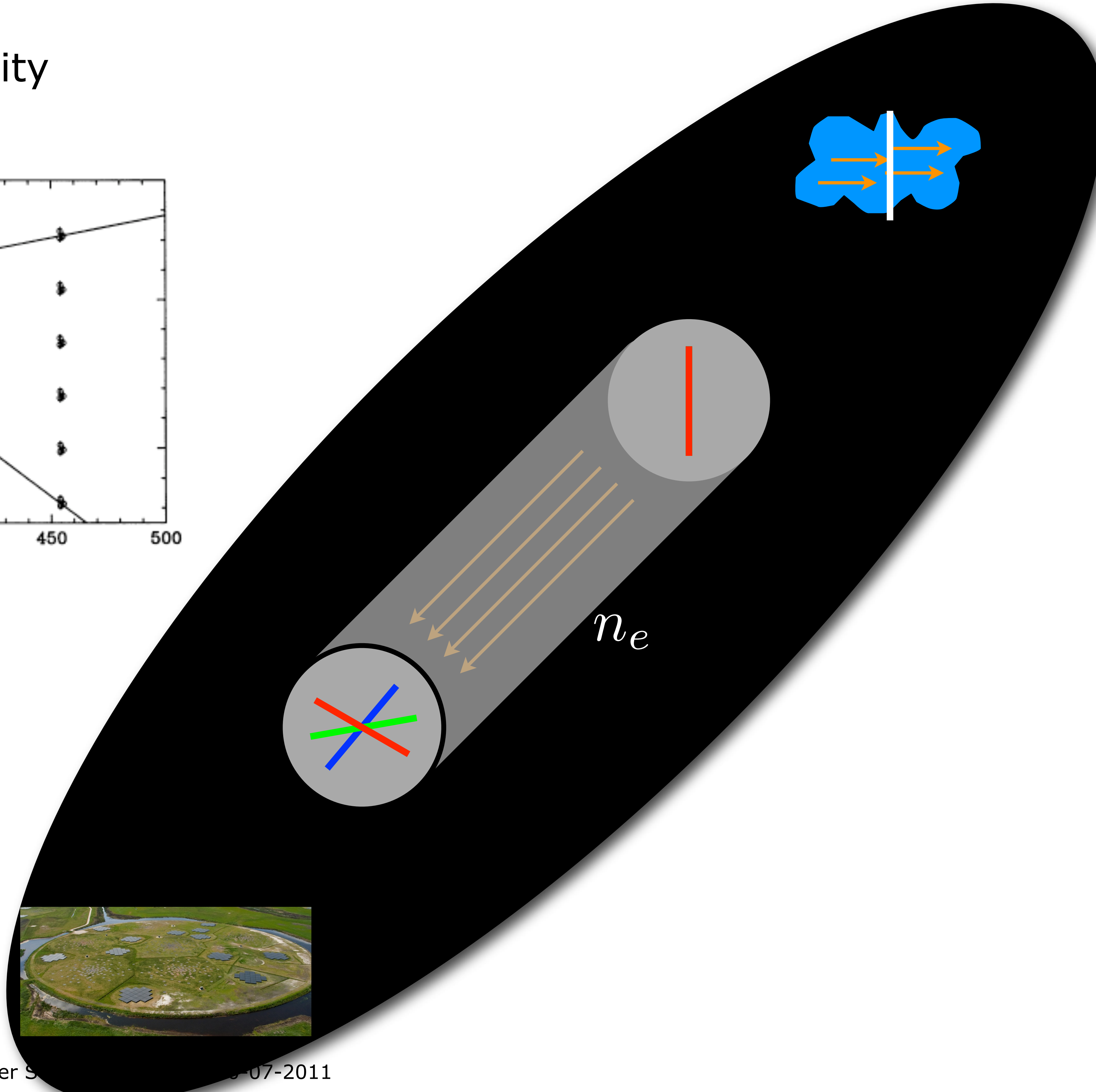
$$\chi = \chi_0 + \mathrm{RM} \times \lambda^2$$



- The nπ ambiguity



Rand & Lyne (1994)



RM synthesis was developed to combat this problem.

RM Synthesis: In theory

- Polarization vector:

$$\vec{P} = p\,I\,e^{2i\chi}$$

but since

$$\chi = \mathrm{RM} \times \lambda^2$$

and redefining

$$\phi \equiv \mathrm{RM}$$

where ϕ is the “Faraday depth”, we can write a more general expression

$$P(\lambda^2) = \int_{-\infty}^{+\infty} F(\phi) e^{2i\phi\lambda^2} d\phi$$

(Burn 1966)

RM Synthesis: In theory

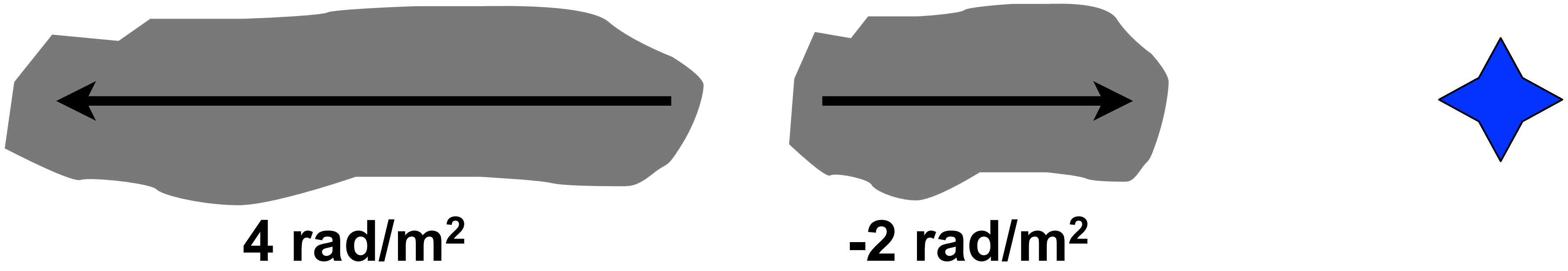
- Generic form of the expression for rotation measure is (Burn 1966):

$$P(\lambda^2) = \int_{-\infty}^{+\infty} F(\phi) e^{2i\phi\lambda^2} d\phi$$

where ϕ (the ***Faraday depth***) has taken the place of RM, and F is the ***Faraday dispersion function***.

- Faraday dispersion function of a Burn slab is a tophat function.

- Note: Faraday depth is ***not*** the same as physical depth! (Hence “2.5D”) Nor is it like optical depth.



RM Synthesis: In theory

- The equation

$$P(\lambda^2) = \int_{-\infty}^{+\infty} F(\phi) e^{2i\phi\lambda^2} d\phi$$

  is like a Fourier transform, and can (in principle) be inverted to determine the physical situation from the observables.

- However there are some complications:
  - We do not measure $\lambda^2 < 0$ *at all*
  - We have finite bandwidth, so we do not measure all values of $\lambda^2 > 0$

- This leads to a sampling (window) function, and to a finite point spread function (called the ***Rotation Measure Spread Function, or RMSF***). Examples are shown in following slides.
- The formal expression

$$P(\lambda^2) = \int_{-\infty}^{+\infty} F(\phi) e^{2i\phi\lambda^2} d\phi$$

becomes

$$\tilde{P}(\lambda^2) = W(\lambda^2) \int_{-\infty}^{+\infty} F(\phi) e^{2i\phi\lambda^2} d\phi$$

This can be inverted (note the addition of λ_0^2):

$$\tilde{F}(\phi) = K \int_{-\infty}^{+\infty} \tilde{P}(\lambda^2) e^{-2i\phi(\lambda^2 - \lambda_0^2)} d\lambda^2 = F(\phi) * R(\phi) \neq F(\phi)$$

RM Synthesis: In practice

- The expression for the (reconstructed) Faraday dispersion function

$$\tilde{F}(\phi) = K \int_{-\infty}^{+\infty} \tilde{P}(\lambda^2) e^{-2i\phi(\lambda^2 - \lambda_0^2)} d\lambda^2$$

  can be written as a sum (if channel width is small),

$$\boxed{\tilde{F}(\phi) = K \sum_{c=1}^{N} \tilde{P}_c e^{-2i\phi(\lambda_c^2 - \lambda_0^2)}}$$

("trial RM" interpretation)

- The RMSF is then

$$\boxed{R(\phi) = K \sum_{c=1}^{N} w_c e^{-2i\phi(\lambda_c^2 - \lambda_0^2)}}$$

$$K = \left(\sum_{c=1}^{N} w_c \right)^{-1}$$

- This operation can be done for a whole field of view (producing an ***RM-cube***).

Key RM Synthesis terms

- Faraday depth: physical quantity, in case of 1 component measured as "RM"
- Faraday dispersion function:
  Intrinsic polarization as function of Faraday depth; see Figure
- RMSF: like "dirty beam" in synthesis imaging
- FWHM of RMSF gives RM precision

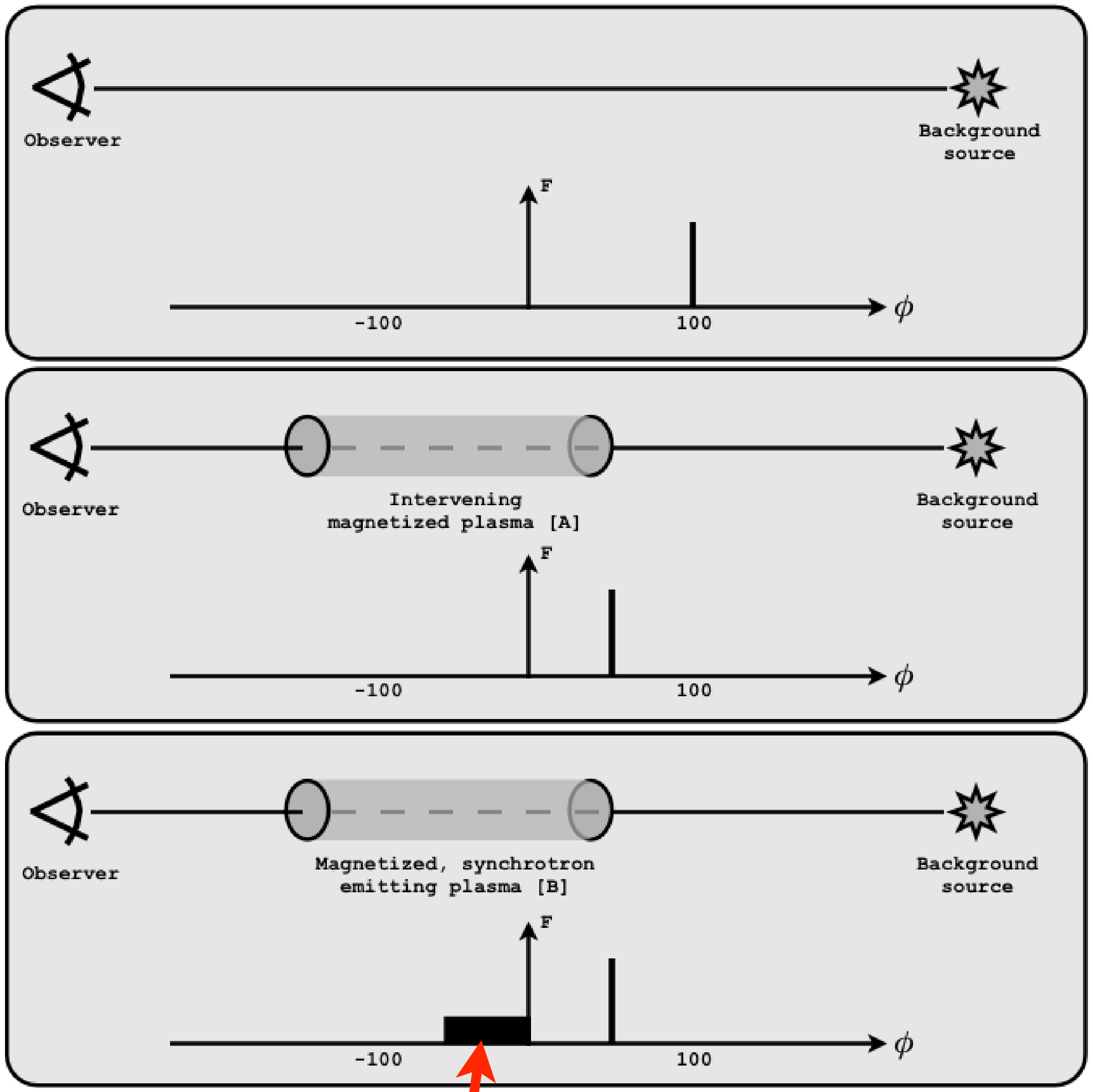
$$\delta\phi \approx \frac{2\sqrt{3}}{\Delta\lambda^2}$$

- Maximum accessible RM

$$\phi_{\max} \approx \frac{\sqrt{3}}{\delta\lambda^2}$$

- Maximum RM scale

$$\mathrm{maxscale} \approx \frac{\pi}{\lambda^2_{\min}}$$



Heald (2009)

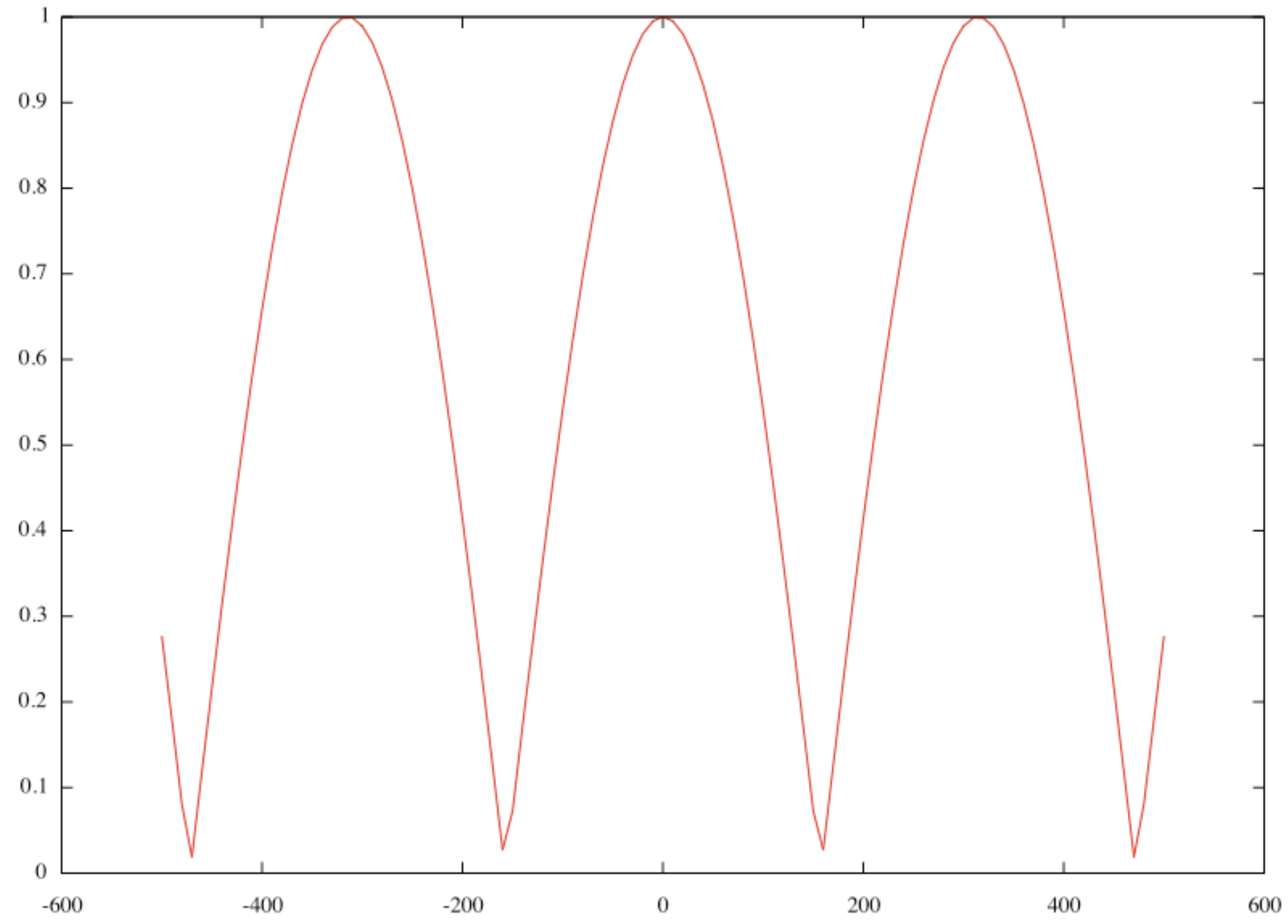
These together allow resolving Faraday thick structures

RM Spread Function (RMSF)

LOFAR ASTRON

- Example, two observing bands: 18cm + 21cm λ^2 = .0355, .0455 m^2
- If observed $d\chi = \pi$
- n π ambiguity becomes $\pi/.01$= 314 rad/m^2
- Fringes in RMSF are exactly spaced by this amount.
- If RM synthesis gives same as traditional method, why bother?
- As with visibilities, do ***not*** need to detect sources in every channel. The transform maximizes S/N.

$$R(\phi) = K \sum_{c=1}^{N} w_c e^{-2i\phi(\lambda_c^2 - \lambda_0^2)}$$

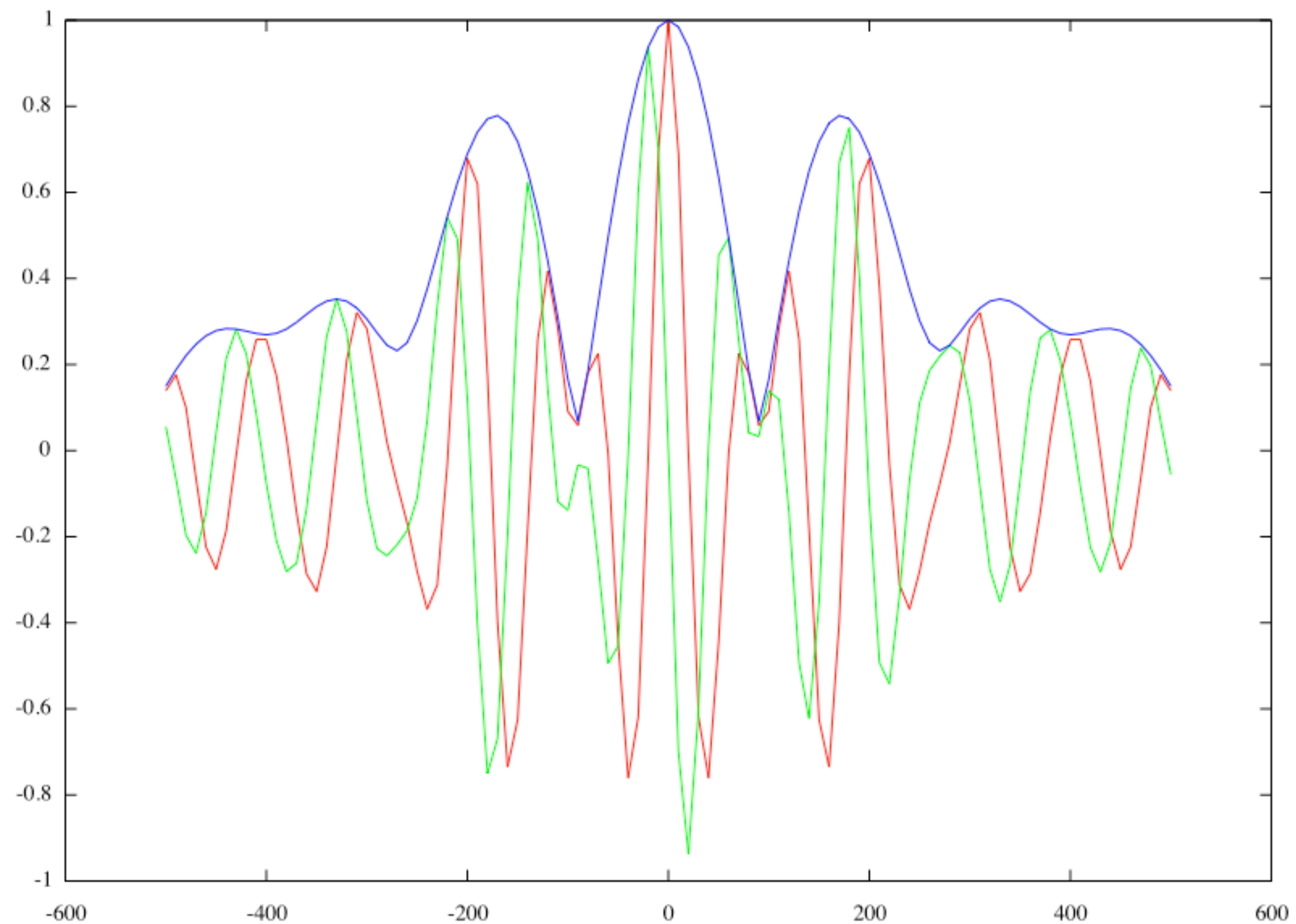


RM Spread Function (RMSF)

- Example, two observing bands: 18cm + 22cm with 512 channels per band

$$R(\phi) = K \sum_{c=1}^{N} w_c e^{-2i\phi(\lambda_c^2 - \lambda_0^2)}$$

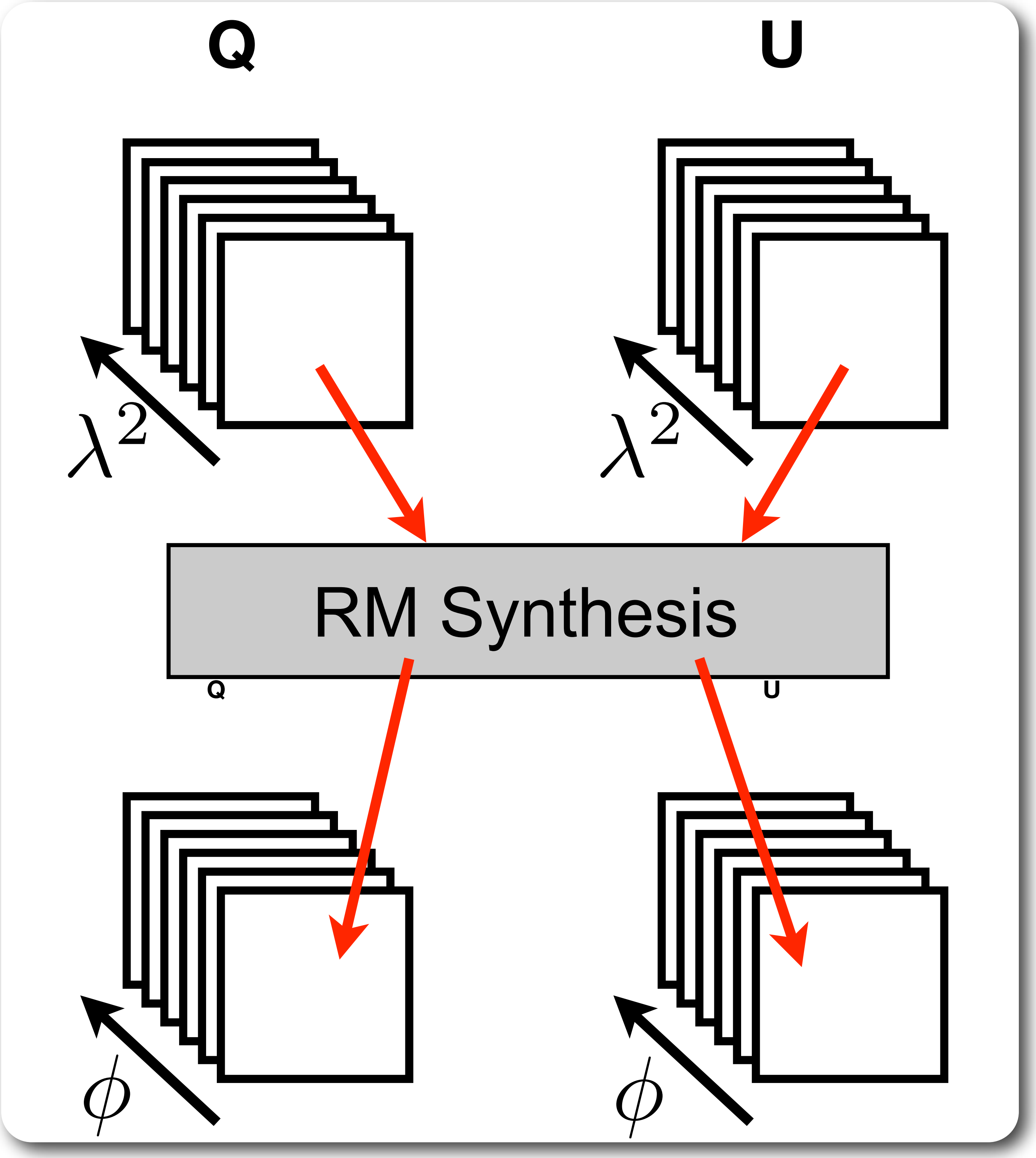
real (Q)
imaginary (U)
absolute value (P)



- One can show that the optimal value of λ_0^2 is the weighted mean of the sampled λ^2 values (Brentjens & de Bruyn 2005)

RM Synthesis: In practice

- RM synthesis works on observed Q,U cubes to produce ***RM-cubes***:



Cubes before and after

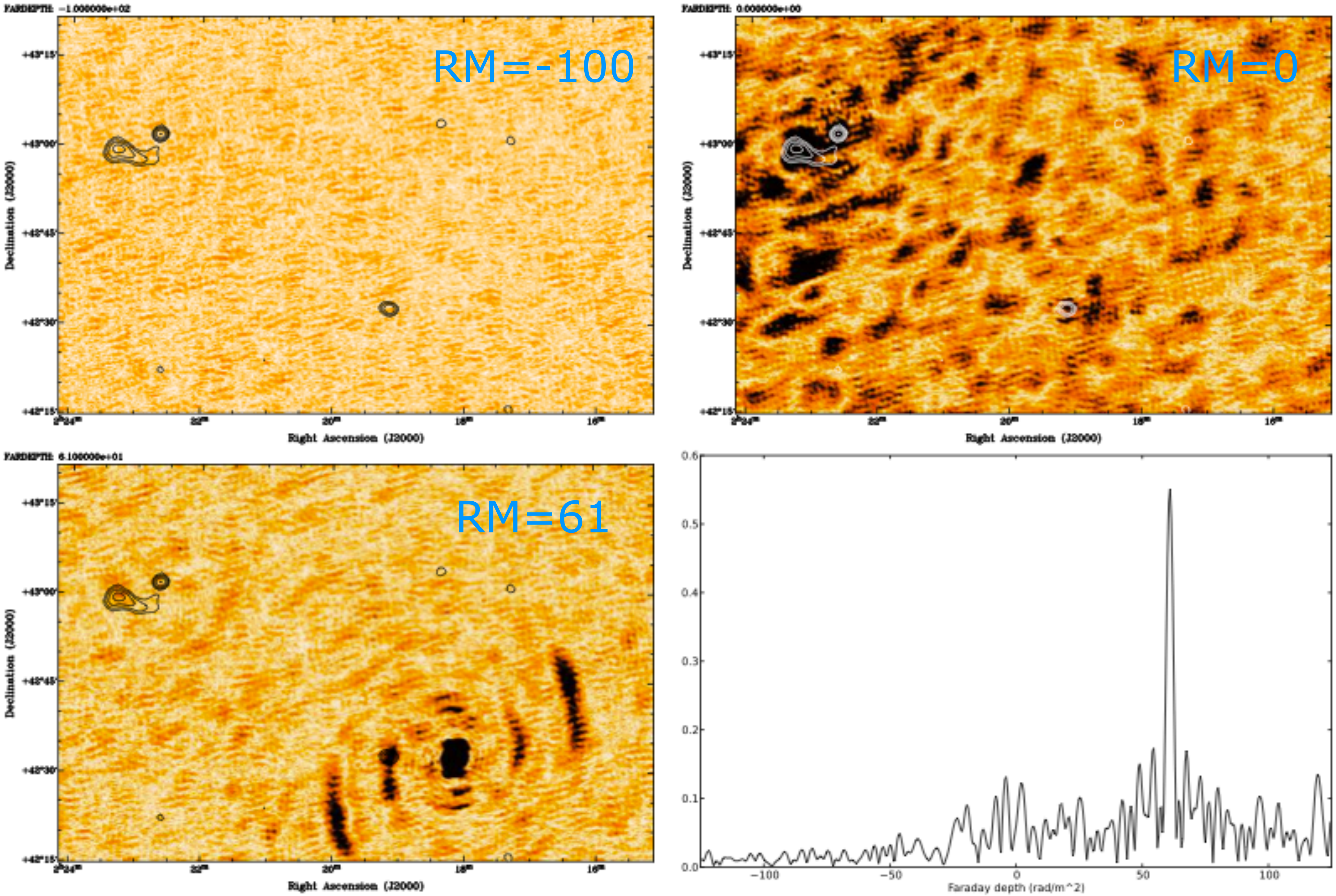
- See movies!

  - qmovie.mpeg and umovie.mpeg -- Q,U frequency frames
    (can you guess which galaxy this is??)
    Note that first ~half of movie is 18cm channels, and these have much higher noise than the second half, which is from 22cm data

  - prmmovie.mpeg -- RM-cube (P) frames
    (now can you guess ...?)

LOFAR PSR0218 RM frames

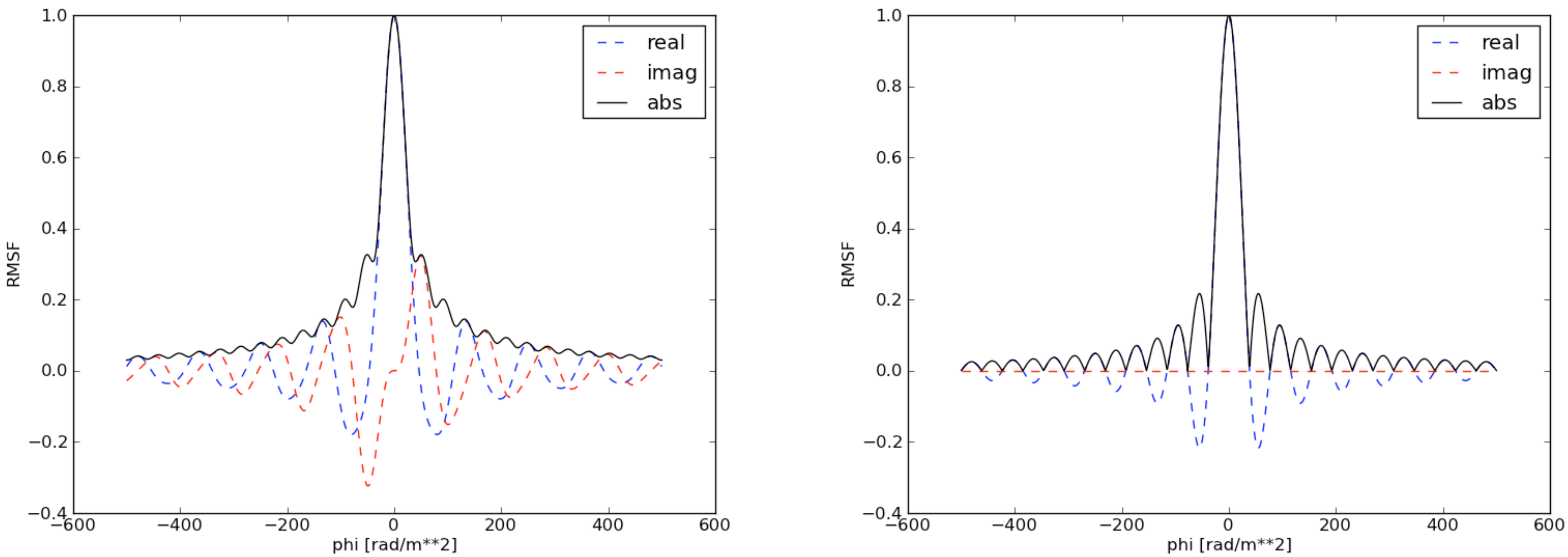
- Illustrates how instrumental polarization appears in RM cubes .....



 Heald et al. (2011); images by Bell, Horneffer

Going from DFT to FFT

- Sidelobe structure well behaved because spacing in λ^2 is constant
- As in imaging case, this is achieved by gridding:
  - grid size = 2/RMRANGE
  - put complex(Q,U) values into bins and do FFT
- Implemented in the RM Synthesis code developed by Mike Bell...



How well can RM be determined?

- From Brentjens & de Bruyn (2005):

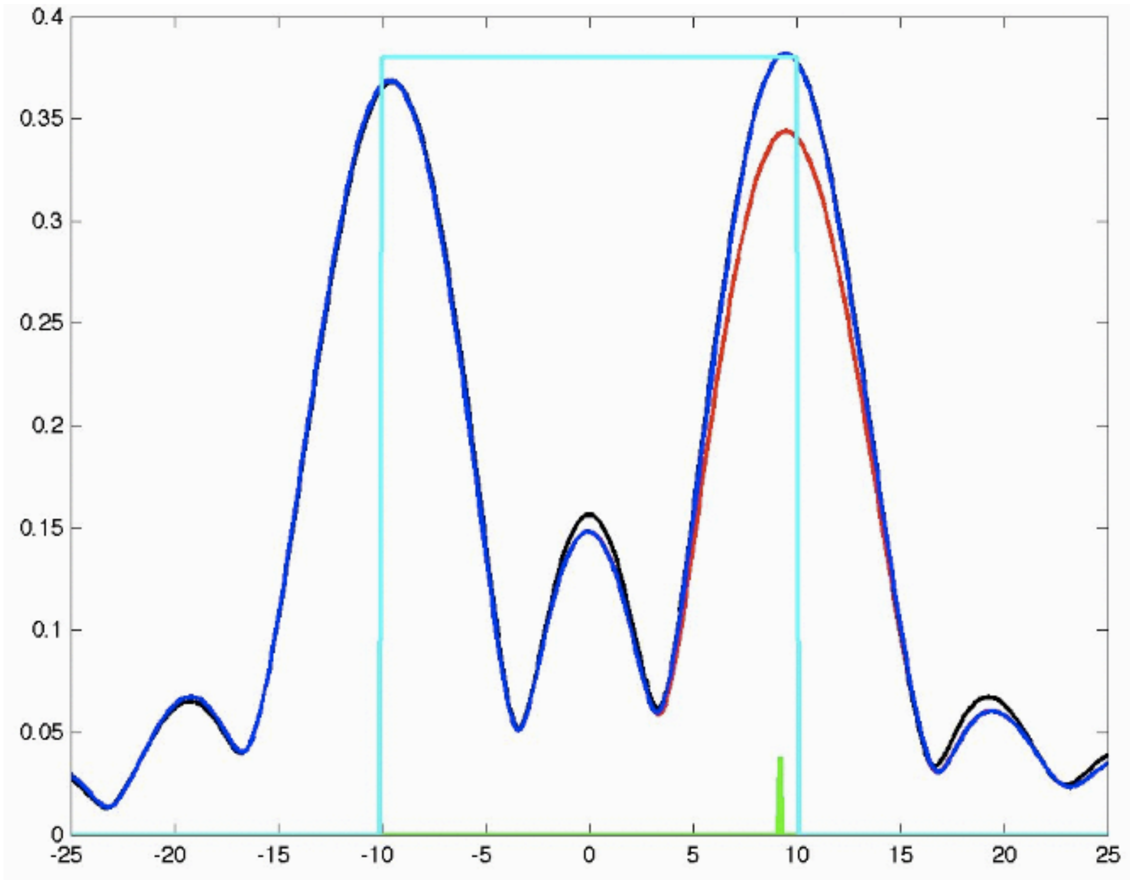
$$\sigma_\phi^2 = \frac{\sigma^2}{4(N-2)||P||^2\sigma_{\lambda^2}^2}$$

- Precision in RM determination is achieved by
  - low noise per channel
  - many channels
  - broad frequency (actually, wavelength squared) coverage

Observational design issues

- The performance of RM synthesis is ***strongly*** dependent on the frequency coverage of the observations.
- Key characteristics:
  - Width of RMSF: sensitivity to weak magnetic fields: $\propto \Delta\lambda^2$ (full coverage)
  - Sensitivity to extended Faraday structures (e.g. Burn slabs): $\propto 1/(\min \lambda^2)$
  - Maximum rotation measure before bandwidth depolarization: $\propto 1/\delta\lambda^2$
  - Sidelobe level: "smoothness" of λ^2 coverage (avoid gaps!!)
  - See movie burnslab.mpeg ....

black = "dirty"
red = residuals
blue = "clean"
green = model



RMSF Deconvolution

- Because of the RMSF, the recovered FDF **must** be deconvolved in situations where multiple components are present

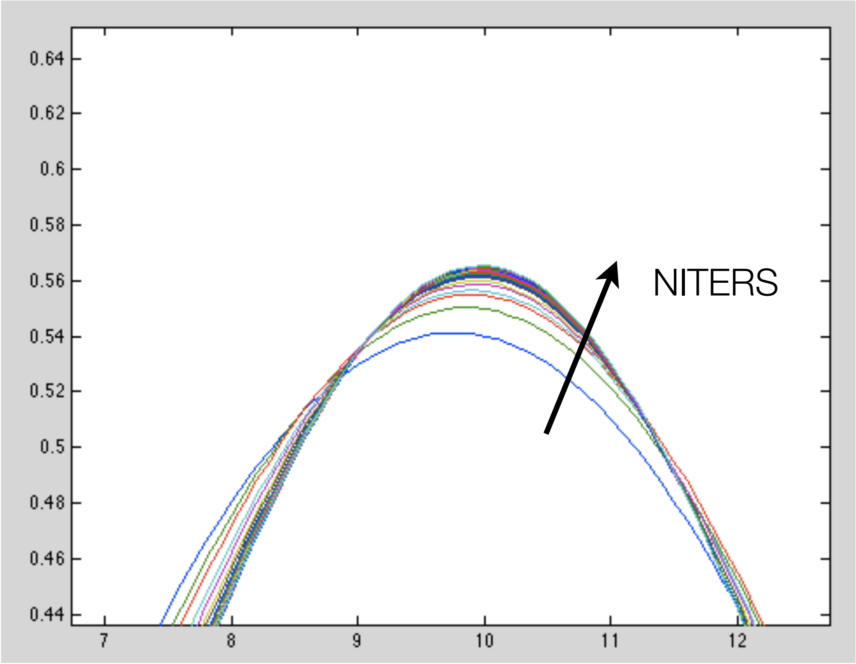
- Technique (just like Högbom CLEAN in the case of interferometric imaging):
  - Find the peak in the reconstructed Faraday dispersion function
  - Subtract a scaled version of the RMSF, centered at that location
  - Store a delta-function component at that location
  - Iterate until noise floor is reached
  - Convolve delta-function model with restoring RMSF, and add residuals

RM CLEAN

- Because of the RMSF, the recovered FDF **must** be deconvolved in situations where multiple components are present

  - See movie rmclean.mpeg .....

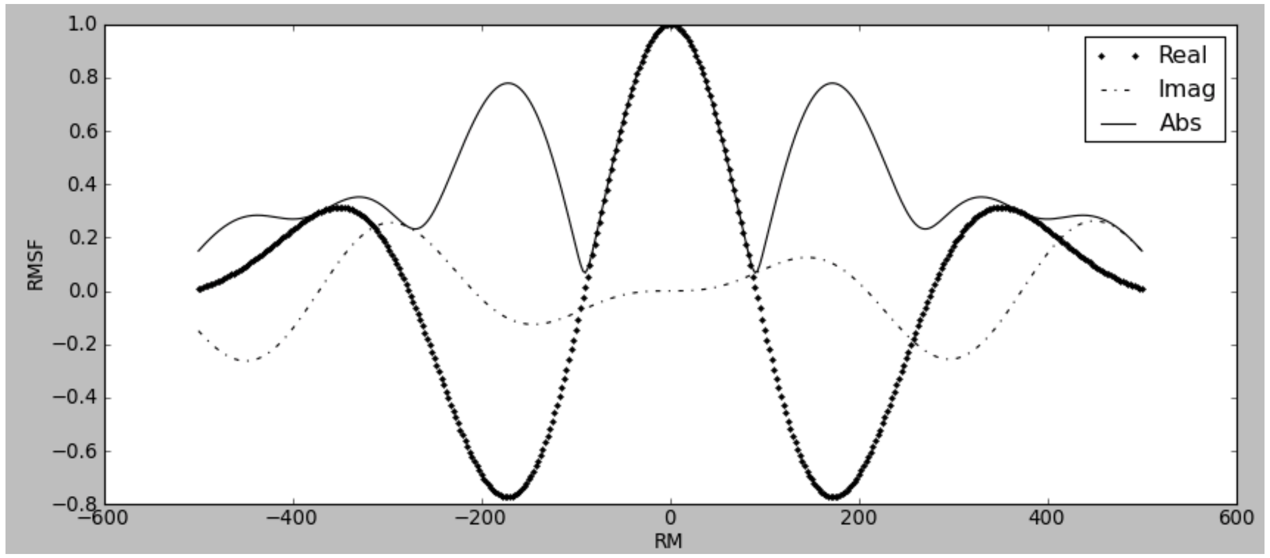
black = “dirty”
red = residuals
blue = “clean”
green = model



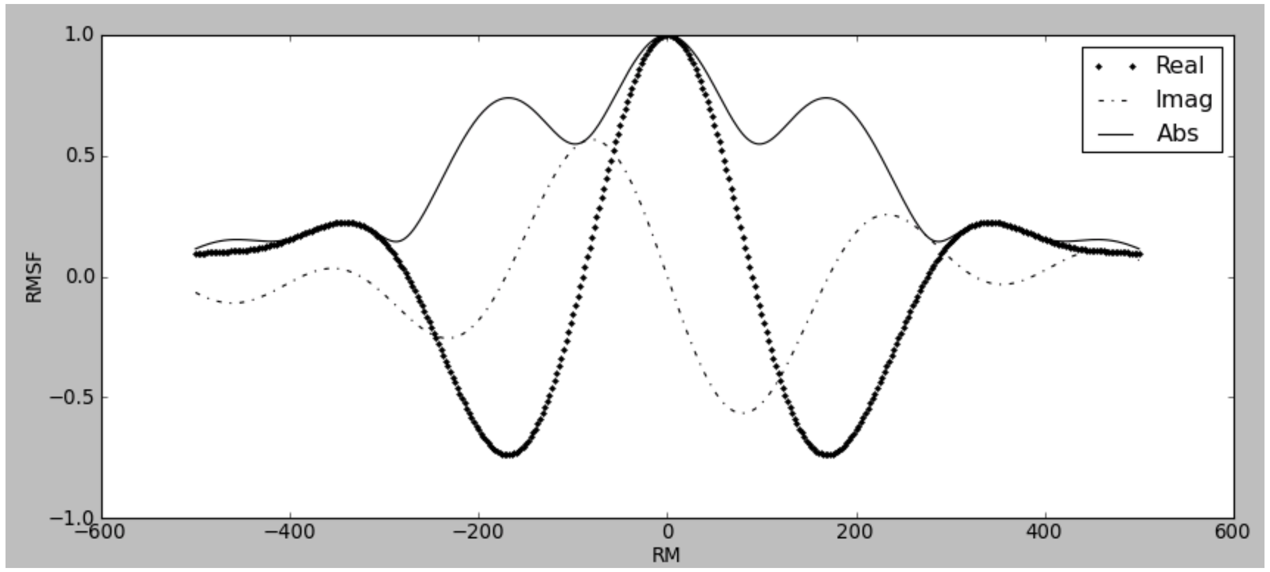
Practical aspects

- All Q,U channels should be imaged independently:
  - consistent resolution ***also in the residuals***
  - primary beam correction per image
  - weighting scheme in FT?

$$R(\phi) = K \sum_{c=1}^{N} w_c e^{-2i\phi(\lambda_c^2 - \lambda_0^2)}$$



RMSF: "uniform" weighting

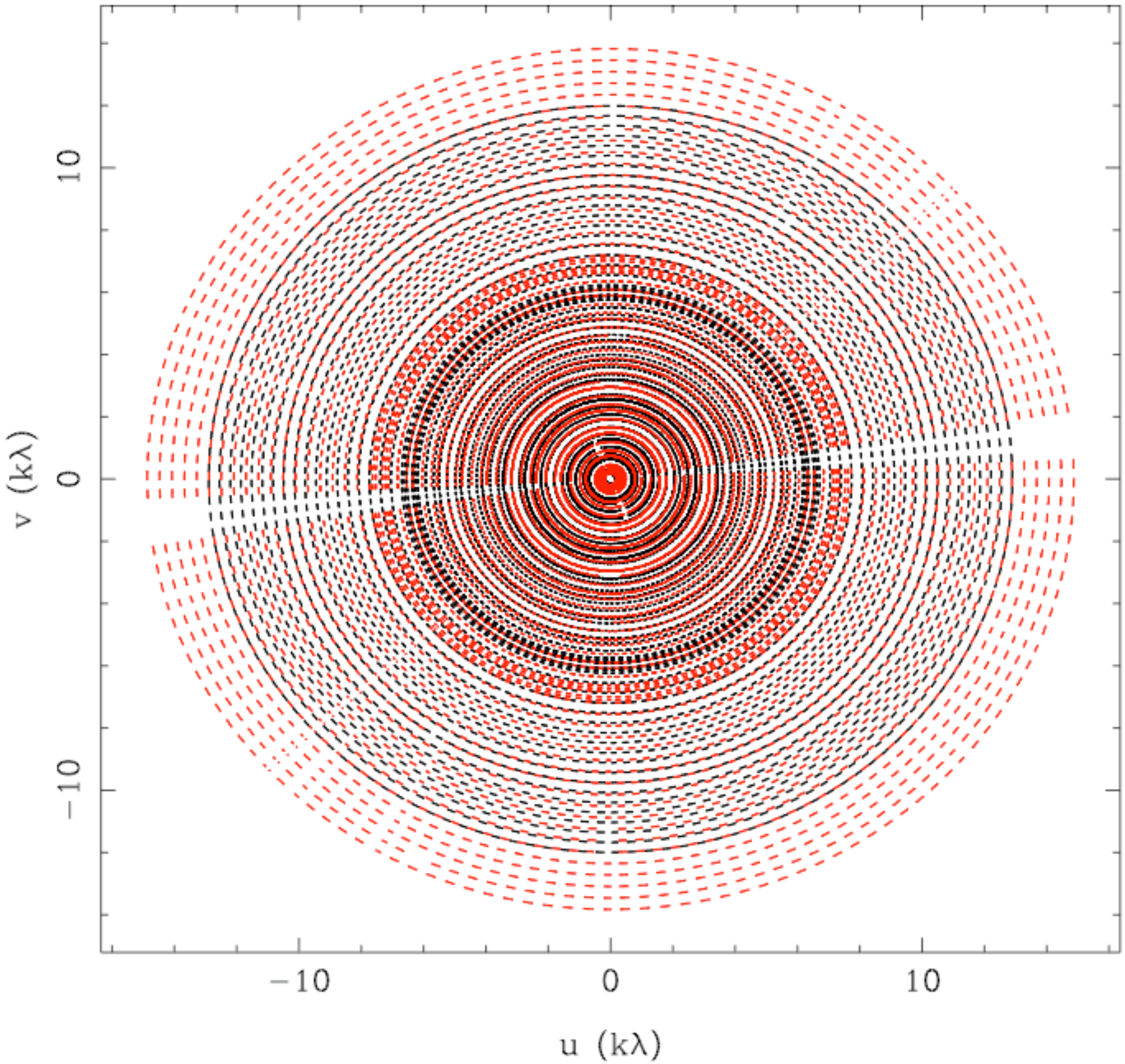


RMSF: "natural" weighting

- Using various weighting schemes may enhance S/N but has not yet been tested (as far as I know...)
  - In particular giving extra sensitivity to extended (in RM) features?

WSRT data

- 2 broad (160 MHz) bands at 18cm and 22cm (high Faraday depth regime)
- Typical noise levels ~10 μJy/beam rms (6h/galaxy/band)

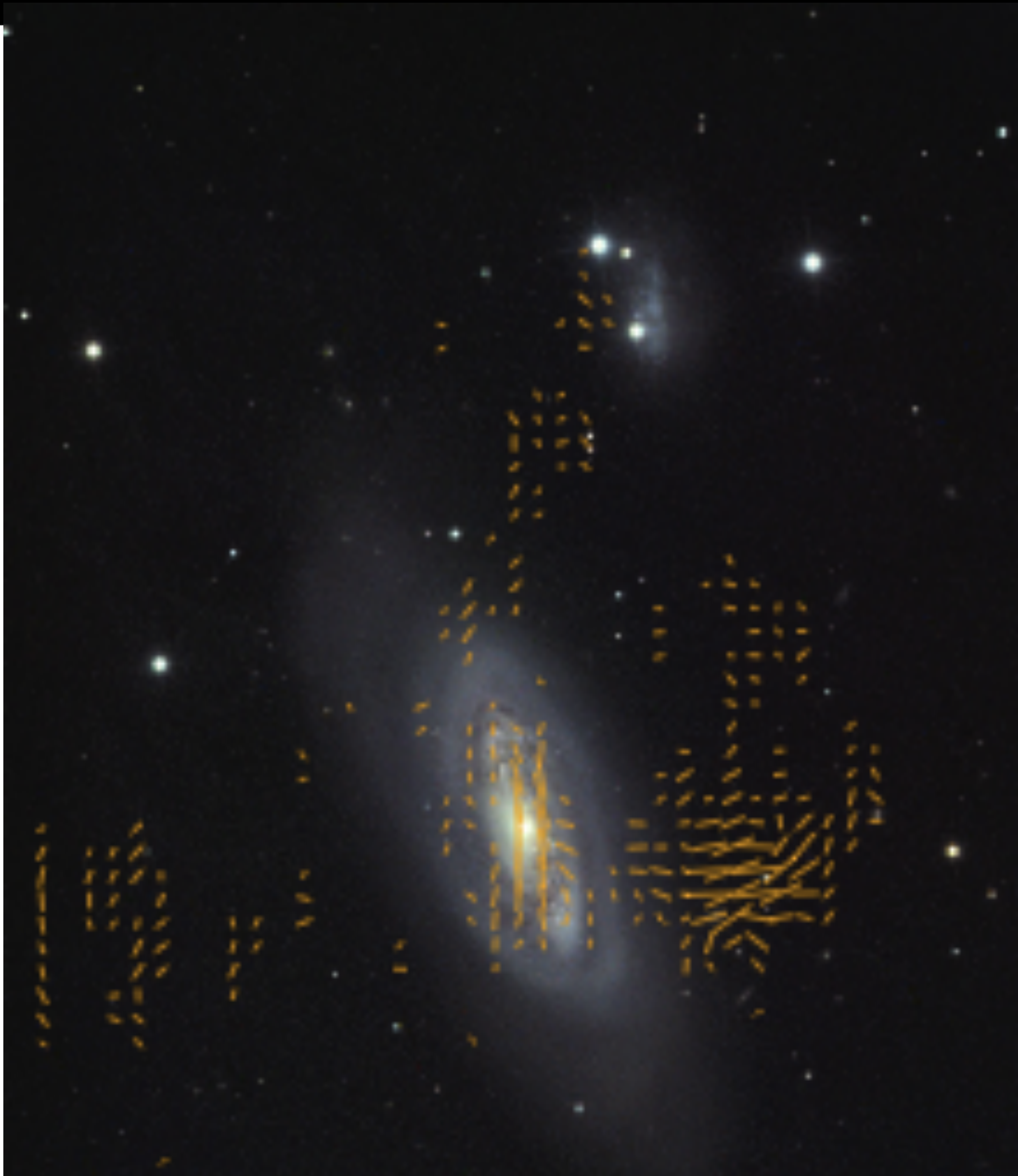


WSRT data analysis

- Data analysed using Rotation Measure Synthesis (RM Synthesis) (see Brentjens & de Bruyn 2005)
  - Fourier transform of observed Stokes (Q,U) over wavelength-squared; provides complex polarization vector as a function of Faraday depth
  - Avoids nπ ambiguity problems (given sufficient frequency sampling)
  - Coherently adds across the full band, optimizing sensitivity regardless of rotation measure value
- RMSF FWHM ~ 144 rad/m^2
- Faraday dispersion functions deconvolved using RM-CLEAN (see Heald et al. 2009; code available online)
- Polarized flux and rotation measure values extracted using moment-map techniques standard in the emission line (e.g. HI) community

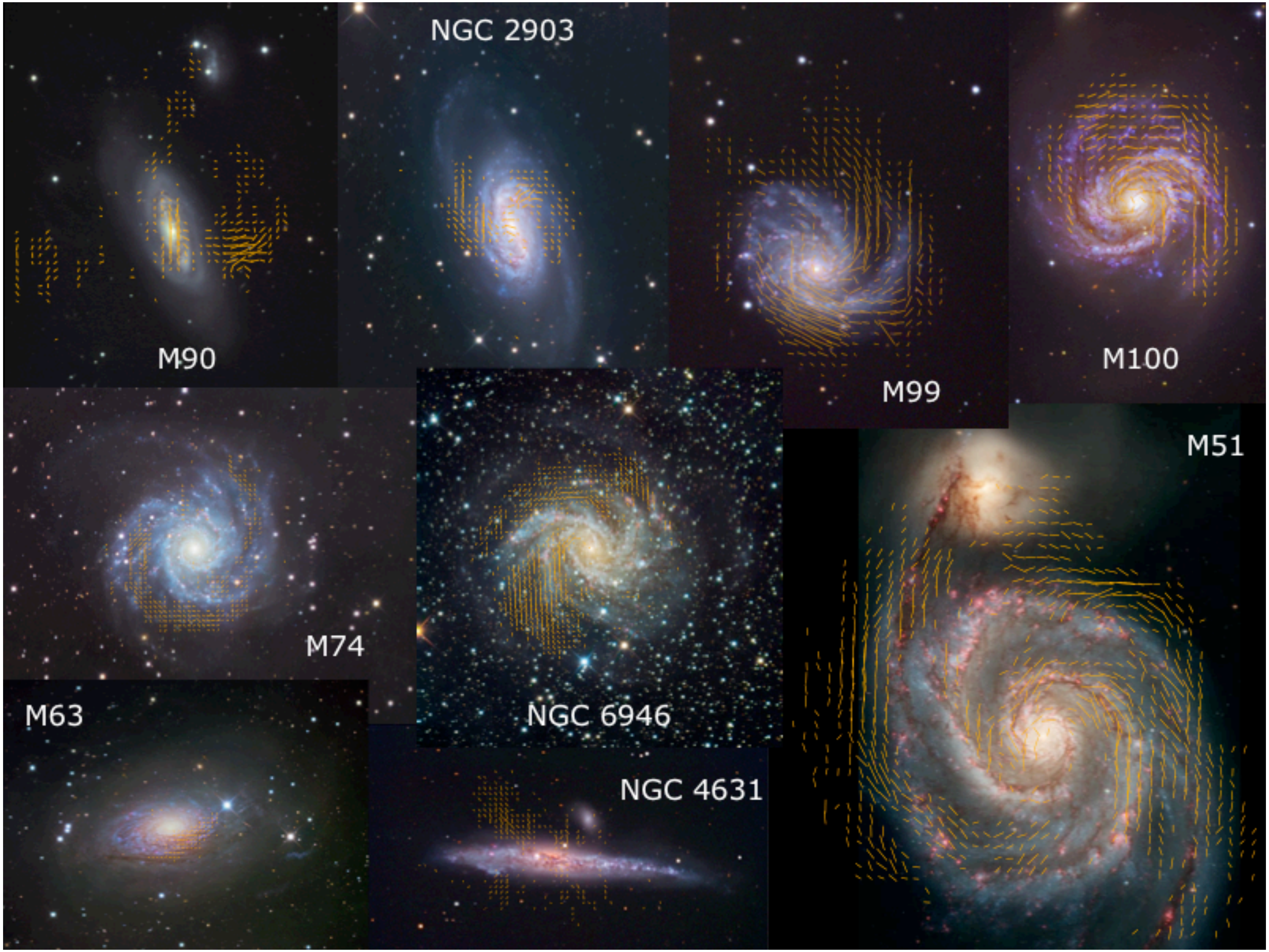
Results

- 28 galaxies studied
  21 detected in polarization
  - 0/4 Magellanic/elliptical
  - 21/24 spirals



Optical image courtesy Robert Gendler

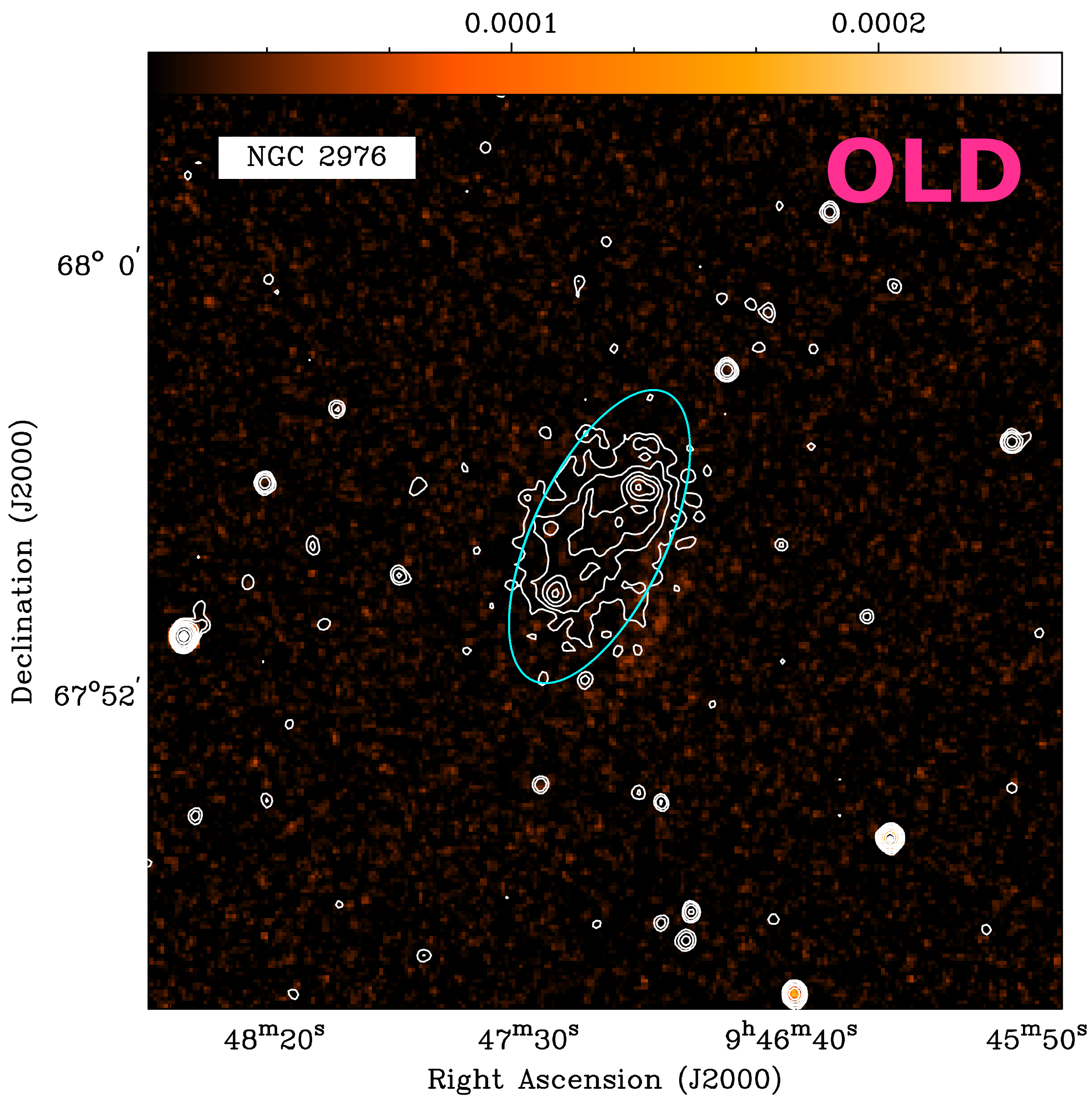
Resulting images

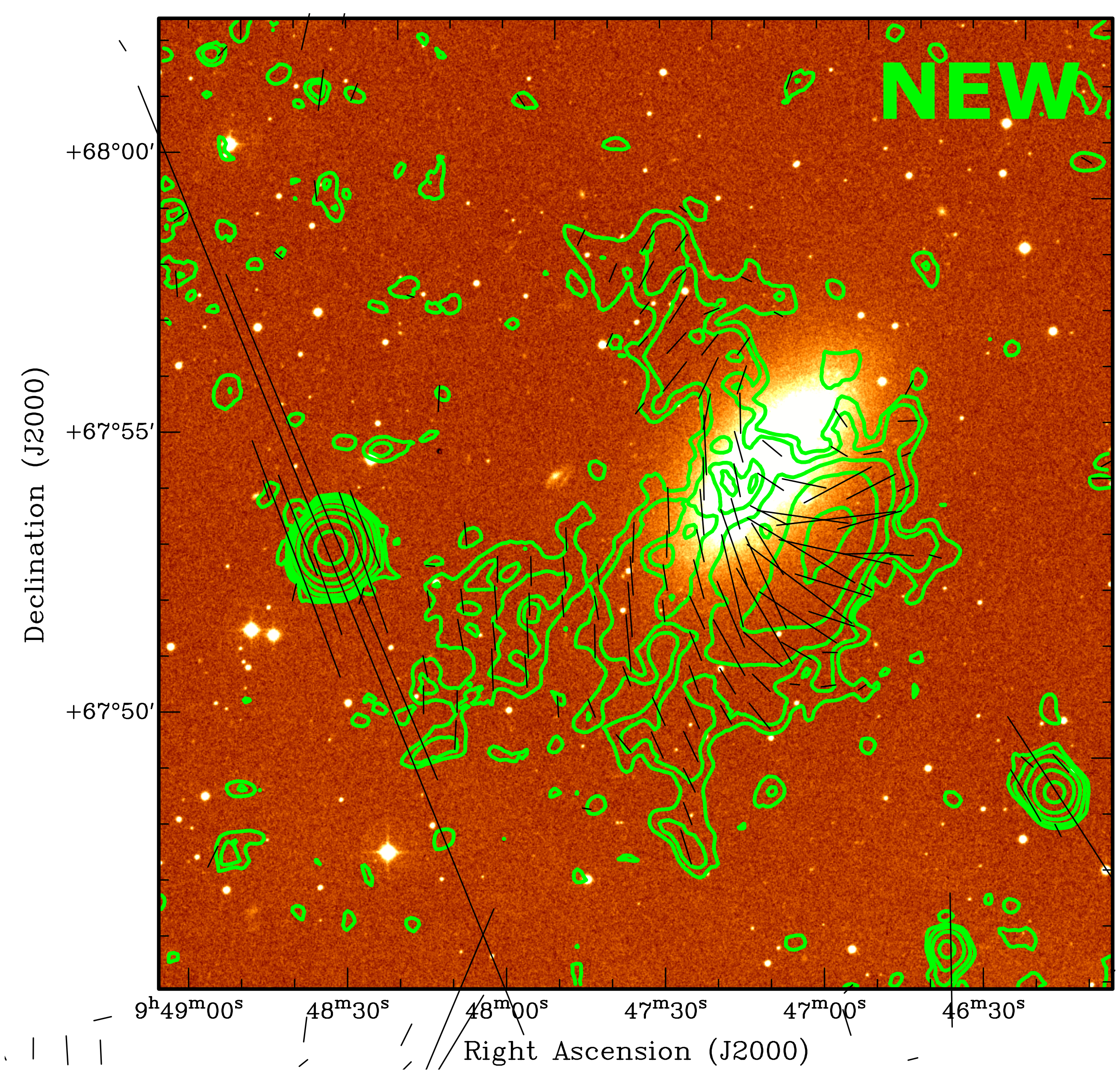


M51 image = Hubble Heritage
All others courtesy Robert Gendler

Reanalysis: imaging at low res

- Previous results used images with uniform weighting; E-W resolution ~15”
- But sensitivity to extended emission enhanced with robust weighting: now used robust=1 and a Gaussian uv-taper (E-W resolution ~45”)
- New emission shows up in some galaxies: eg. N2976; see also David’s talk!





New techniques

- Several new techniques have been proposed recently to overcome deficiencies in the RM Synthesis method as described here:
  - Wiener filtering: “almost” just like natural weighting: Torsten’s talk!
  - Wavelet RM Synthesis (Frick et al. 2010,2011)
  - Compressive sensing (Li et al. 2011)

- Assumptions inherent to various techniques:
  - “Normal” RM Synthesis: $P(\lambda^2<0)=0$
  - Wavelet RM Synthesis: Features in FD are symmetric

    $$\Rightarrow \quad P(-\lambda^2) = \exp\left(-4\mathrm{i}\phi_0\lambda^2\right) P(\lambda^2),$$

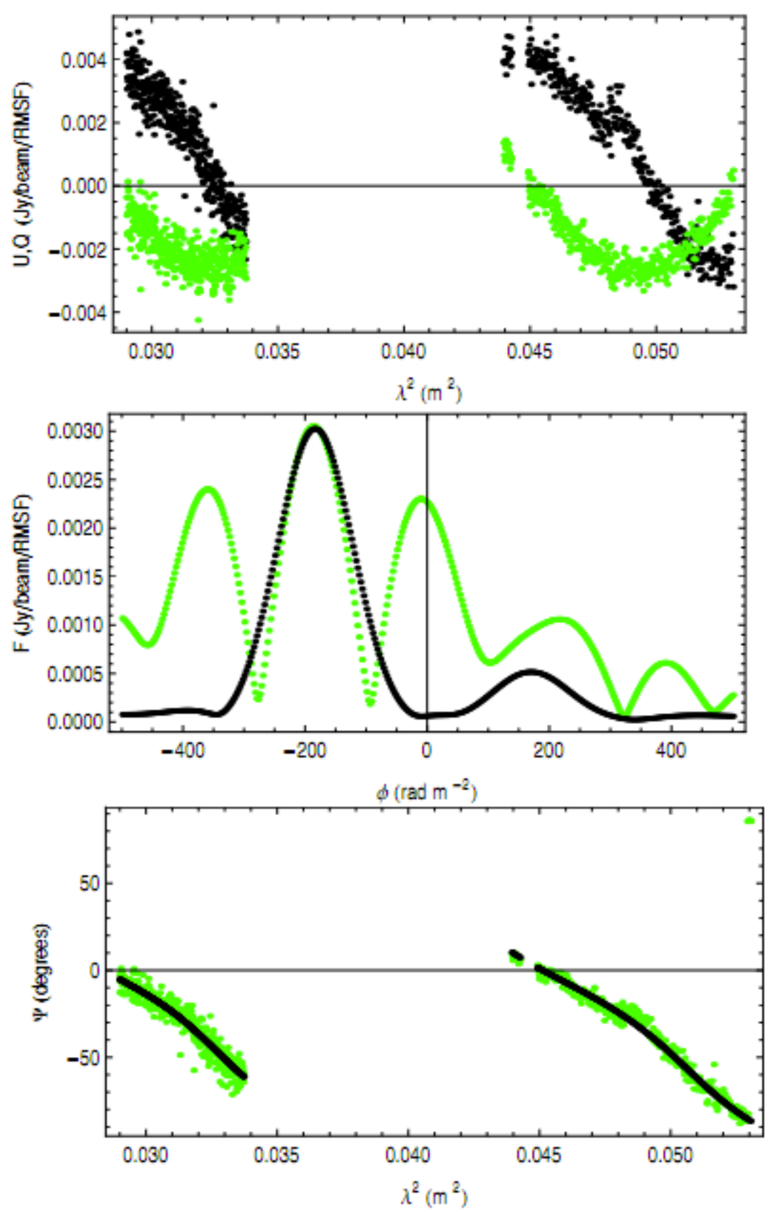
  - Compressive sensing: needs prior information about FD scales ....

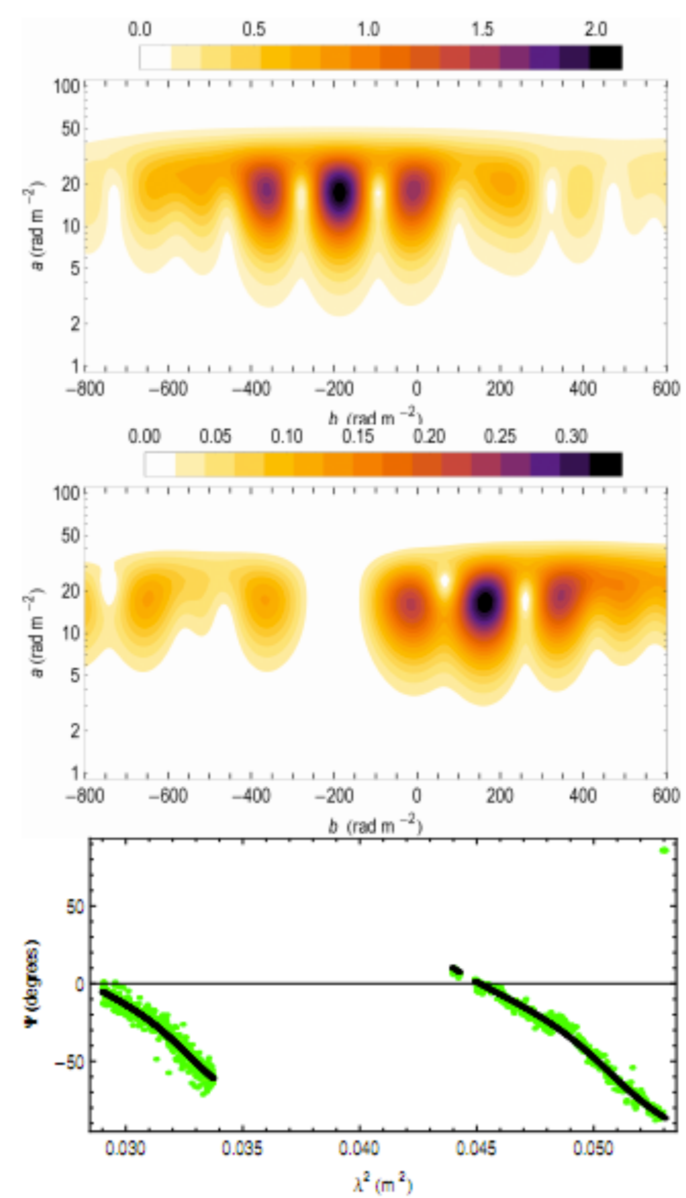
Wavelet RM Synthesis

- Recasts relation between complex polarization (measurements) and the Faraday dispersion function as a wavelet transform (recall Andrew's talk)

$$w_F(a,b) = \frac{1}{|a|}\int_{-\infty}^{\infty} F(\phi)\psi^*\left(\frac{\phi-b}{a}\right)d\phi,$$

$$w_F(a,b) = \frac{1}{\pi}\int_{-\infty}^{\infty} P(\lambda^2)e^{-2ib\lambda^2}\,\hat{\psi}^*\left(-2a\lambda^2\right)d\lambda^2.$$





Frick et al. (2011)

Compressive sensing

- Technique for reconstruction of sparsely sampled signals
- Define matrix Y and model FDF f that predicts the measured signal $\tilde{p}$

$$\mathbf{Y}(j, N/2+k) = e^{2i\phi_k \lambda_j^2}, j = 1, \cdots, m; k = 1 - N/2, \cdots, N/2.$$

$$\mathbf{Y}f = \tilde{p},$$

- then minimize L1 norm of f (i.e. assert that the FDF is ***sparse***):

$$\min\{\|\mathrm{Re}(f)\|_{l_1} + \|\mathrm{Im}(f)\|_{l_1}\} \quad s.t.\ \mathbf{Y}f = \tilde{p},$$

- For extended FD features, a wavelet transform is used (different basis!):

$$\alpha = \mathbf{W}f.$$

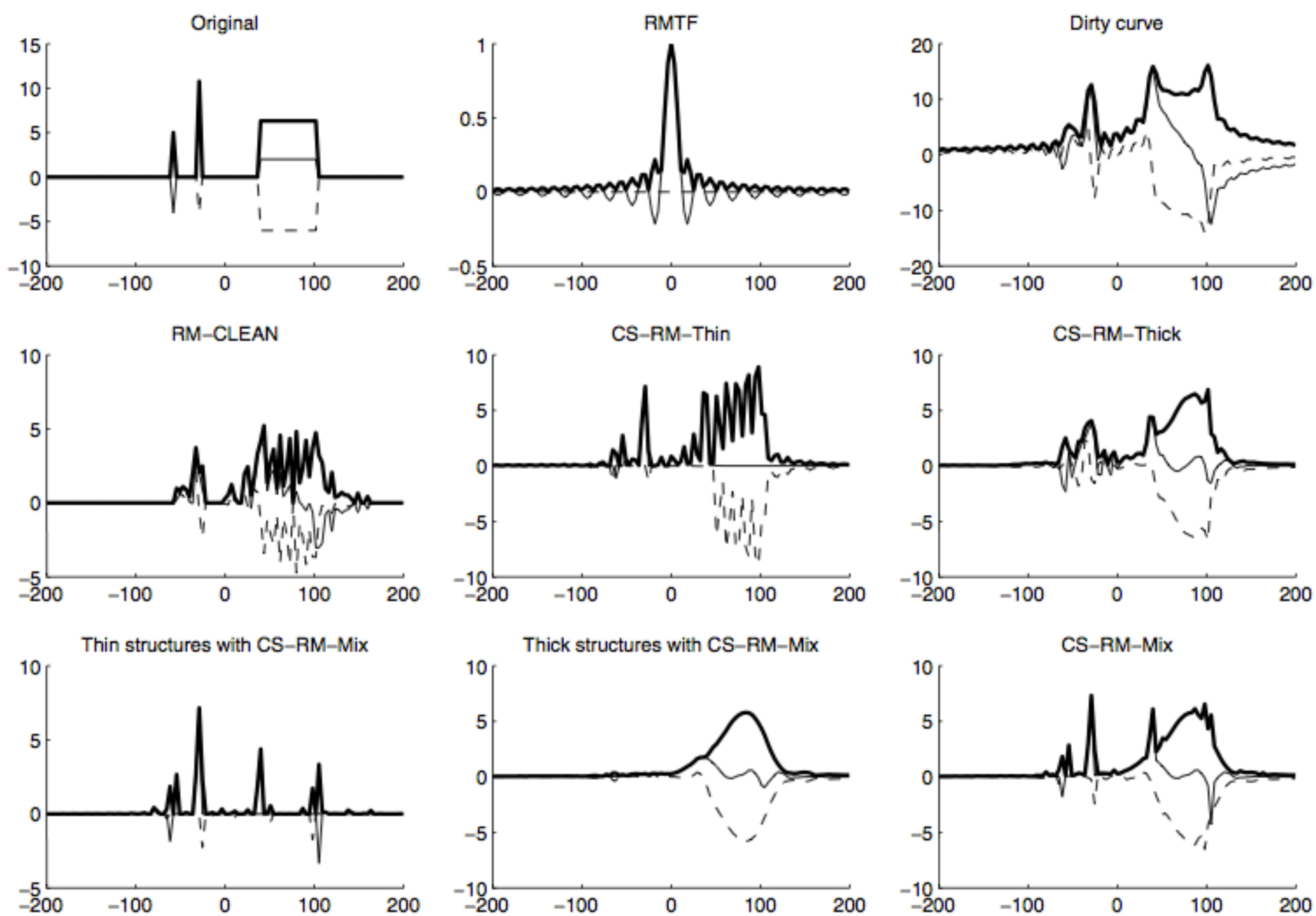
$$\mathbf{Y}\mathbf{W}^{-1}\alpha = \tilde{p},$$

Then:

$$\min\{\|\mathbf{W}\cdot\mathrm{Re}(f)\|_{l_1} + \|\mathbf{W}\cdot\mathrm{Im}(f)\|_{l_1}\} \quad s.t.\ \mathbf{Y}f = \tilde{p}.$$

What is an L1 norm??

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad |\mathbf{x}|_1 = \sum_{r=1}^{n} |x_r|.$$



Li et al. (2011)

Summary

- RM Synthesis provides a way to recover properties of polarized signals from multi-channel wideband data
  - Even when input data quality is not perfect! ... RFI, off-axis sources, instrumental polarization, ...

- Need a good handle on the deficiencies of the particular technique used!
  - Traditional RM Synthesis has trouble with extended FD features, in particular determining polarization angles
  - Wavelet-based RM Synthesis is slow and is based on the assumption of symmetry in the FD features
  - CS RM Synthesis gives better performance than RM-CLEAN (even for point-like FD features!) but in principle requires foreknowledge of the structure of the FDF (note, though, that this is implicit in RM-CLEAN...)