The interpretation of (radio) polarization observations

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Topics

1. Processing the data
    - resolution: best not high
    - filtering: averaging, wavelets
2. Analysis & interpretation
    - look at three approaches:
      a. phenomenological (pick an object)
      b. comparing different models
      c. constraining a parameterised model

The problem/challenge

The complex polarization ...

beam | synchrotron emissivity | angle on emission | Faraday rotaton

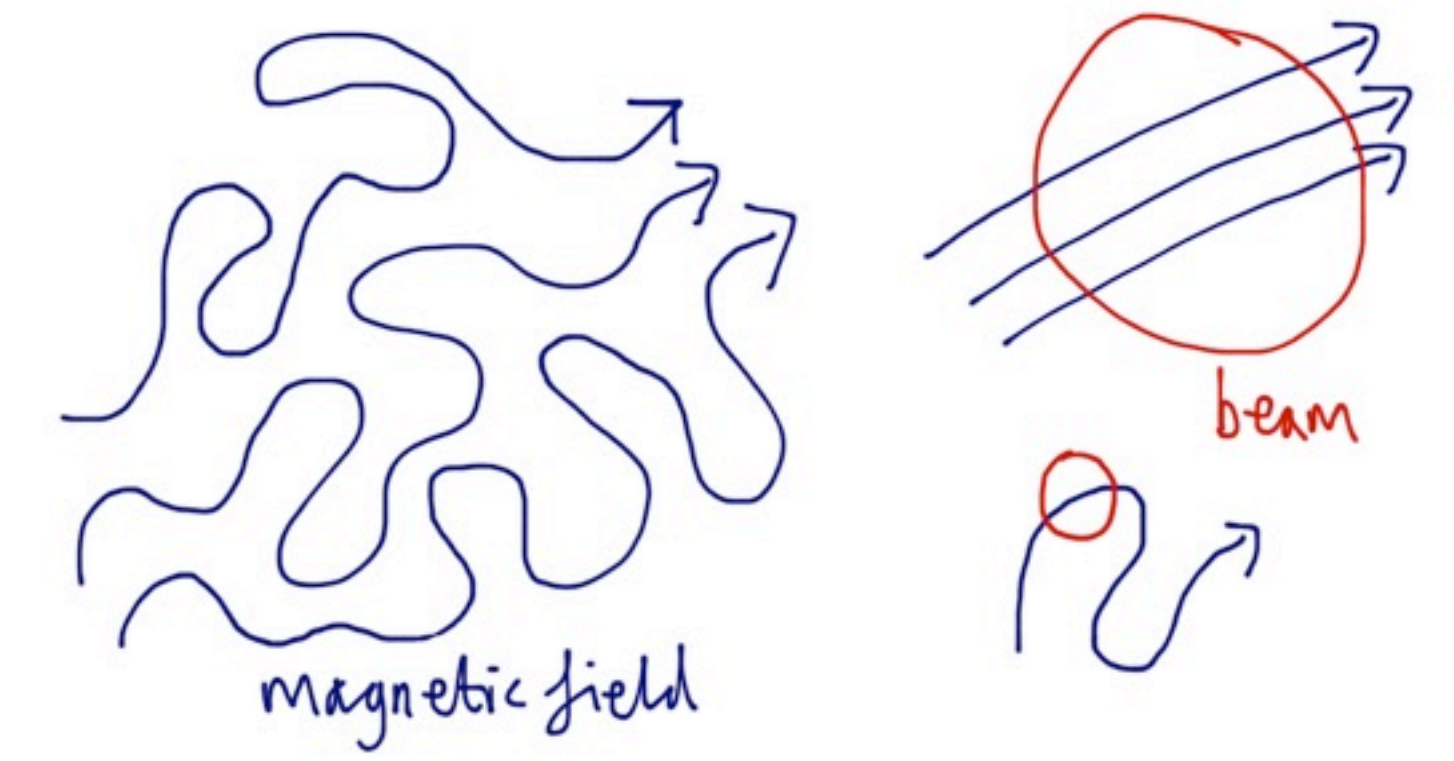
$$\mathcal{P} = p_0 \frac{\int_V D(x,y)\ \epsilon[B_\perp(\mathbf{x}), n_{\mathrm{cr}}(\mathbf{x})]\ \exp\{2\mathrm{i}\,[\psi_0(B_\perp(\mathbf{x})) + 0.81\lambda^2 \int_z n_e(\mathbf{x}) B_\parallel(\mathbf{x})\mathrm{d}z]\}\,\mathrm{d}V}{\int_V D(x,y)\ \epsilon(B_\perp(\mathbf{x}), n_{\mathrm{cr}}(\mathbf{x}))\,\mathrm{d}V}$$

... is well named.

Aim: retrieve (some useful properties of) $\boldsymbol{B}$ from observations of $\mathcal{P}$.

Resolution 1

Require the appropriate resolution to study the scales of B that are of interest.

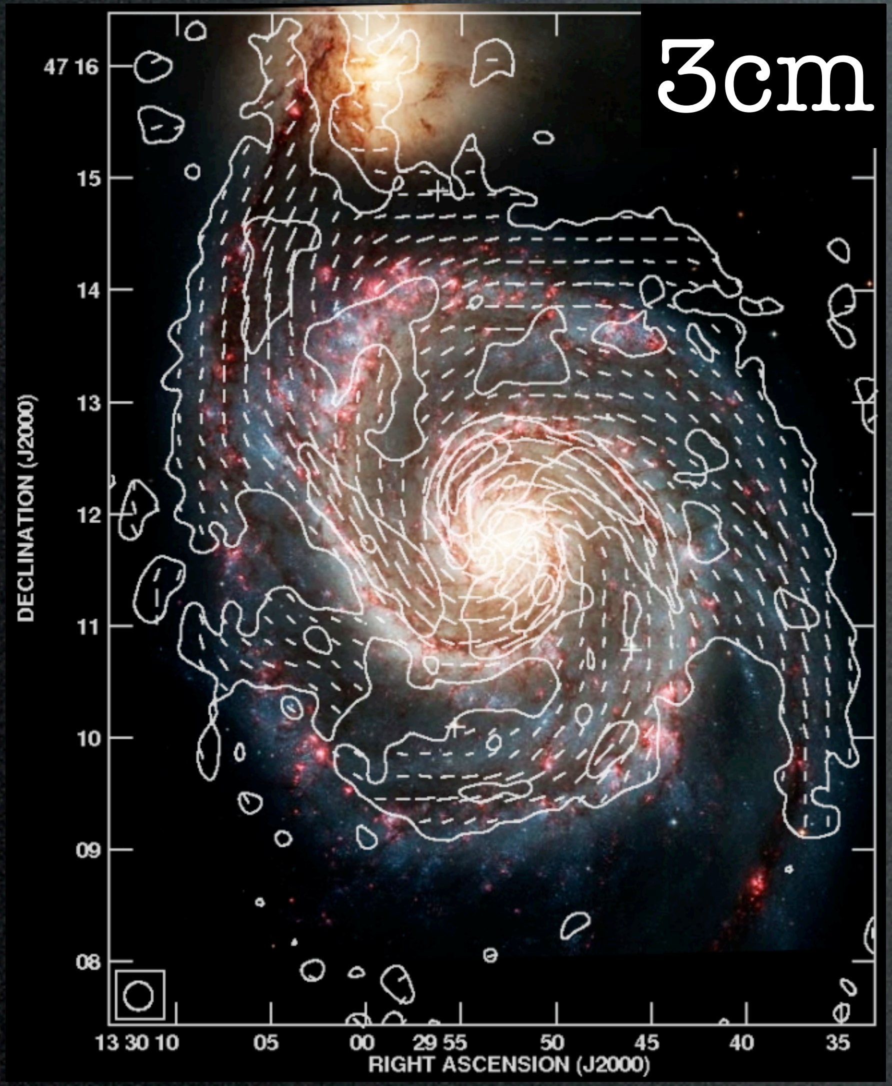


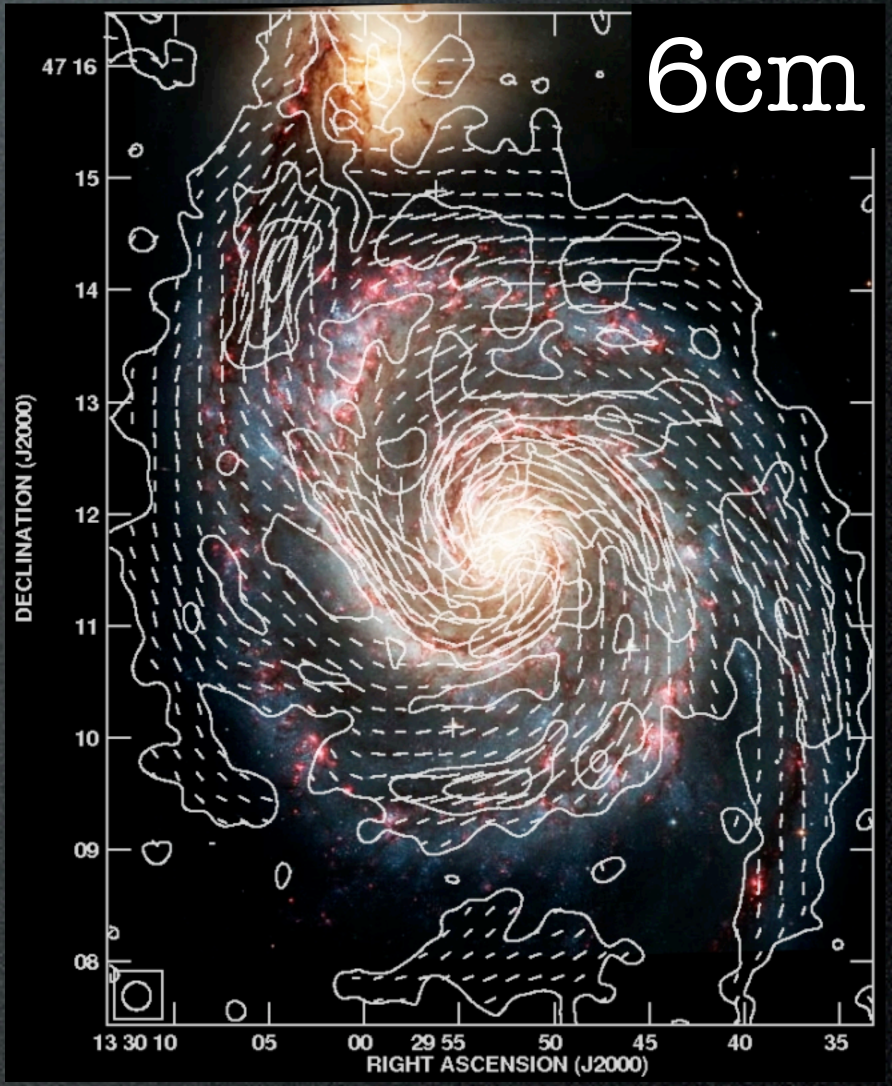
This is not necessarily the highest resolution available.

Resolution 2

Example: Faraday rotation in M51 (3cm & 6cm)

Observations: combined VLA & Effelsberg, resolution 4” – 15”; painstaking reduction.



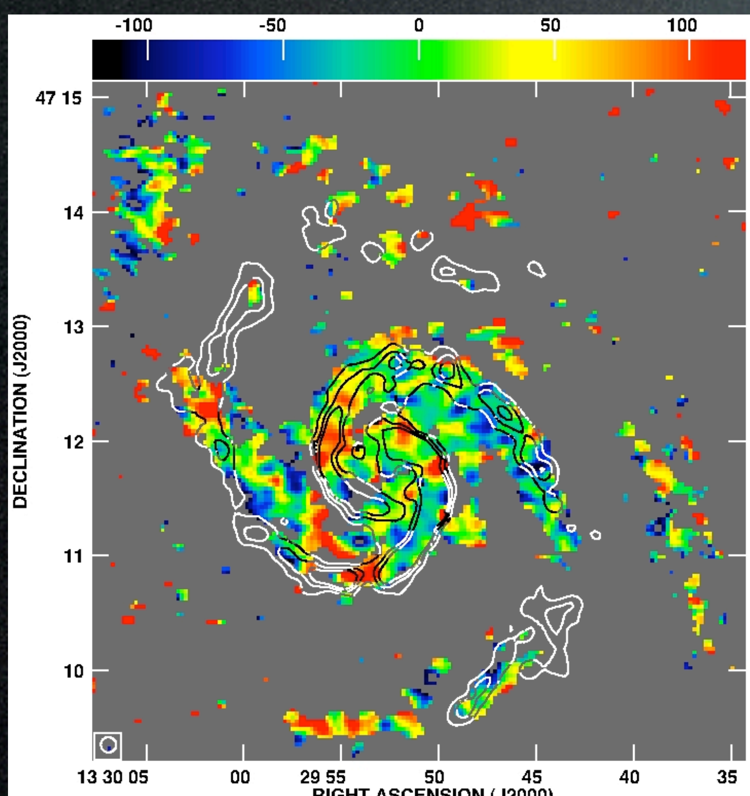


Resolution 3

Example: Faraday rotation in M51 (3cm & 6cm)

8” = 300pc

Fletcher et al. 2011



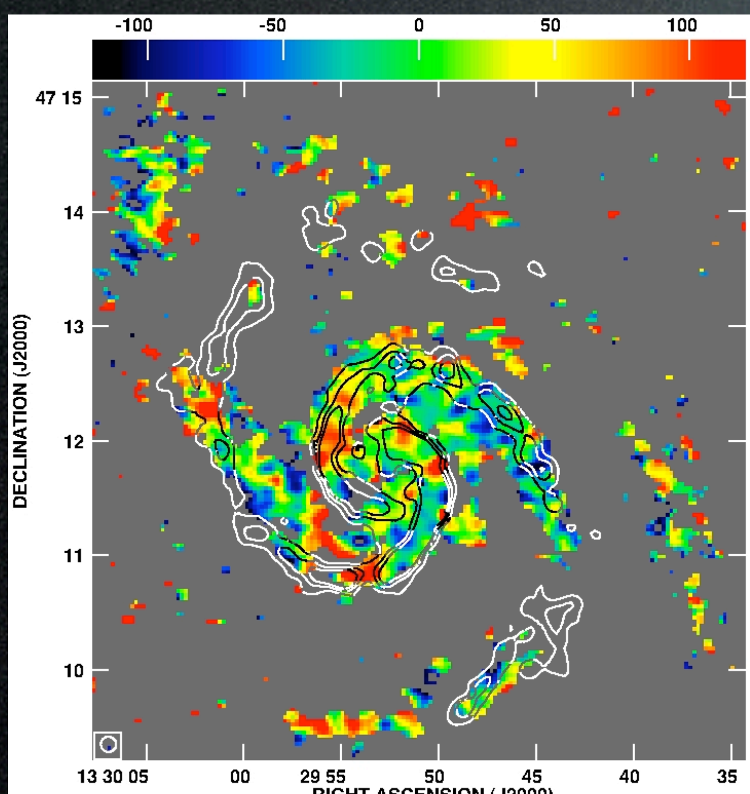
No $\boldsymbol{B}_{\rm reg}$

Resolution 3

Example: Faraday rotation in M51 (3cm & 6cm)

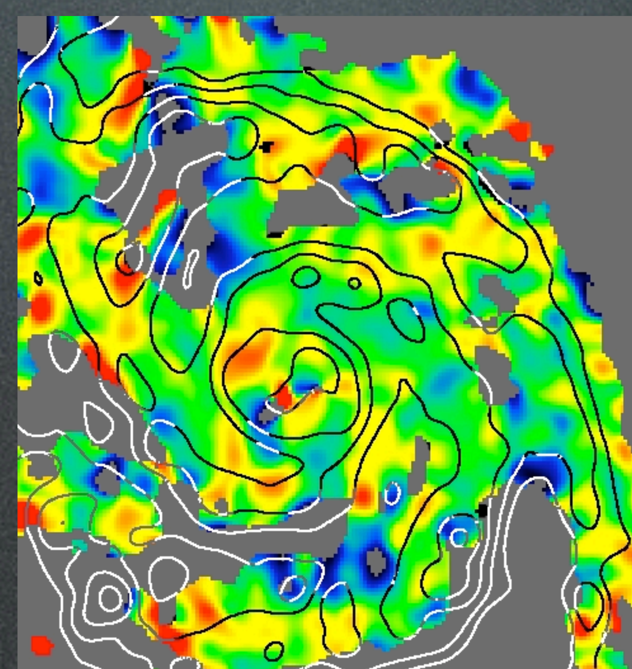
Fletcher et al. 2011

8” = 300pc



No B_{reg}

15” = 600pc



No B_{reg}

Resolution 3

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Fletcher et al. 2011

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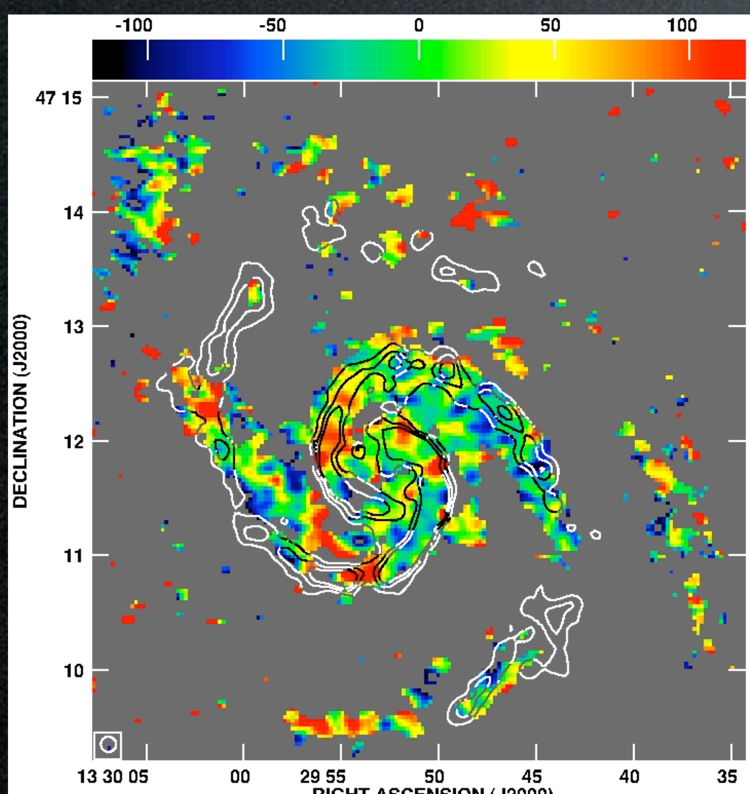
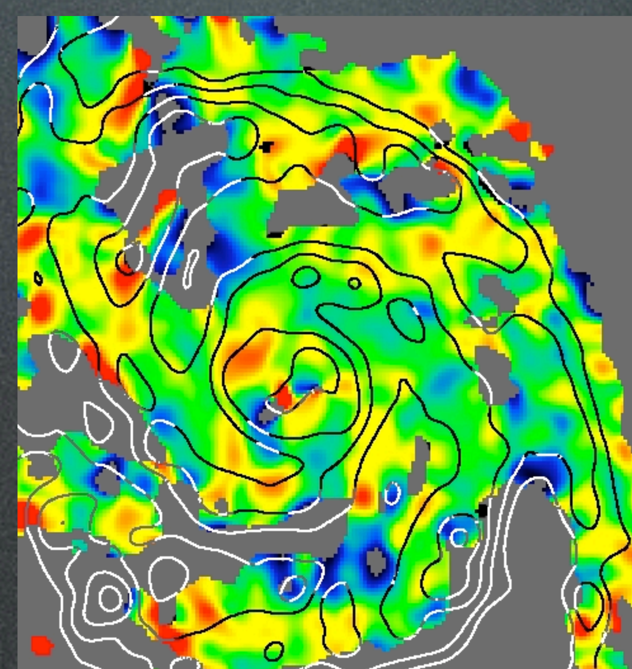


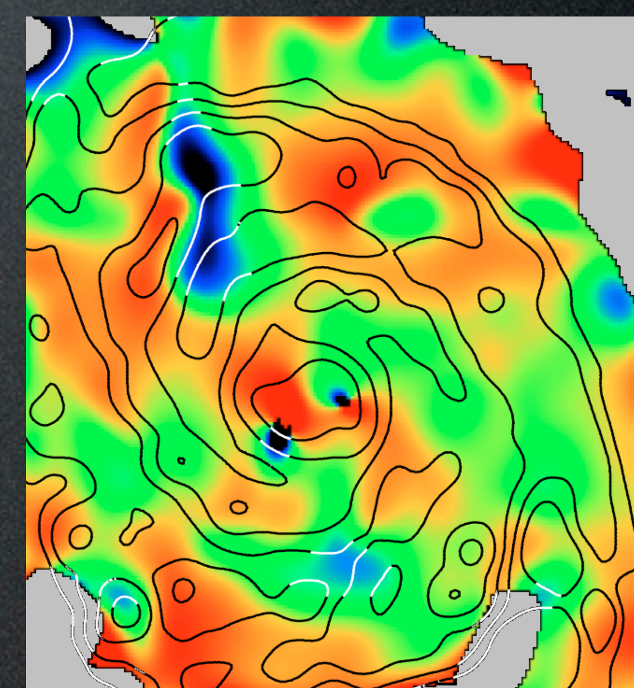
No $\boldsymbol{B}_{\mathrm{reg}}$

15" = 600pc



No $\boldsymbol{B}_{\mathrm{reg}}$

30" = 1100pc



$\boldsymbol{B}_{\mathrm{reg}}$ ~ 1to 3 μG

Smoothing & filtering

Suppress outliers, isolate trend, extract regions, separate scales

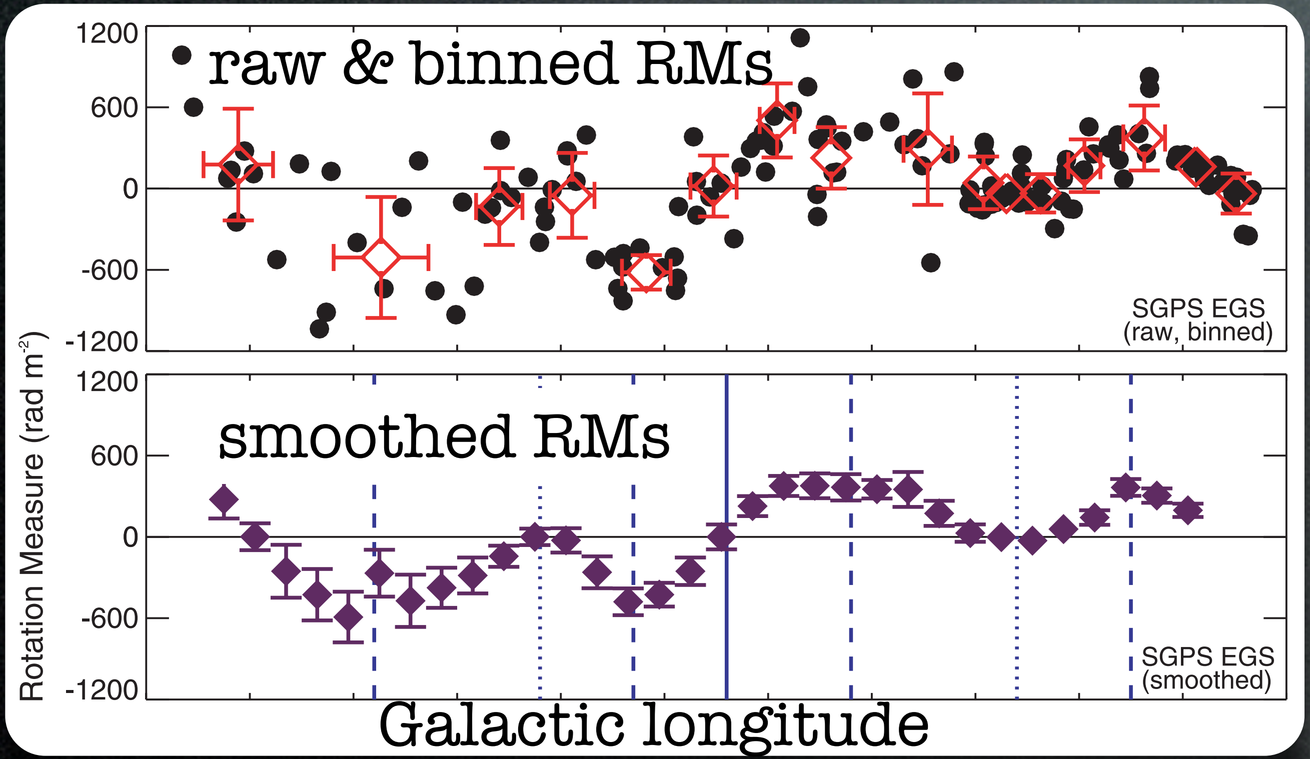
1. Processing the data

Smoothing & filtering

Suppress outliers, isolate trend, extract regions, separate scales

Point source RMs

Brown et al. 2007



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Brown et al. 2007

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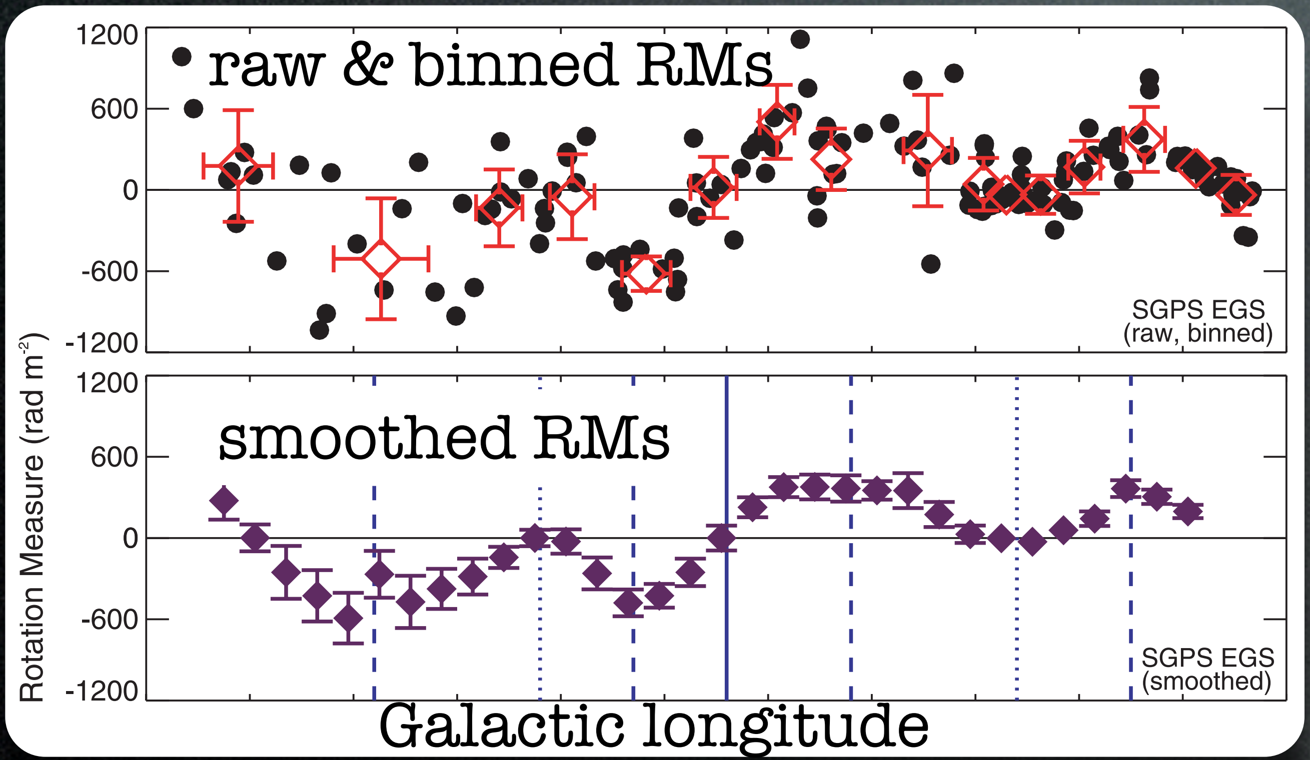


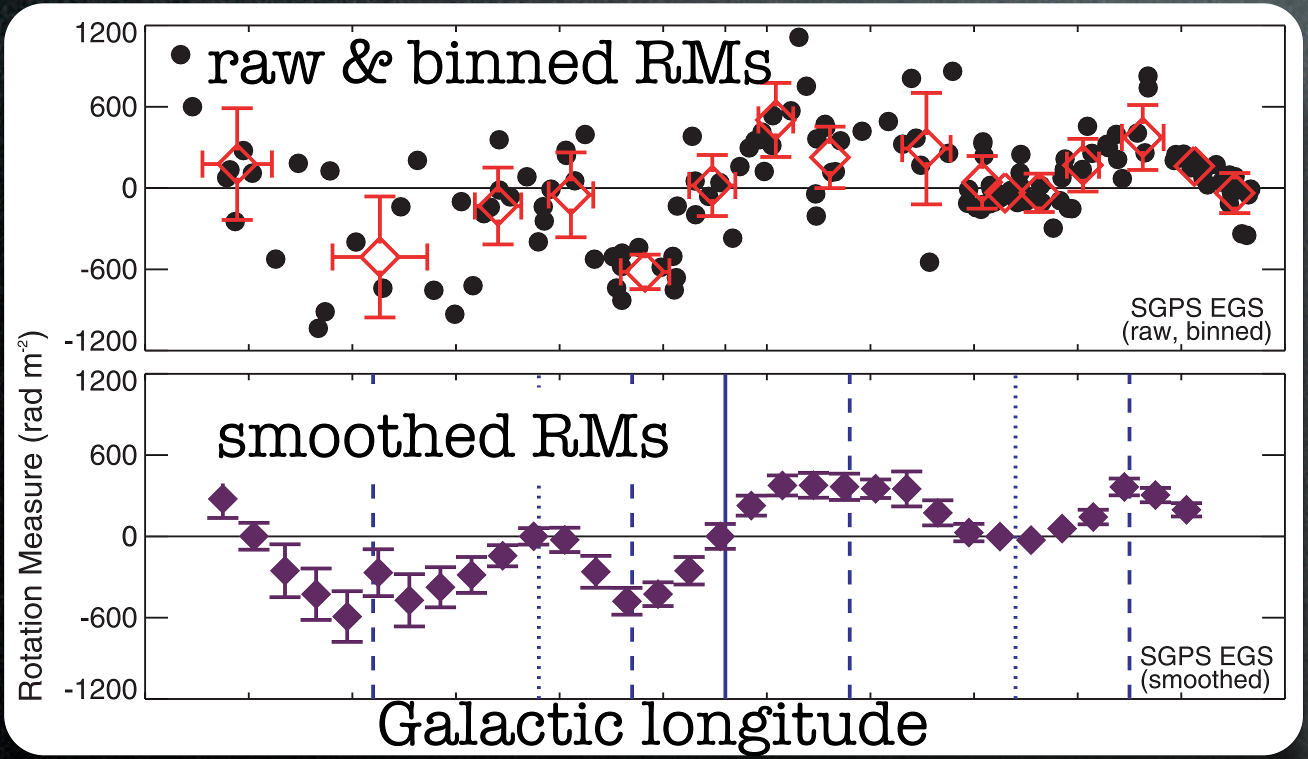
Diffuse emission

1. Processing the data

Smoothing & filtering

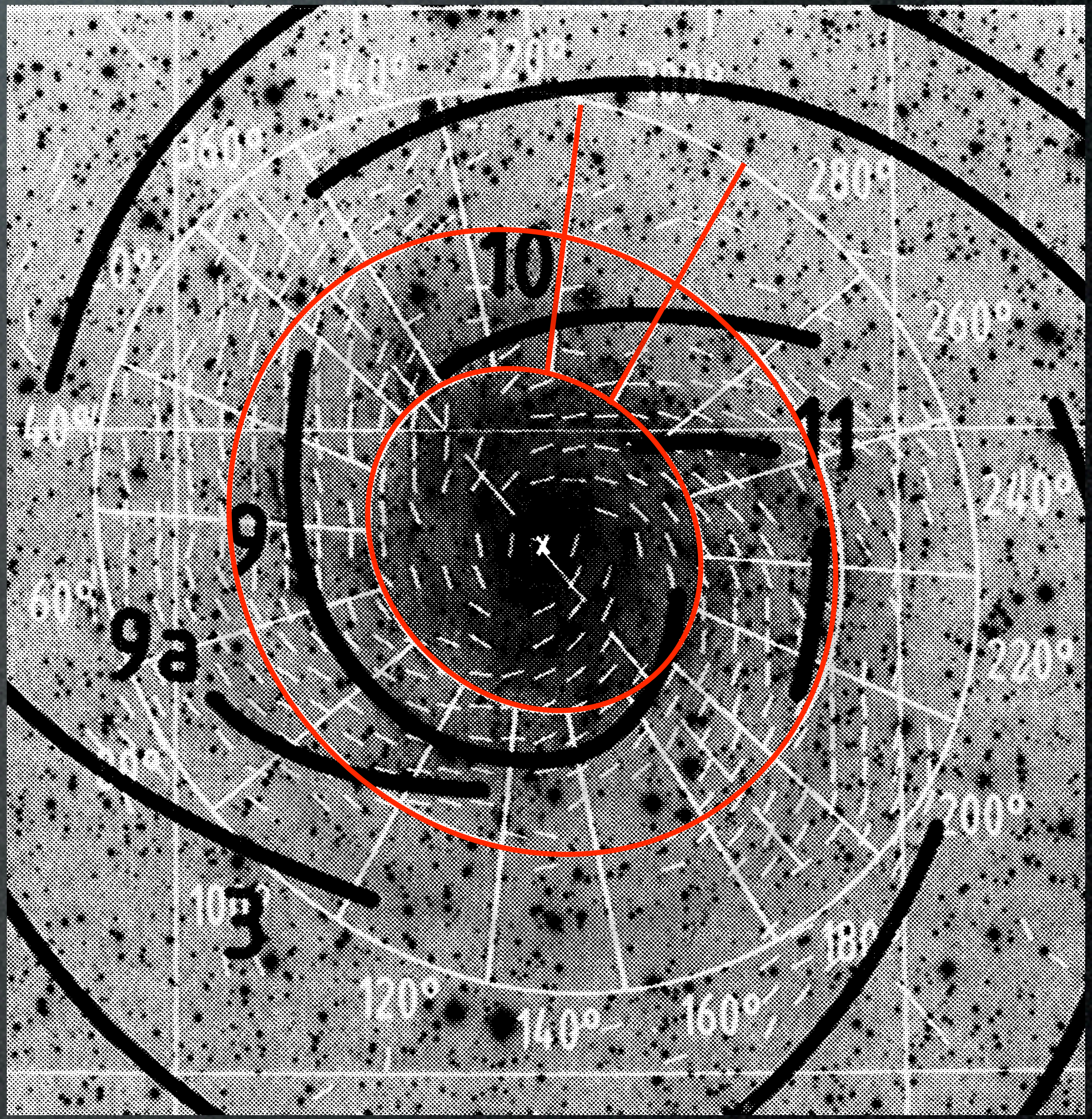
Suppress outliers, isolate trend, extract regions, separate scales

Point source RMs



Brown et al. 2007

Diffuse emission



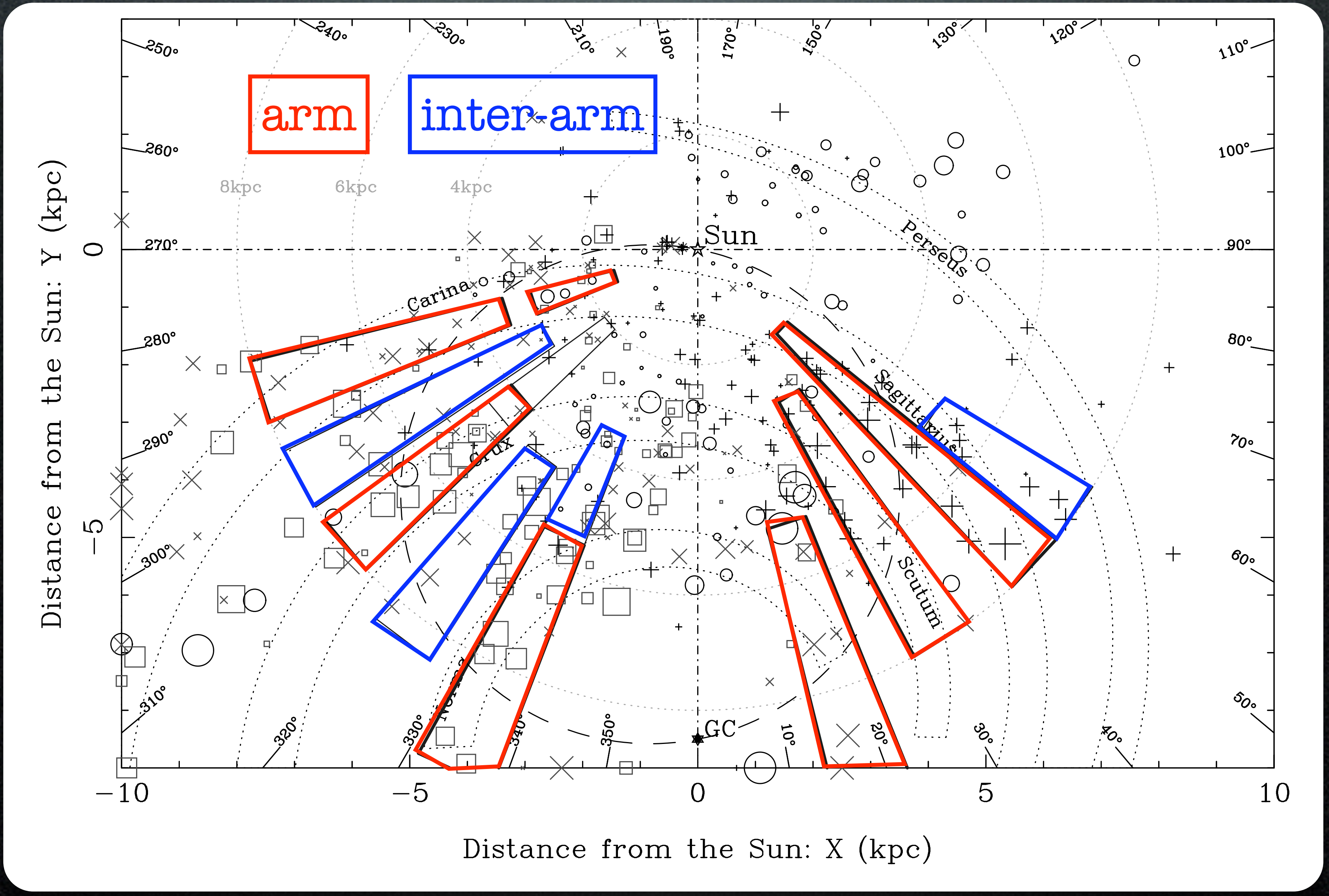
Krause et al. 1989

1. Processing the data

Smoothing & filtering

Suppress outliers, isolate trend, extract regions, separate scales

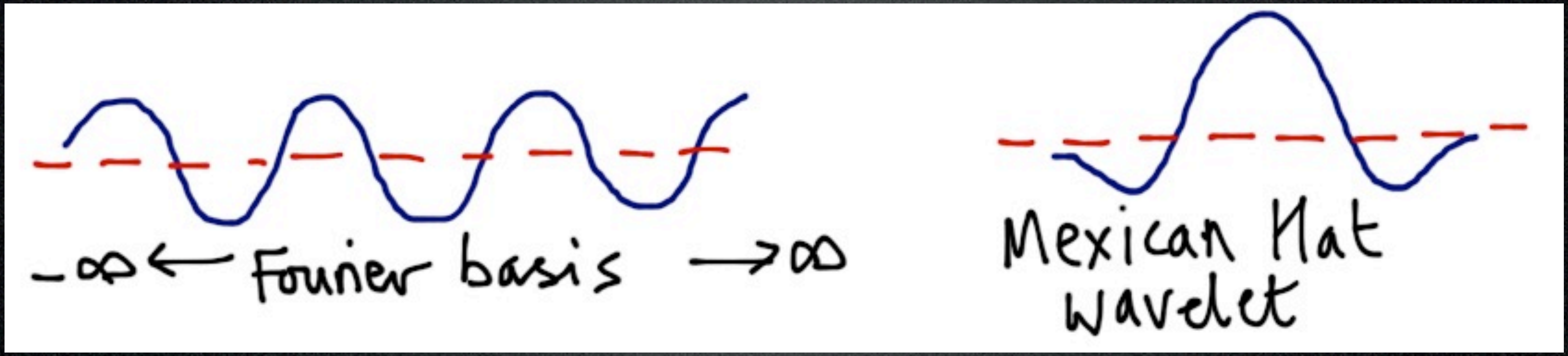
Han et al. 2006



Smoothing & filtering

Suppress outliers, isolate trend, extract regions, separate scales

Wavelet transform: similar to Fourier transform, except basis functions are localised.



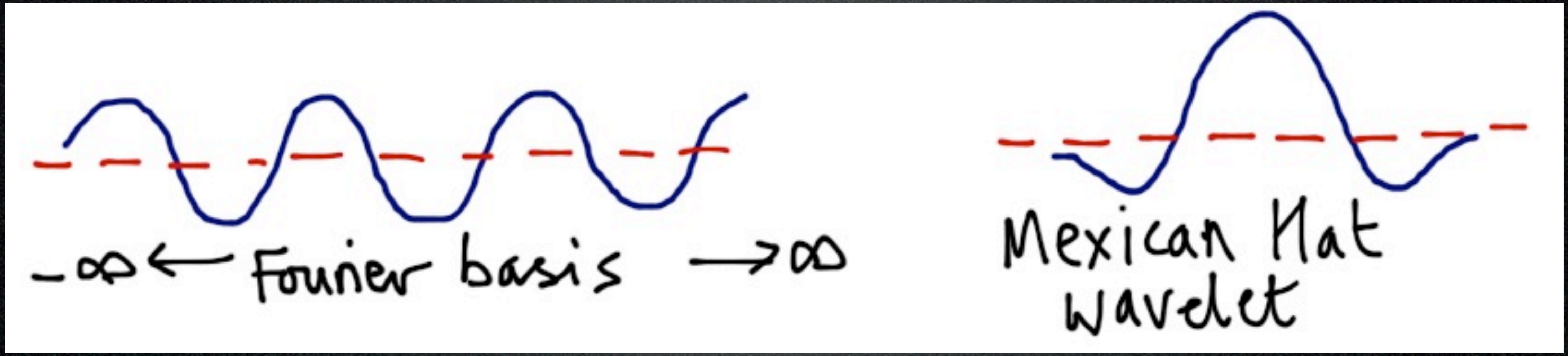
1. Processing the data

Smoothing & filtering

Suppress outliers, isolate trend, extract regions, separate scales

Wavelet transform: similar to Fourier transform, except basis functions are localised.

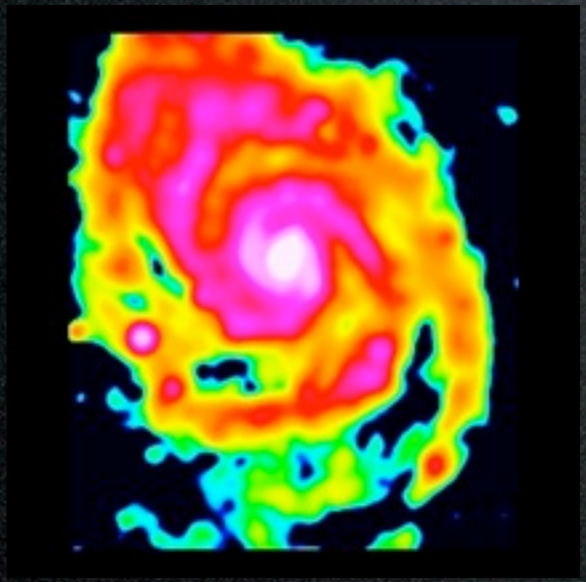
$$\underbrace{W}_{\text{wavelet coefficient}}(\underbrace{a}_{\text{scale}},\underbrace{\mathbf{x}}_{\text{position}}) = \frac{1}{a^k}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underbrace{f(\mathbf{x}')}_{\text{image}}\underbrace{\psi^*[(\mathbf{x}'-\mathbf{x})/a]}_{\text{wavelet}}\mathrm{d}\mathbf{x}'$$



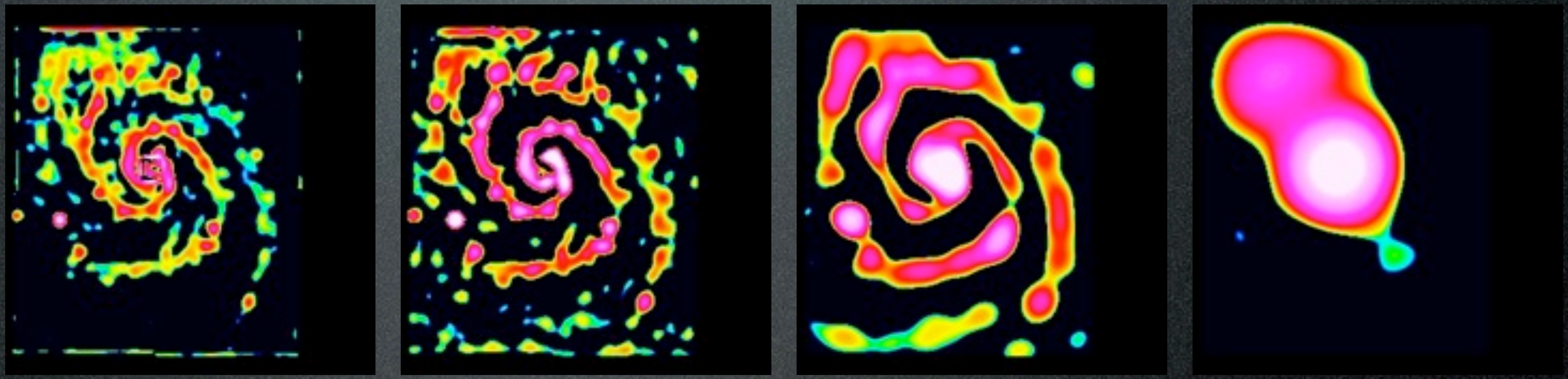
Smoothing & filtering

Suppress outliers, isolate trend, extract regions, separate scales

Original



Wavelet coefficients at different scales



Many uses, e.g.:

- trace spiral arms
- scale-by-scale correlation
- separate arm & inter-arm

Frick et al. 2000

Frick, Beck et al. 2001

Fletcher et al. 2011

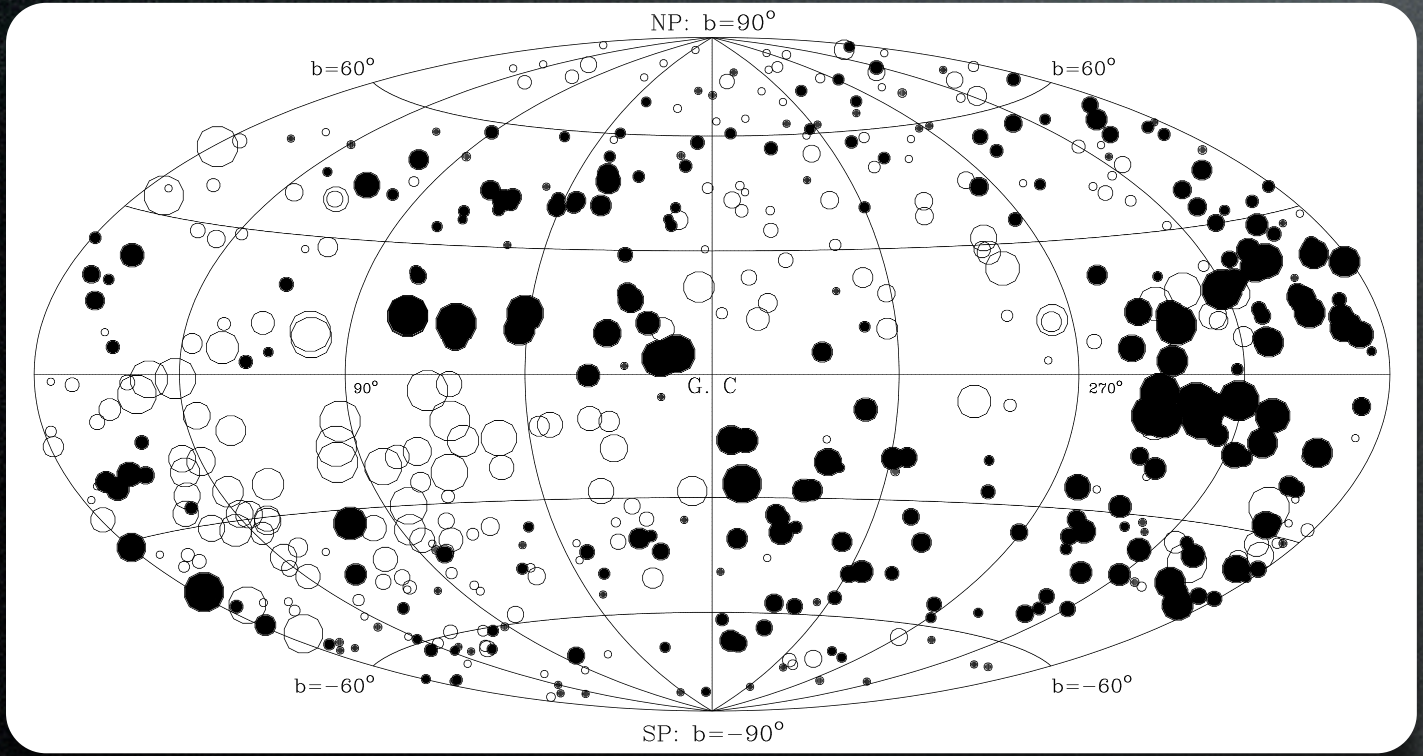
1. Processing the data

Smoothing & filtering

Suppress outliers, isolate trend, extract regions, separate scales

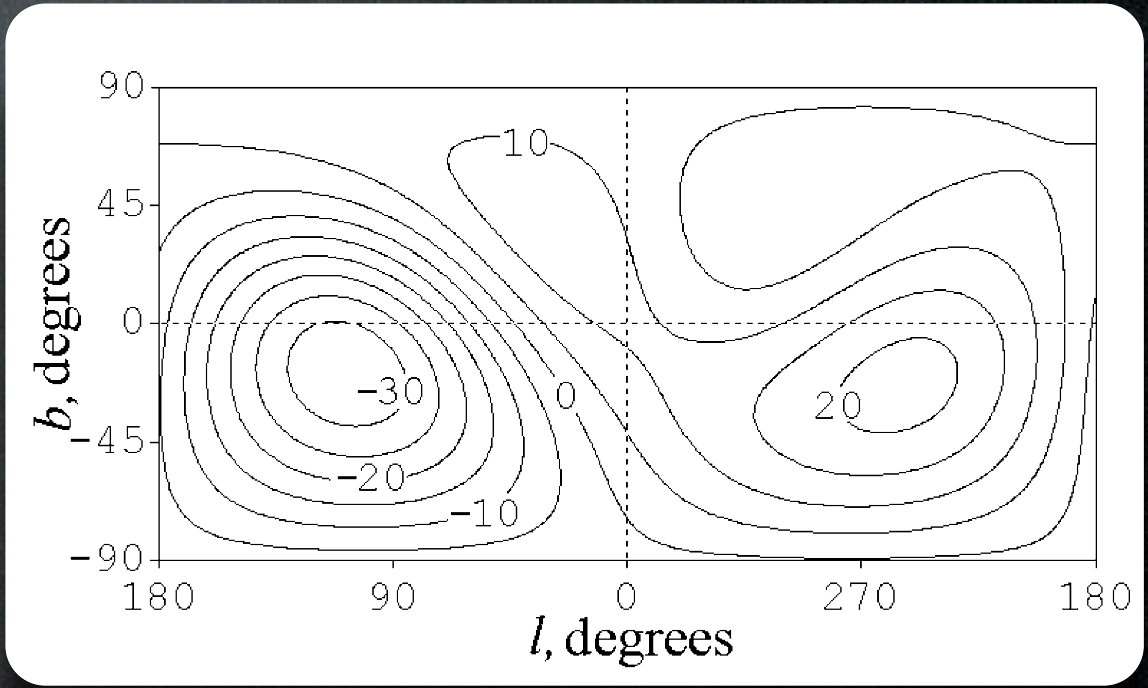
Wavelet transform can be applied to discrete data

Extra-galactic RMs



Broten catalogue in Han et al. 1997

Wavelet coefficients at dominant scale 67°



Units are rad m^{-2}

Frick, Stepanov et al. 2001

1. Processing the data

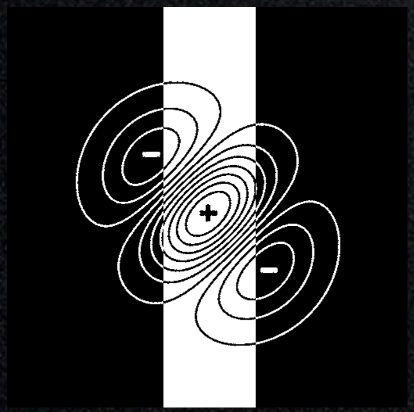
Smoothing & filtering

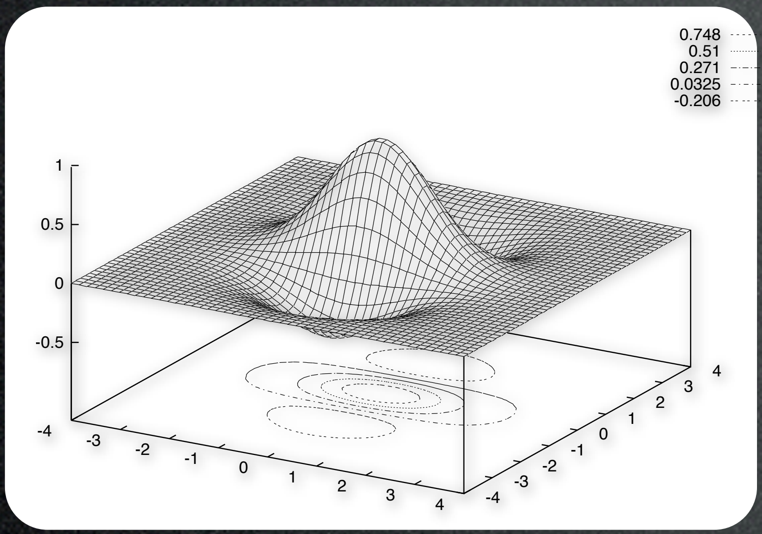
Suppress outliers, isolate trend, extract regions, separate scales

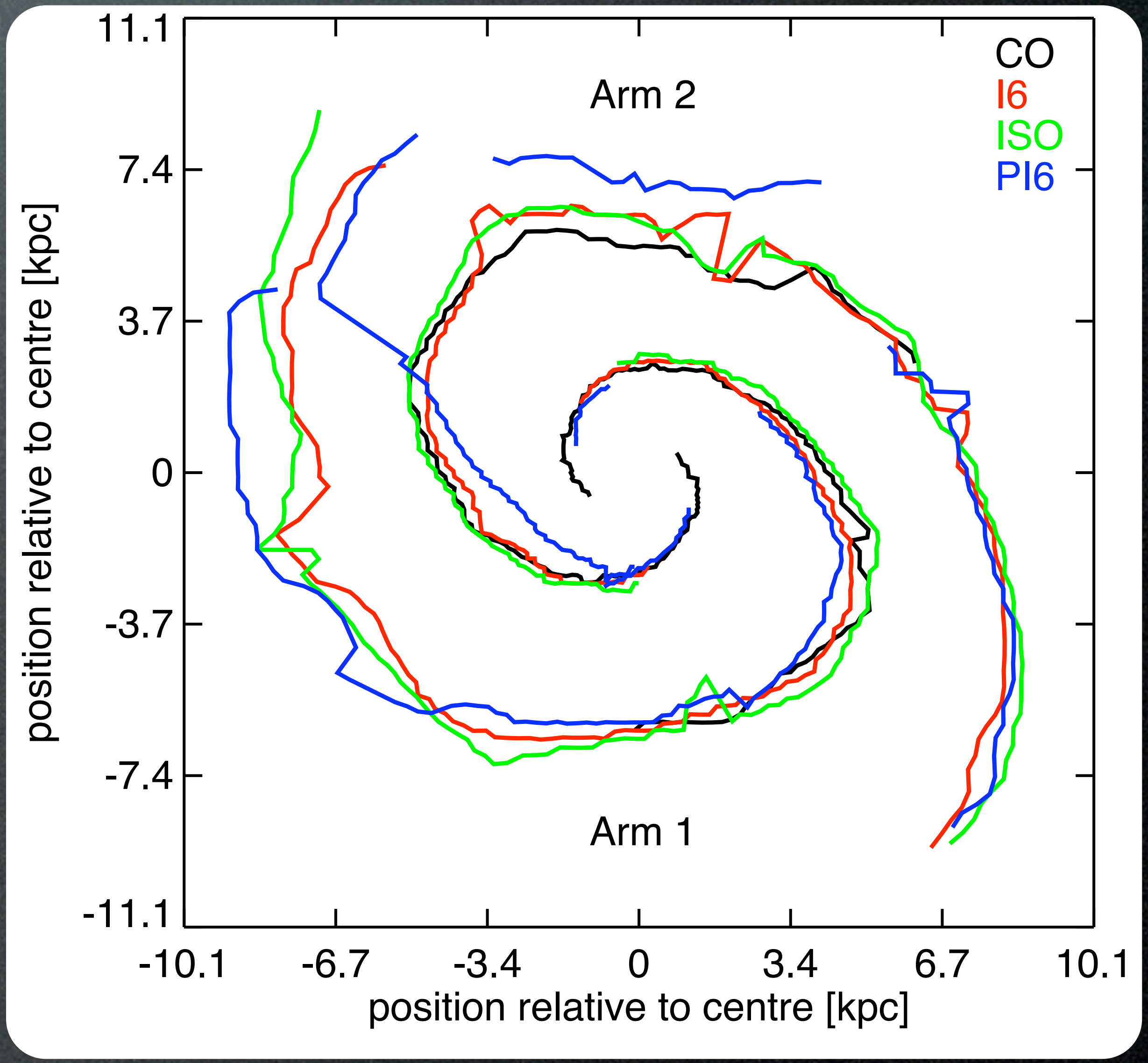
Basic wavelet methods can be extended:

Use anisotropic wavelet to trace and measure elongated structures.

$w(a,\mathbf{x},\theta)$, θ=orientation







Patrikeev et al. 2006

1. Processing the data

Smoothing & filtering

Suppress outliers, isolate trend, extract regions, separate scales

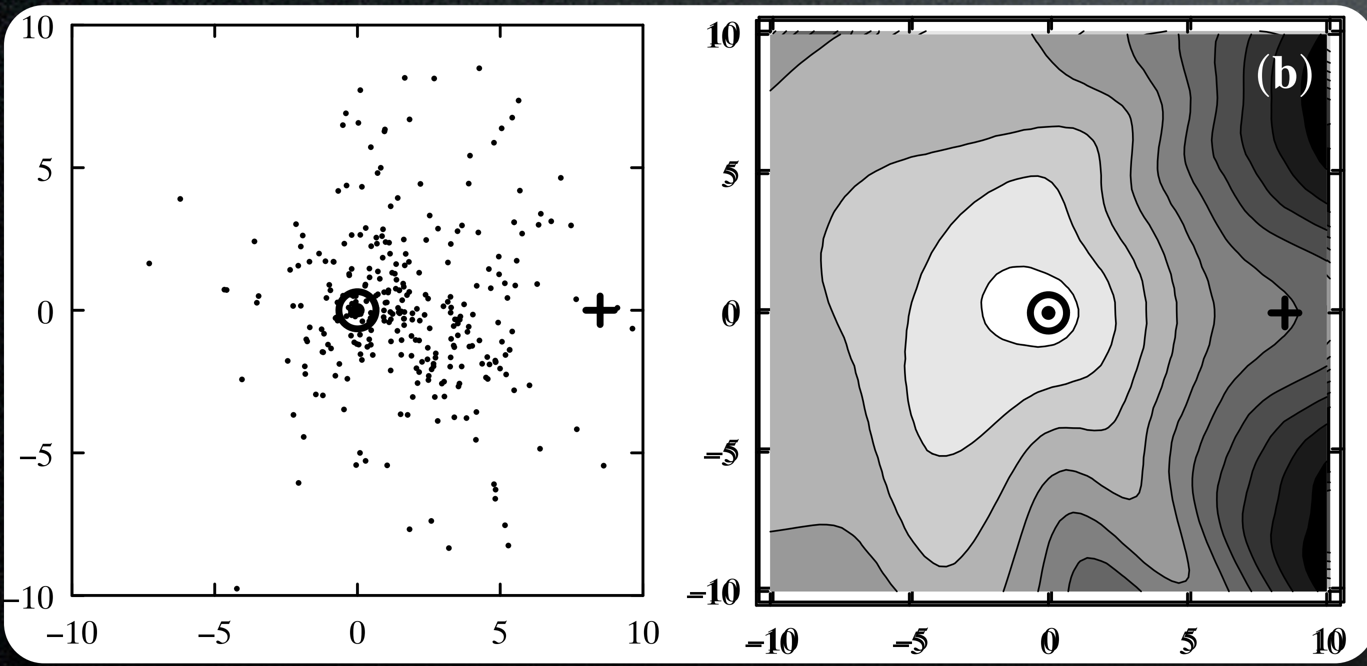
Basic wavelet methods can be extended:
direct reconstruction of an irregularly sampled field

Distribution of pulsars

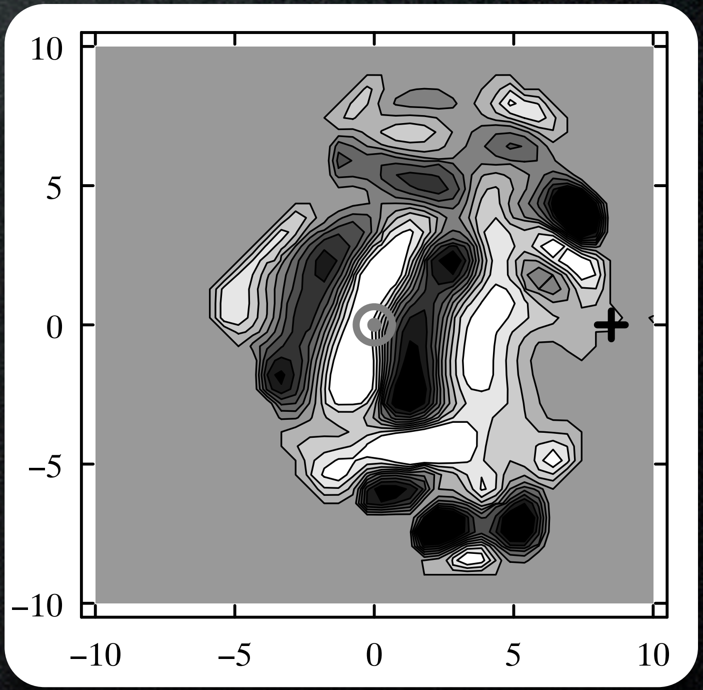
Test DM

$$\mathrm{DM} = \int_l n_e \mathrm{d}l$$

Recovered n_e from test DM



differentiating wavelet



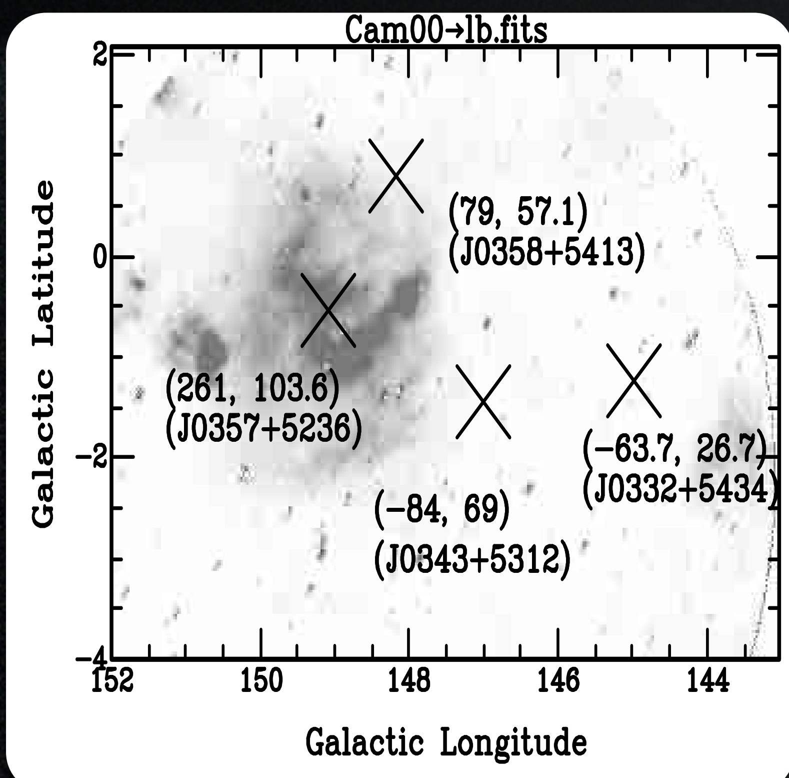
Stepanov et al. 2002

Phenomenological

Association with specific object(s)
"By-hand" approach

The effect of HII regions on rotation measure of pulsars

D. Mitra[1], R. Wielebinski[1], M. Kramer[2], and A. Jessner[1]



2. Analysing the data

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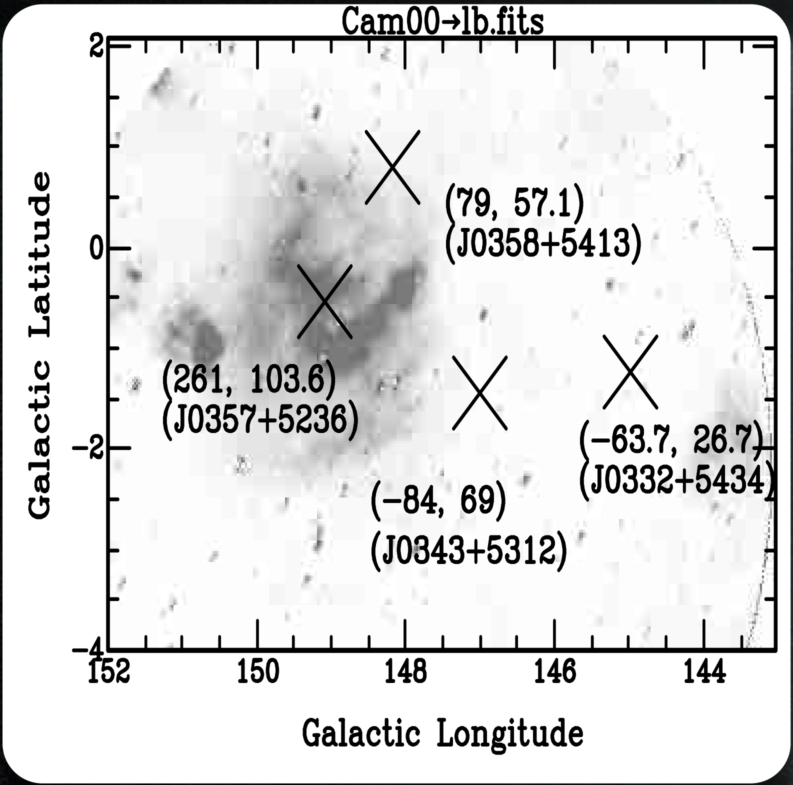

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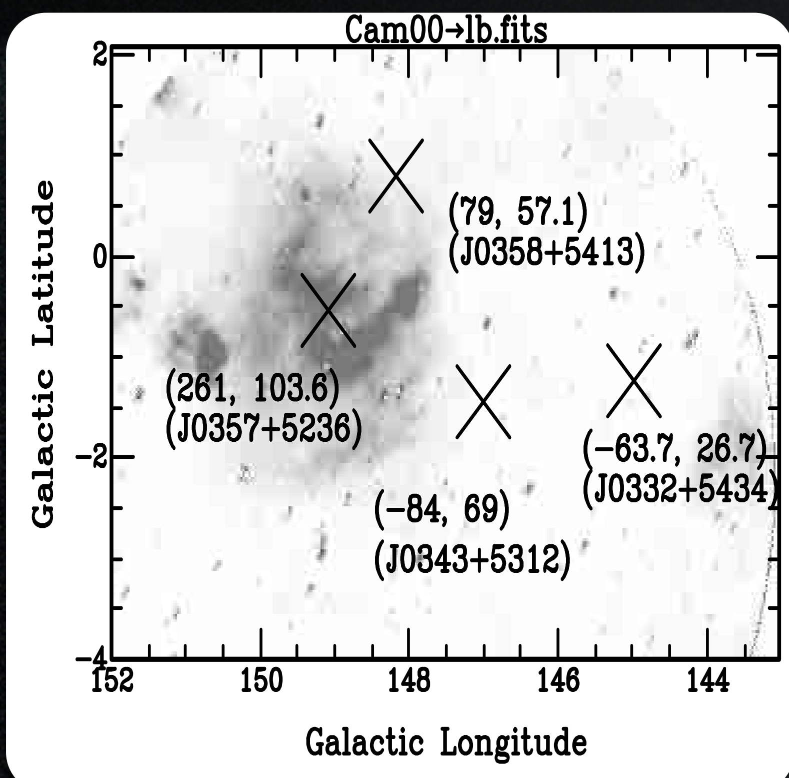

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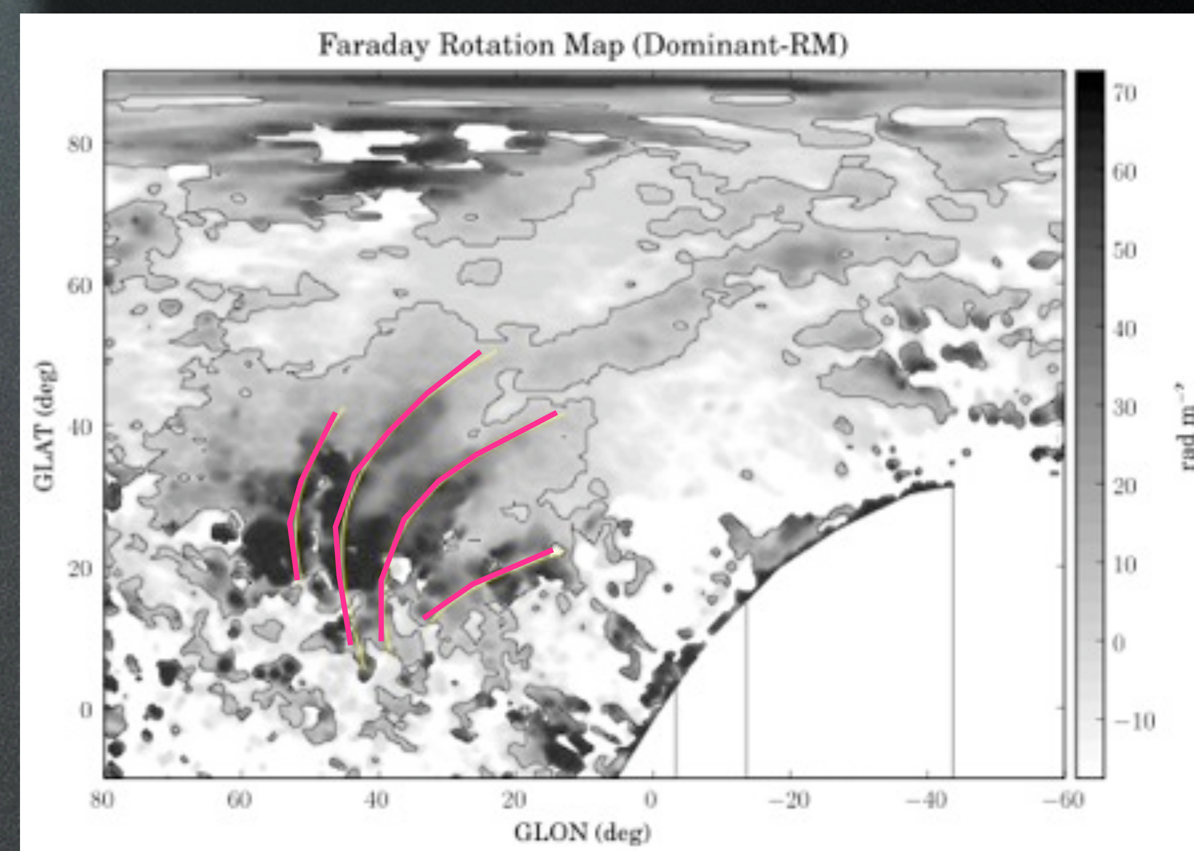

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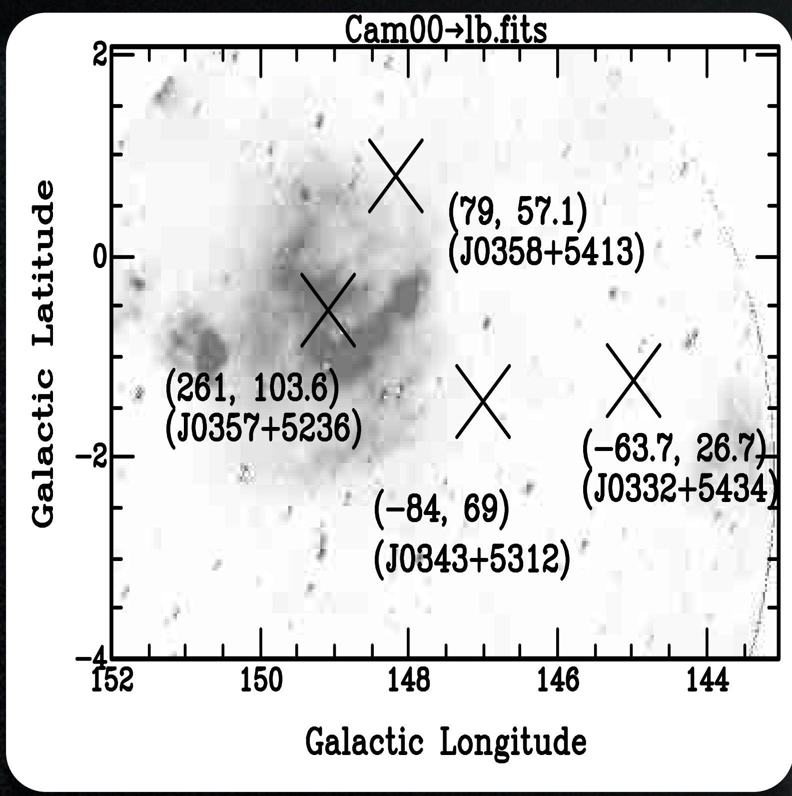

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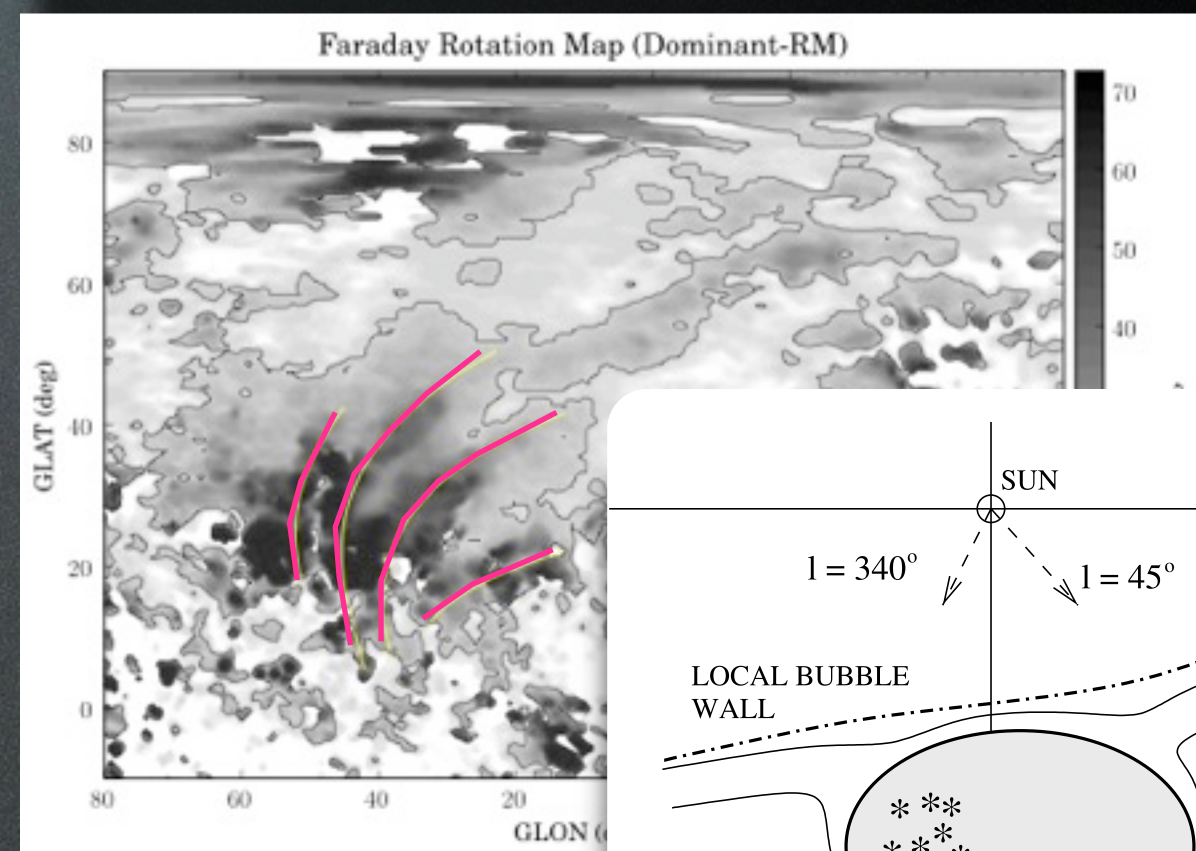
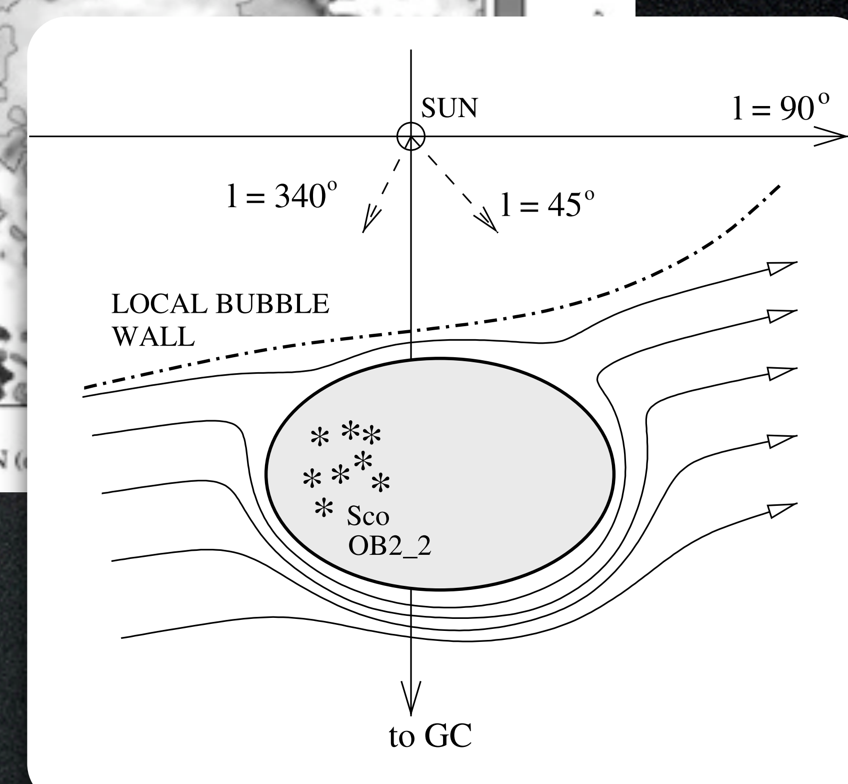


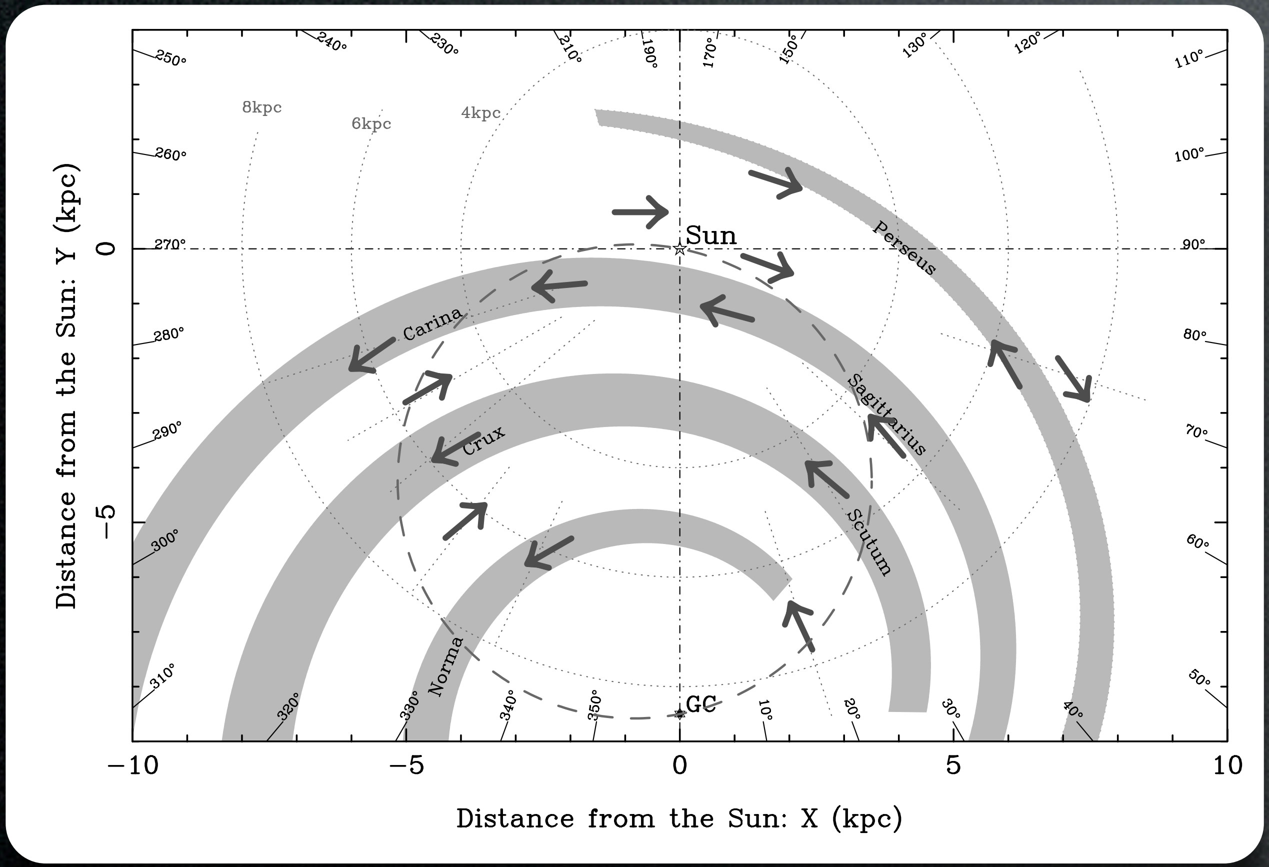


Choose between models

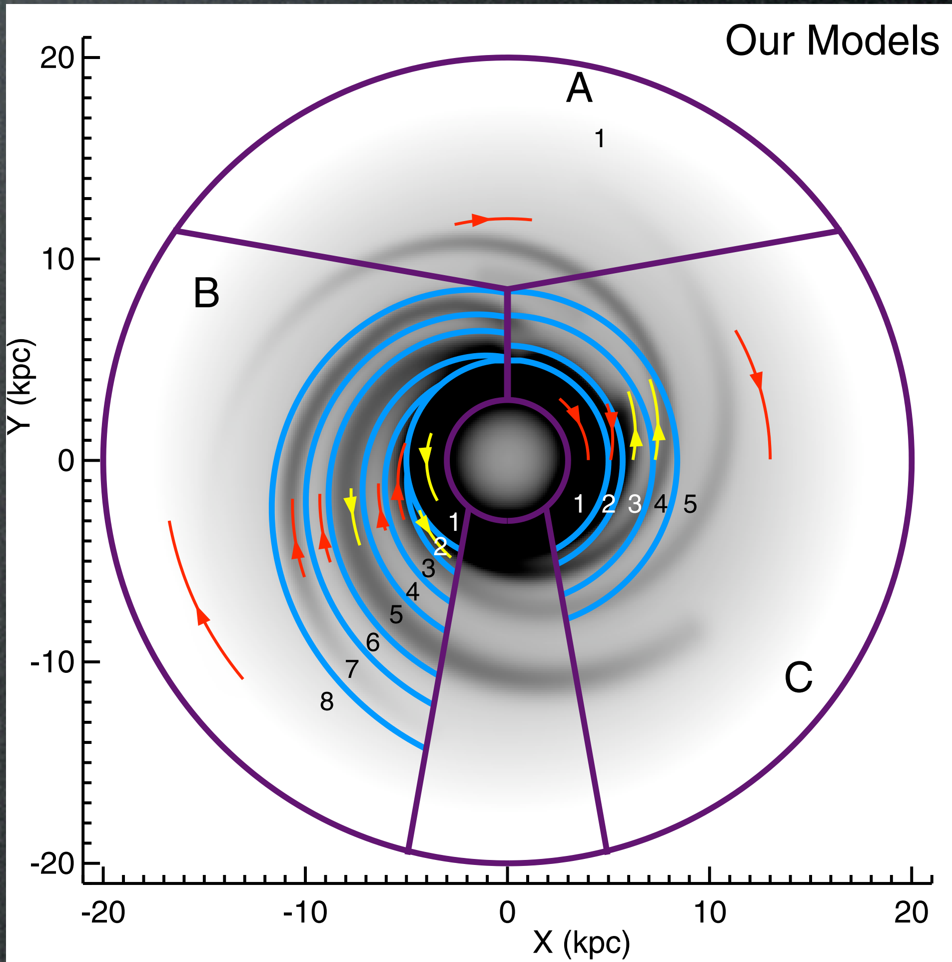
Compare the data to model/expectation
Identify the model which works best

Compare two approaches to Milky Way magnetic field

Han et al. 2006



van Eck et al. 2011



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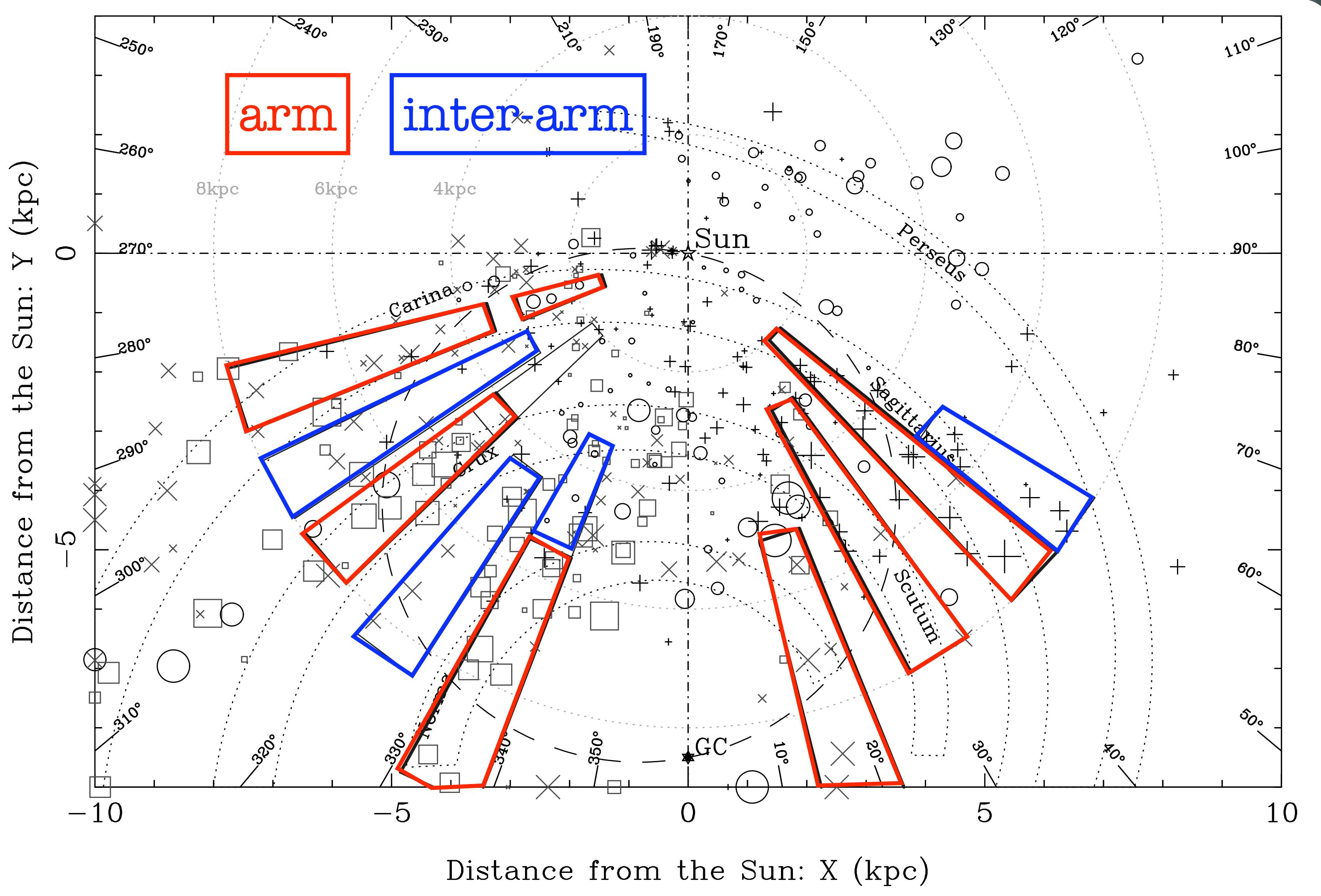

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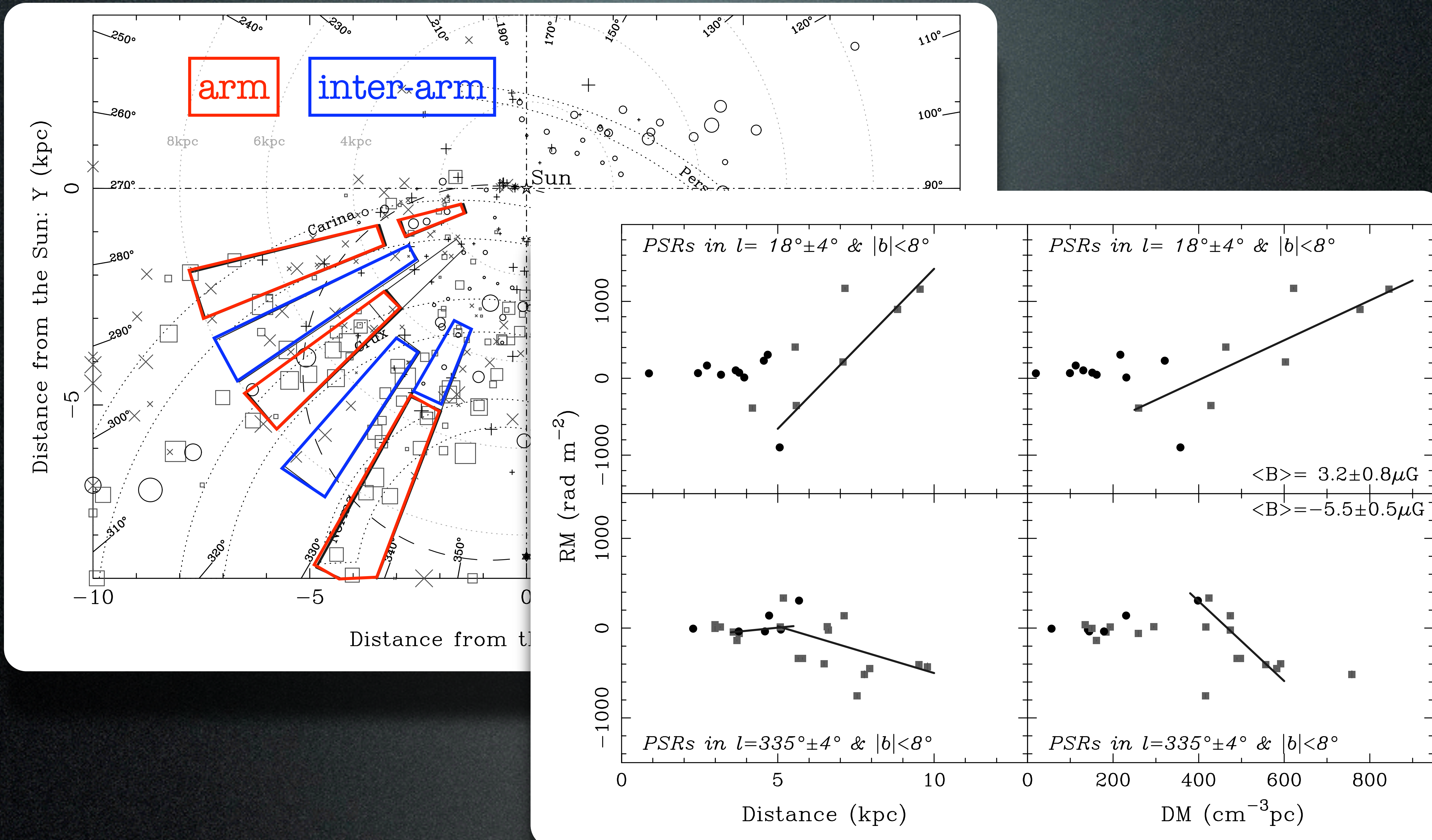

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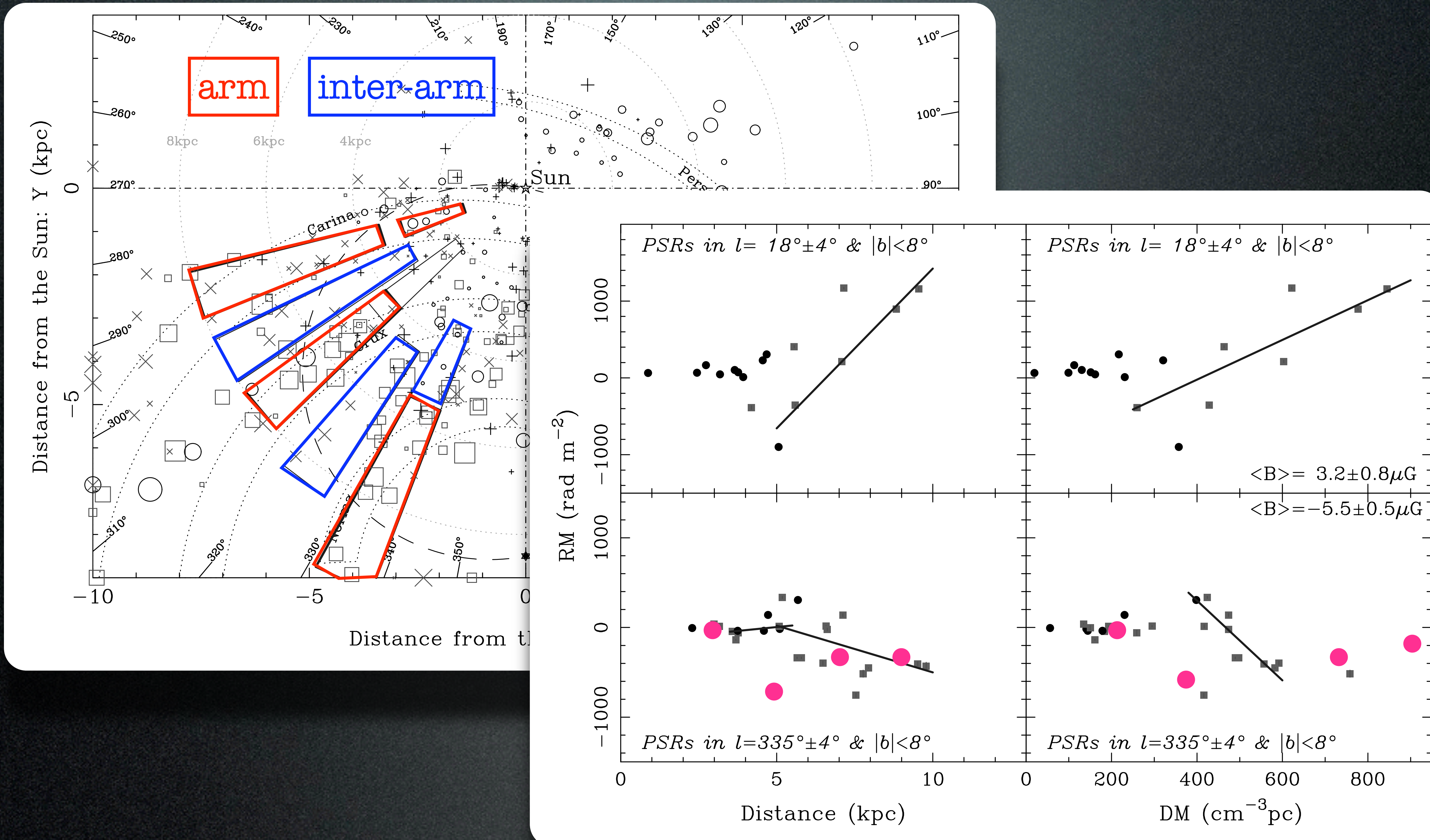

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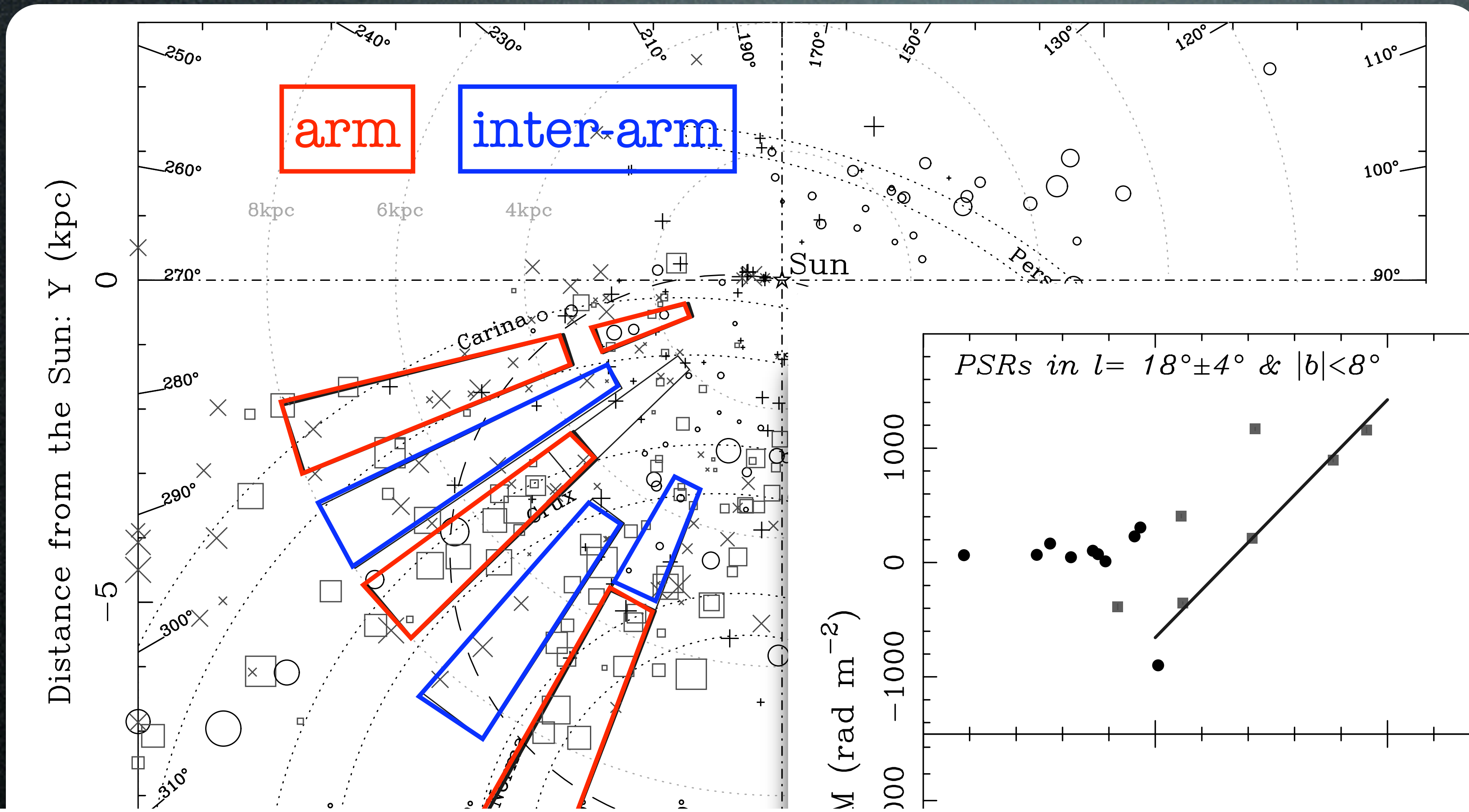

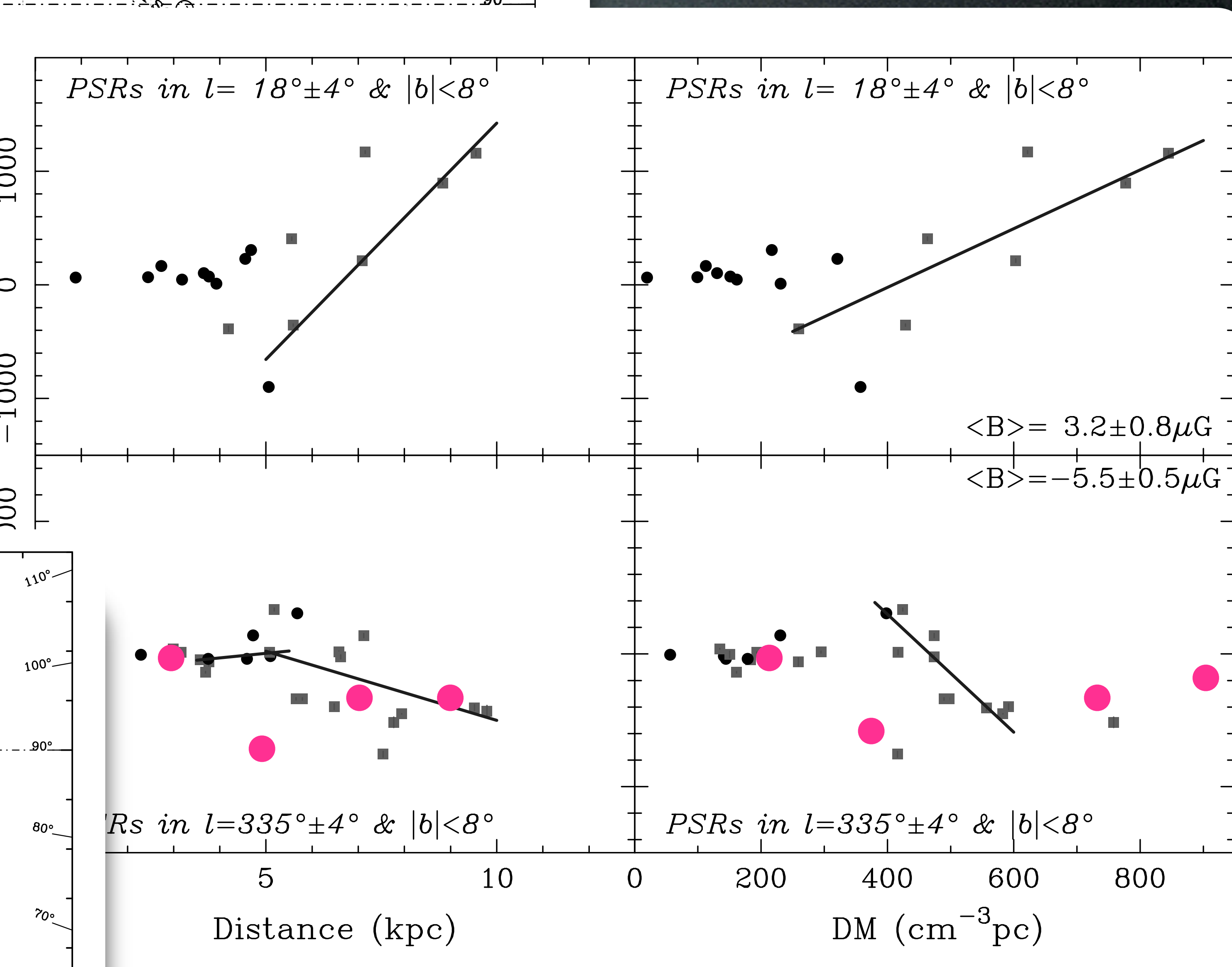

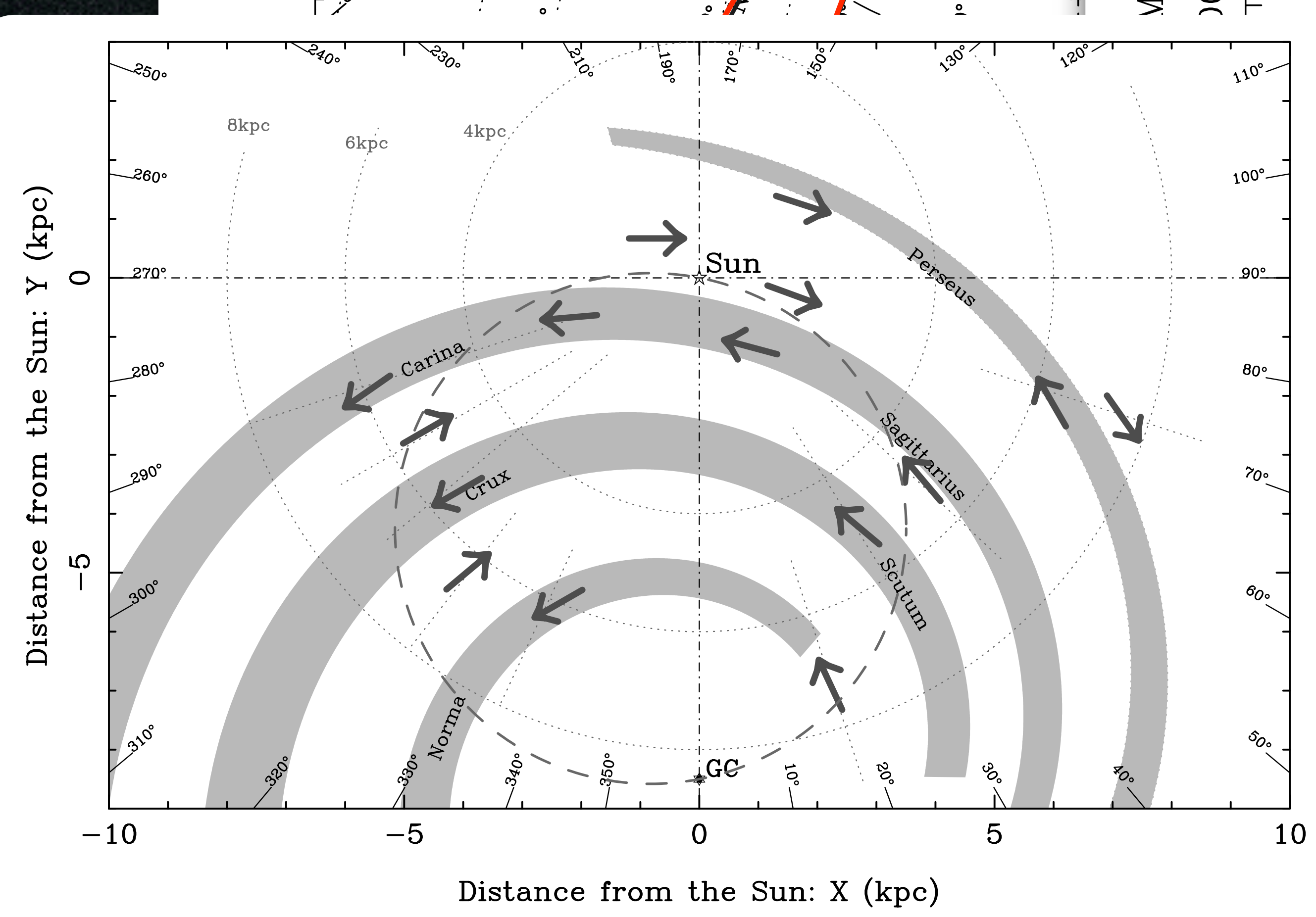

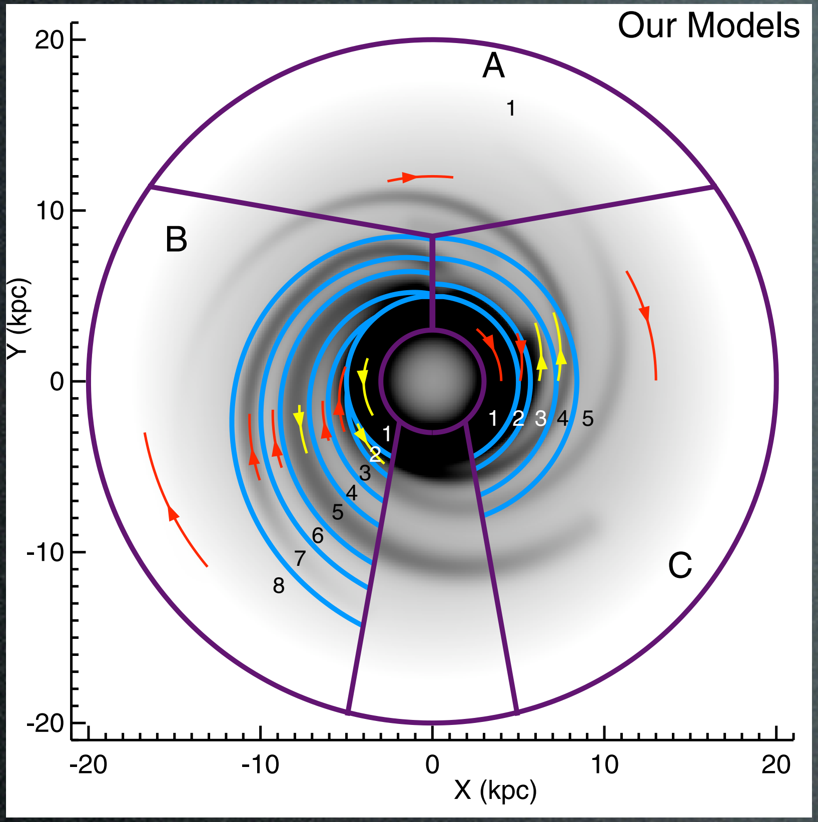

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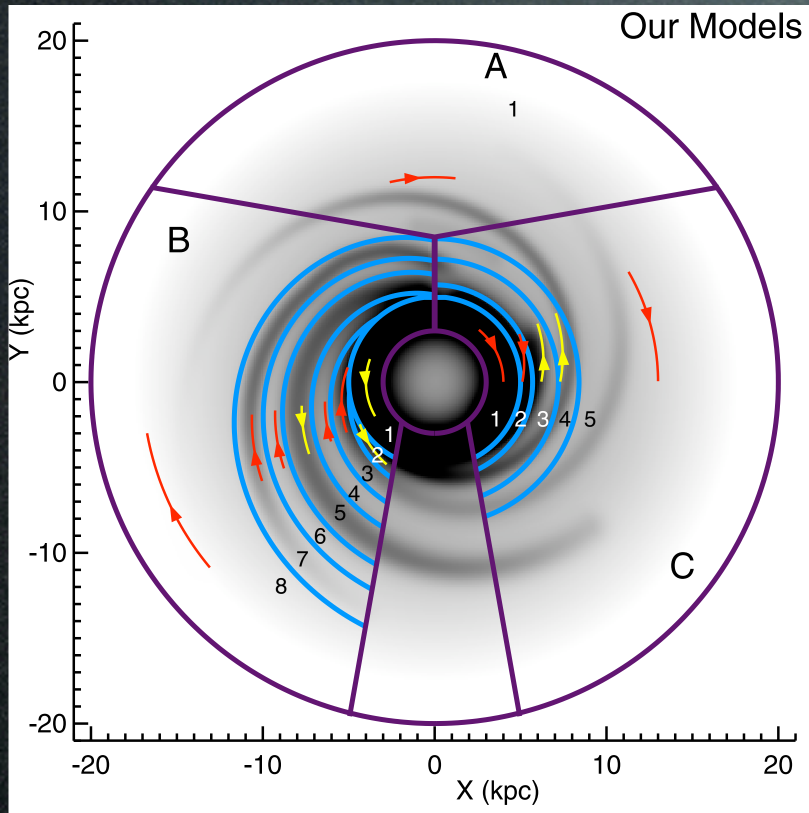

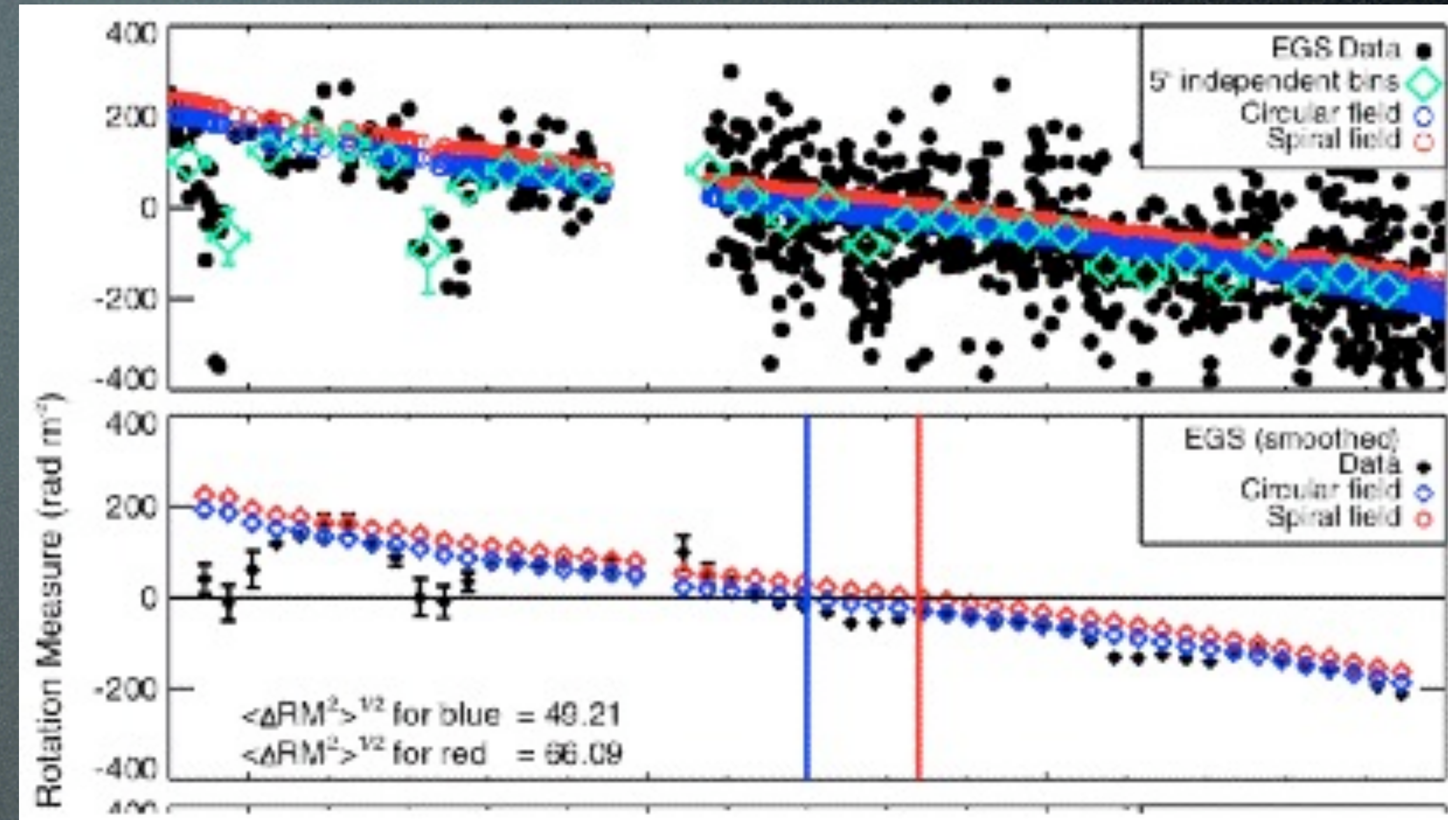

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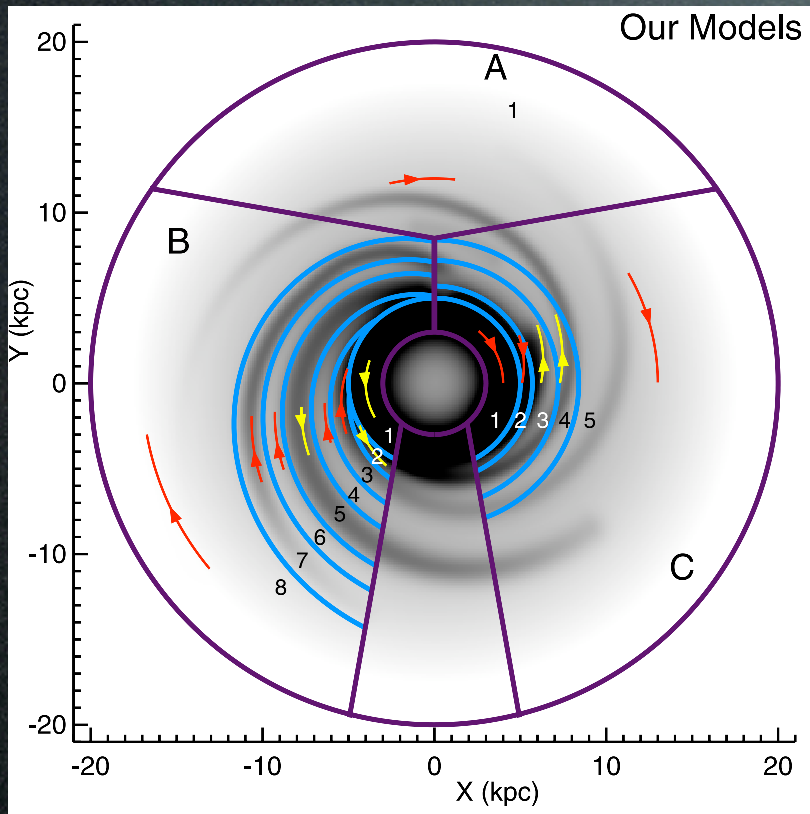


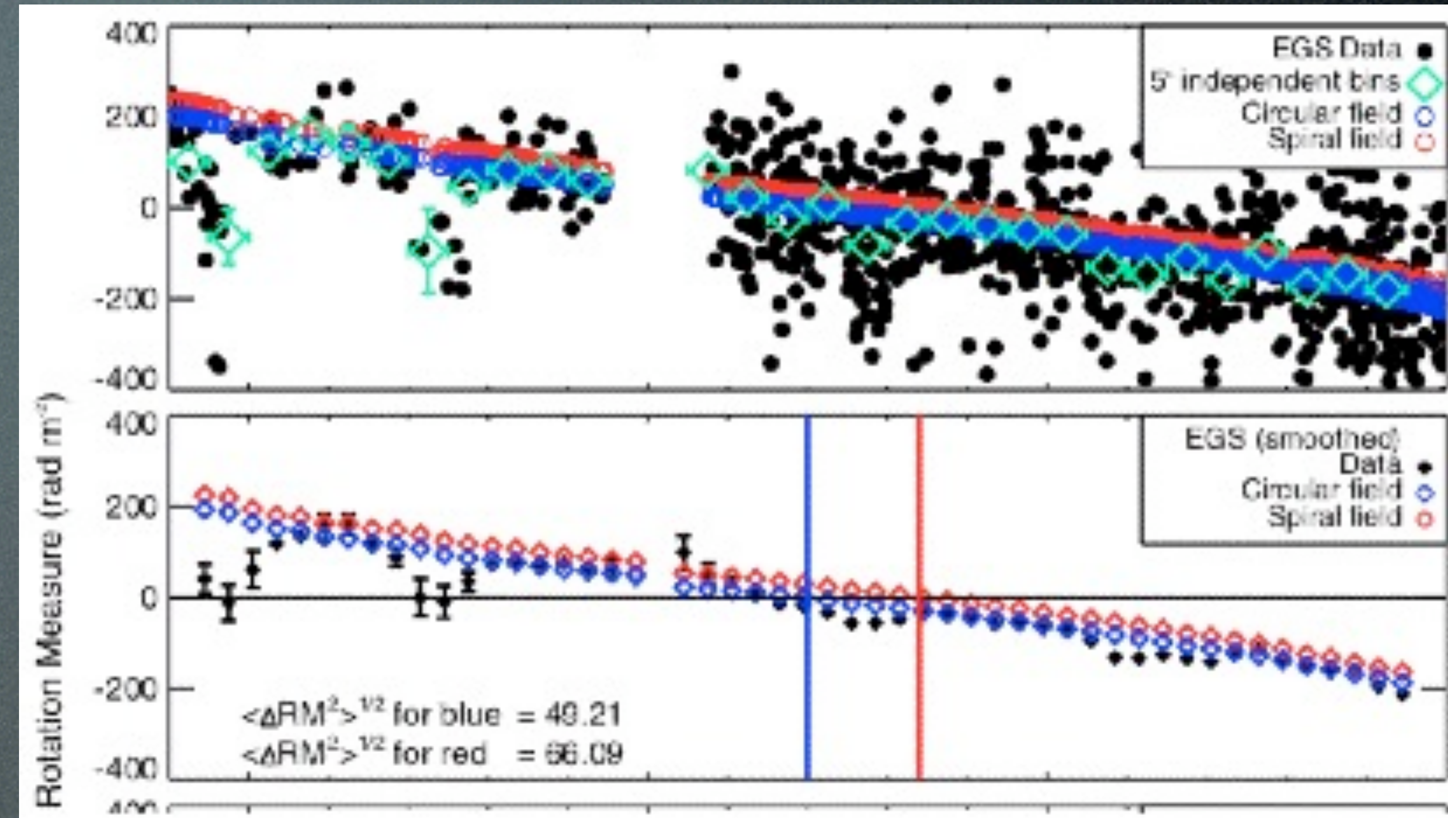
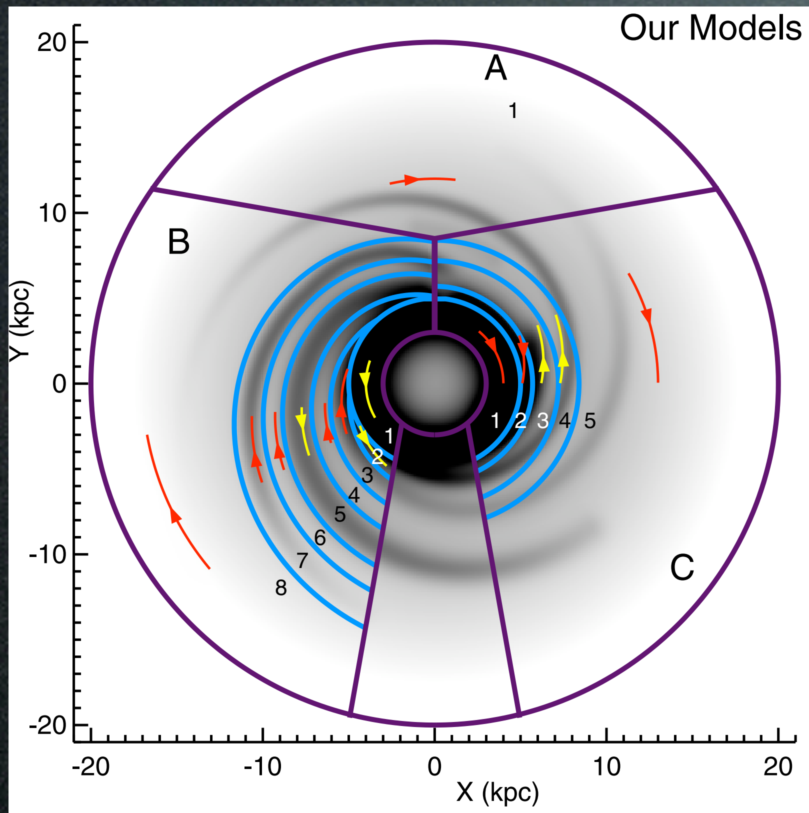


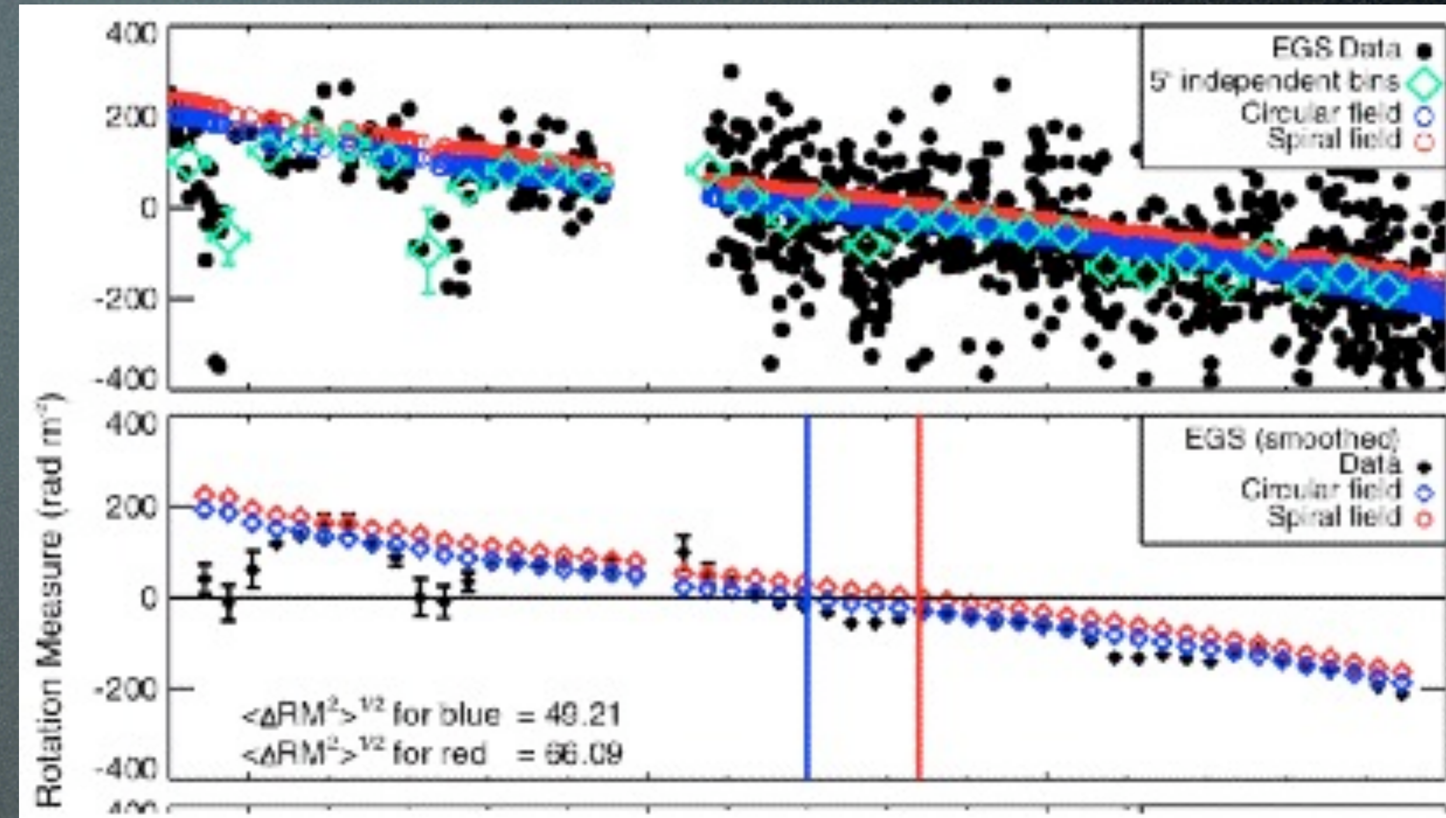
Table 2

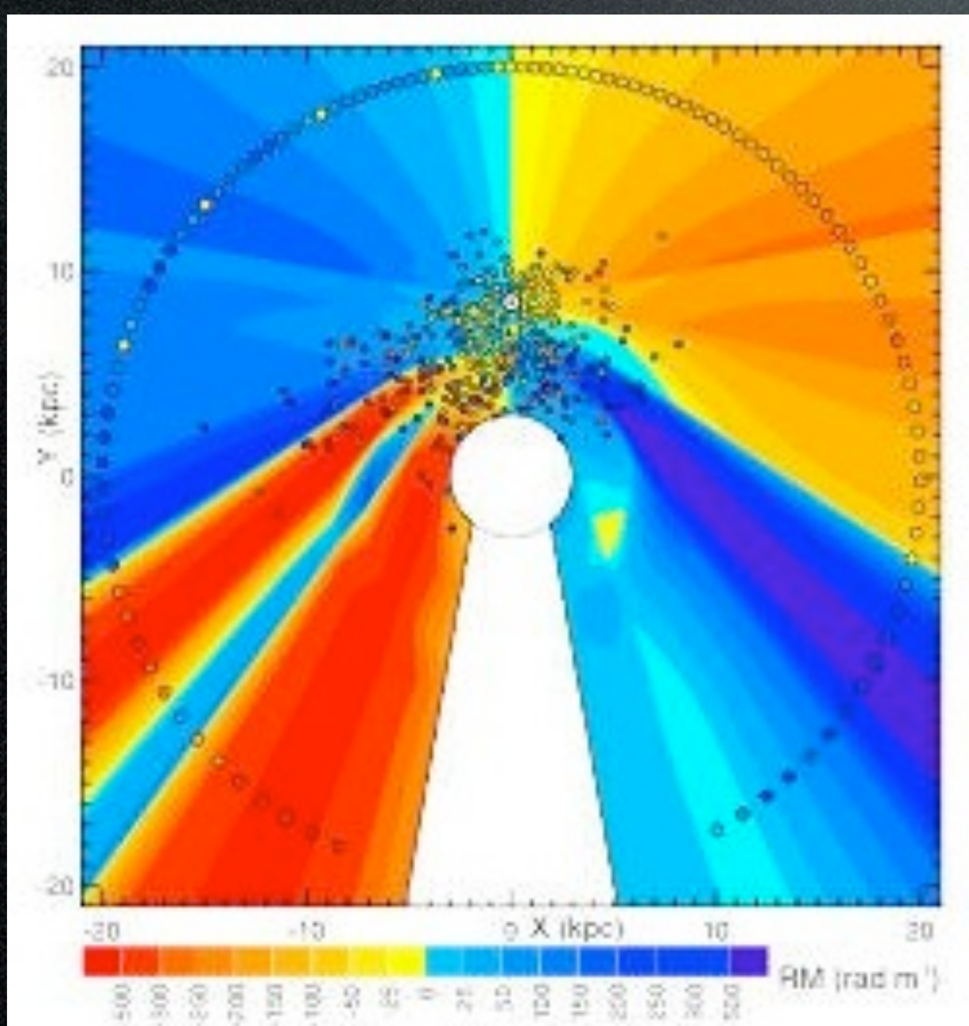
Best-fit *B*-field Values for Our Model Sectors Shown in Figure 6

Sector and Region	Magnetic Pitch Angle	Radial Dependence	B^a (in μG)
Sector A			
1	0°	R^{-1}	-1.20 ± 0.48
Sector B			
1	0°	Constant	0.85 ± 0.06
2	11°.5	Constant	1.0 ± 0.4
3	11°.5	Constant	-0.54 ± 0.07
4	11°.5	Constant	-0.92 ± 0.06
5	11°.5	Constant	1.71 ± 0.06
6	11°.5	Constant	-0.90 ± 0.08
7	11°.5	Constant	-0.34 ± 0.06
8	0°	R^{-1}	-0.78 ± 0.12
Sector C			
1	0°	Constant	-0.15 ± 0.04
2	11°.5	Constant	-0.40 ± 0.01
3	11°.5	Constant	2.23 ± 0.13
4	11°.5	Constant	0.09 ± 0.05
5	0°	R^{-1}	-0.86 ± 0.09

van Eck et al. 2011







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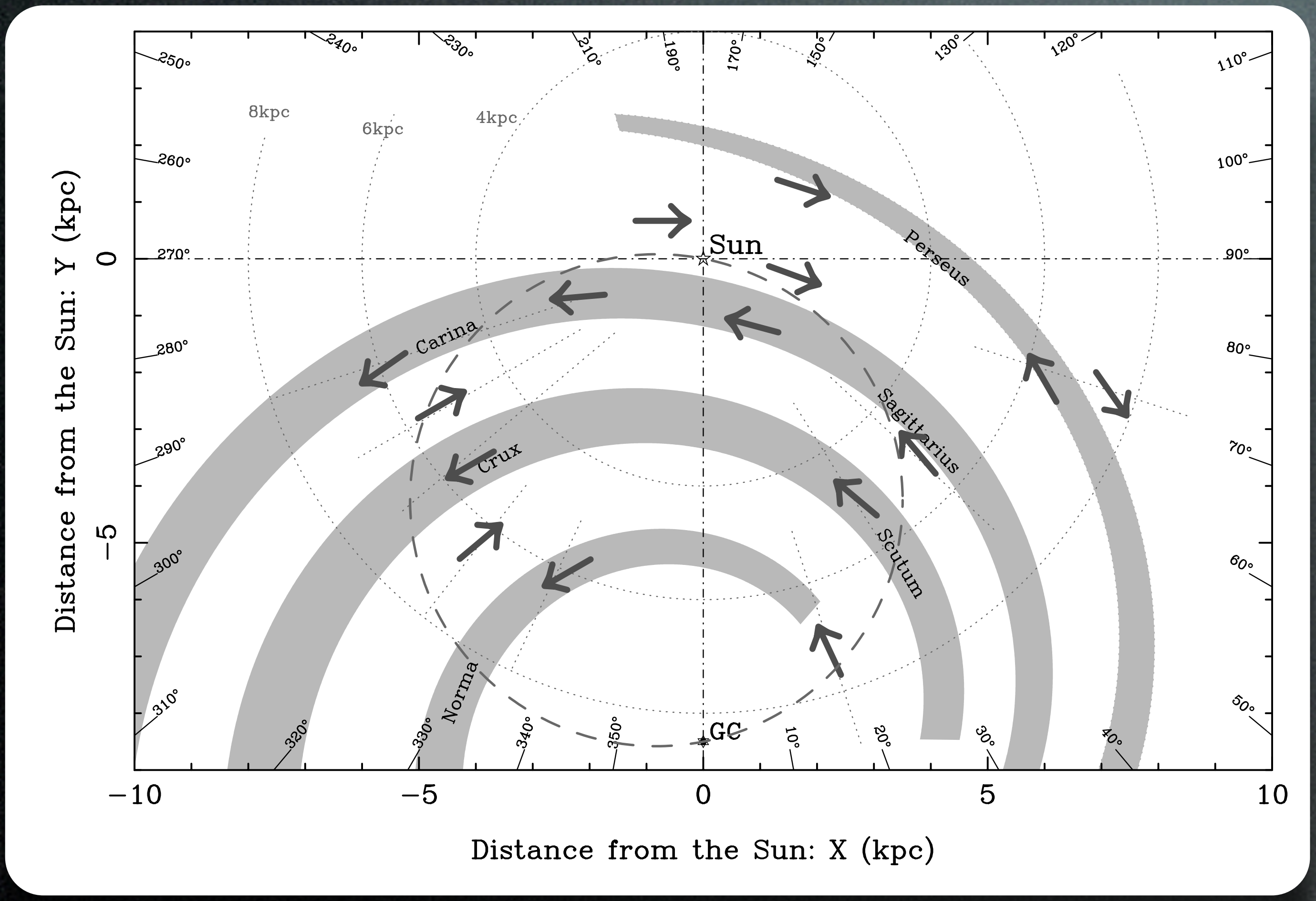
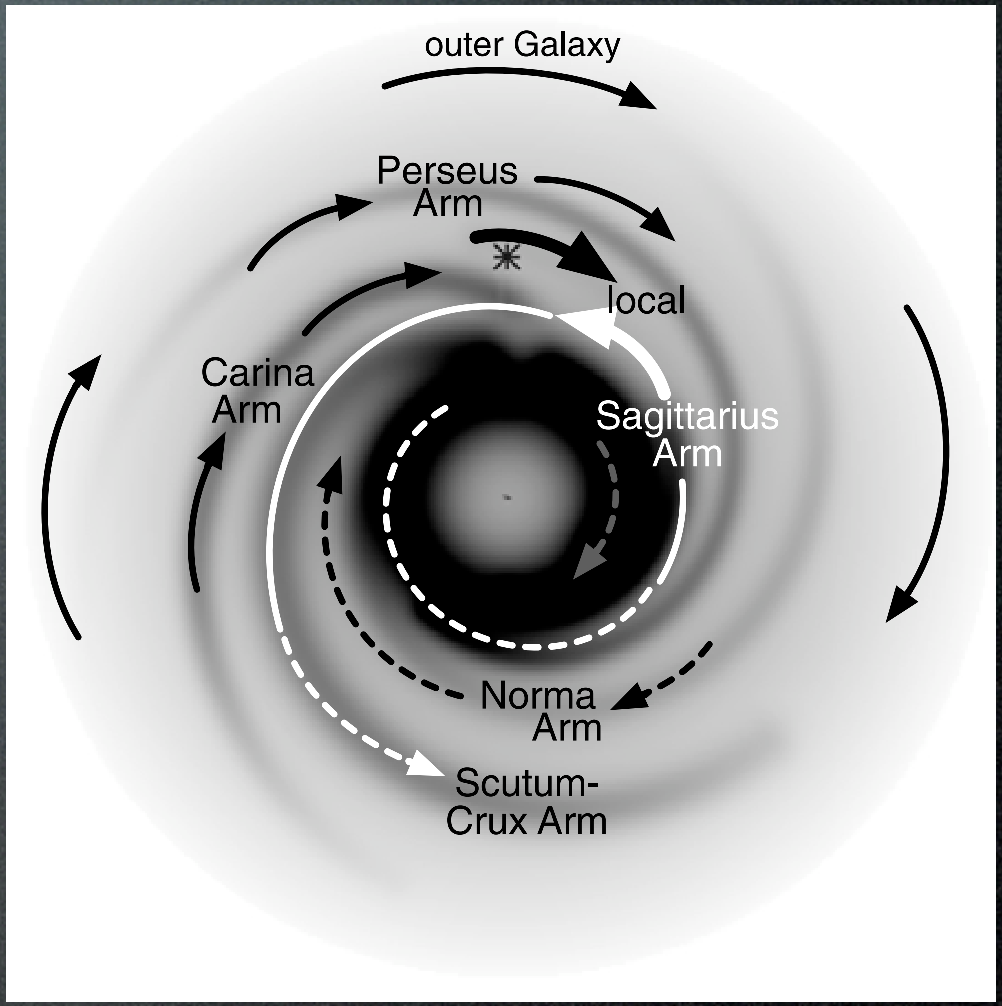
2. Analysing the data

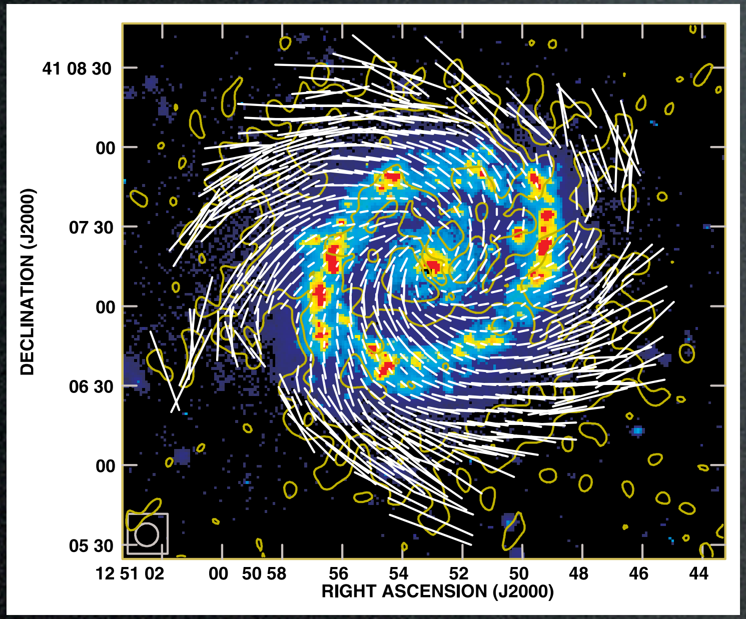
Choose between models

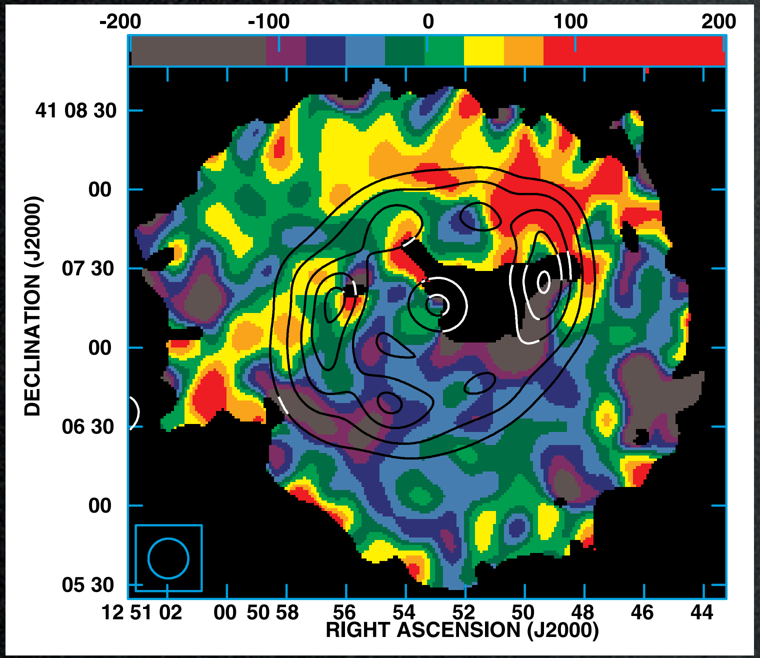




Find the best model

Fit a model to the data
Search for a statistically good fit





Chyzy & Buta 2007

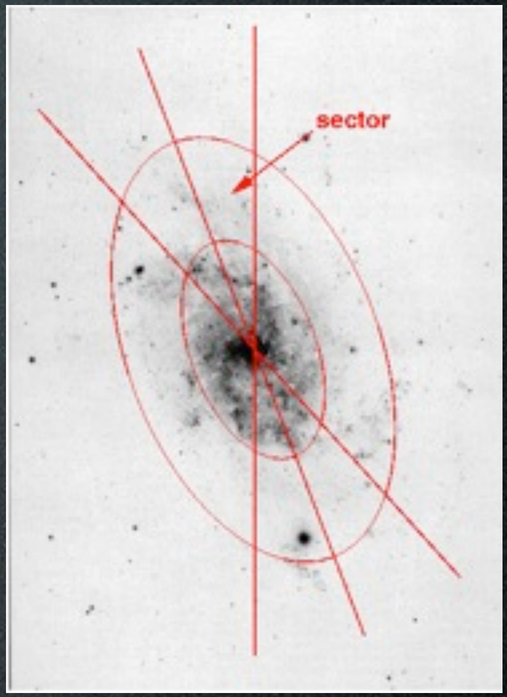
Polarization angle:

$$B_{\perp}(r, \phi)$$

Faraday rotation:

$$B_{||}(r, \phi)$$

Data at several λ, averaged in sectors



$$\psi = \psi_0(B_\perp) + \mathrm{RM}(B_\parallel)\lambda^2 + \mathrm{RM}_{\mathrm{fg}}\lambda^2$$

$$B_r = B_0 \sin(p_0) + B_1 \sin(p_1) \cos(\theta - \beta_1) + \ldots$$

$$B_\theta = B_0 \cos(p_0) + B_1 \cos(p_1) \cos(\theta - \beta_1) + \ldots$$

$$B_z = B_{z0} + B_{z1} \cos(\theta - \beta_{z1}) + \ldots$$

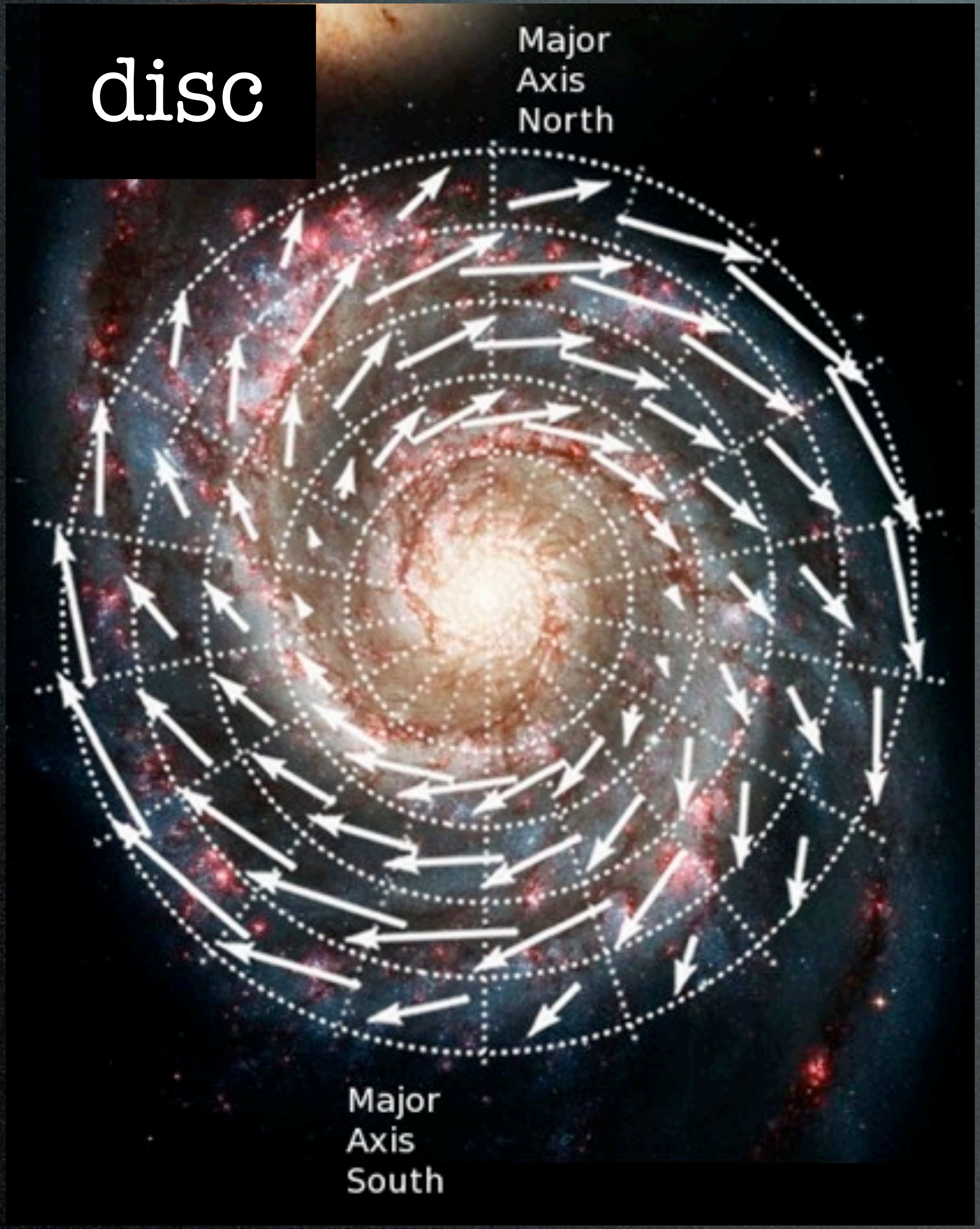
Seek statistically good fits for B_i, p_i, β_i by minimising weighted squared-difference between model and observations.

Two magneto-ionic layers in M51

Fletcher et al. 2011

would never be revealed by looking at RM map.

Two magneto-ionic layers in M51

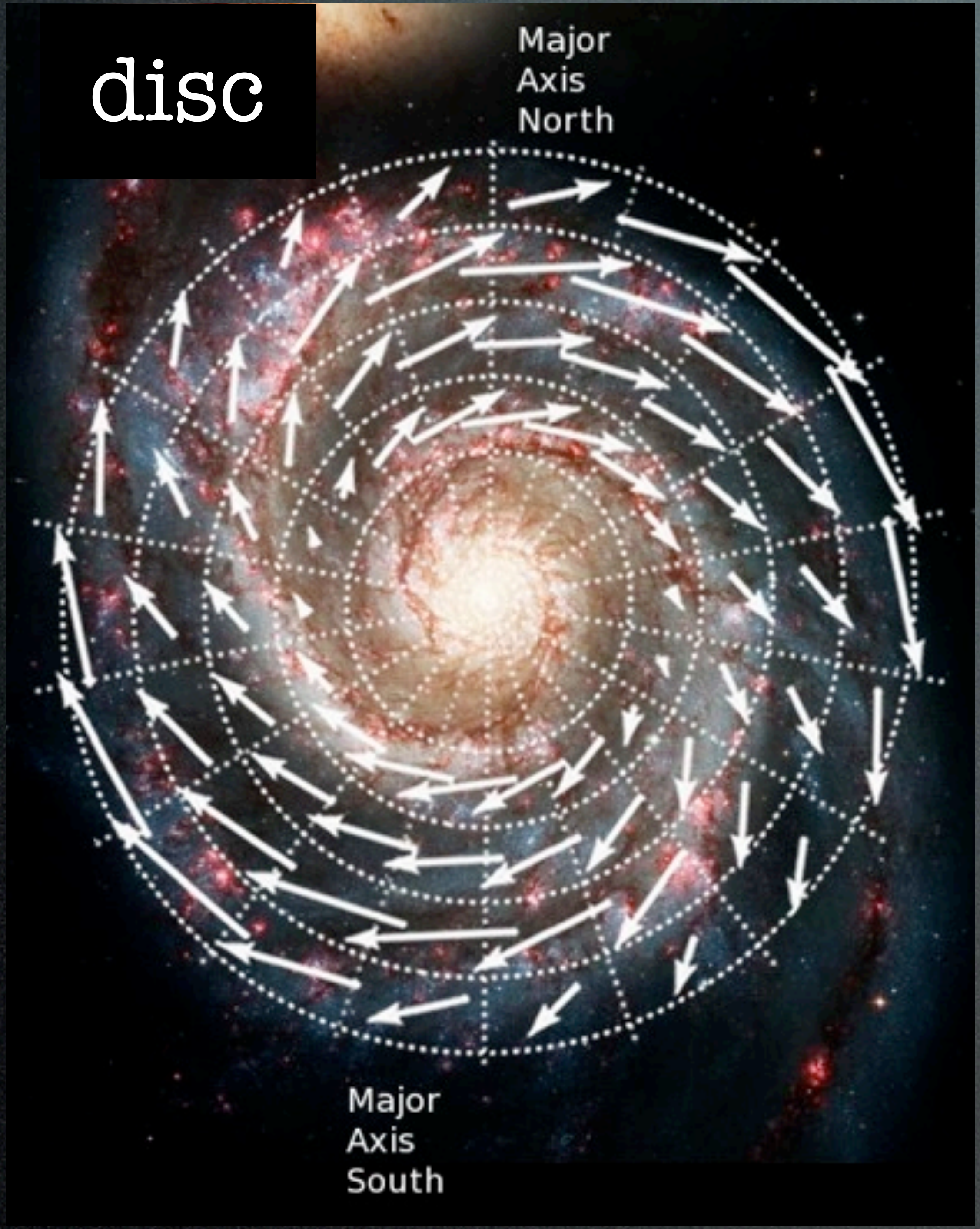


Fletcher et al. 2011

would never be revealed by looking at RM map.

Two magneto-ionic layers in M51

Fletcher et al. 2011



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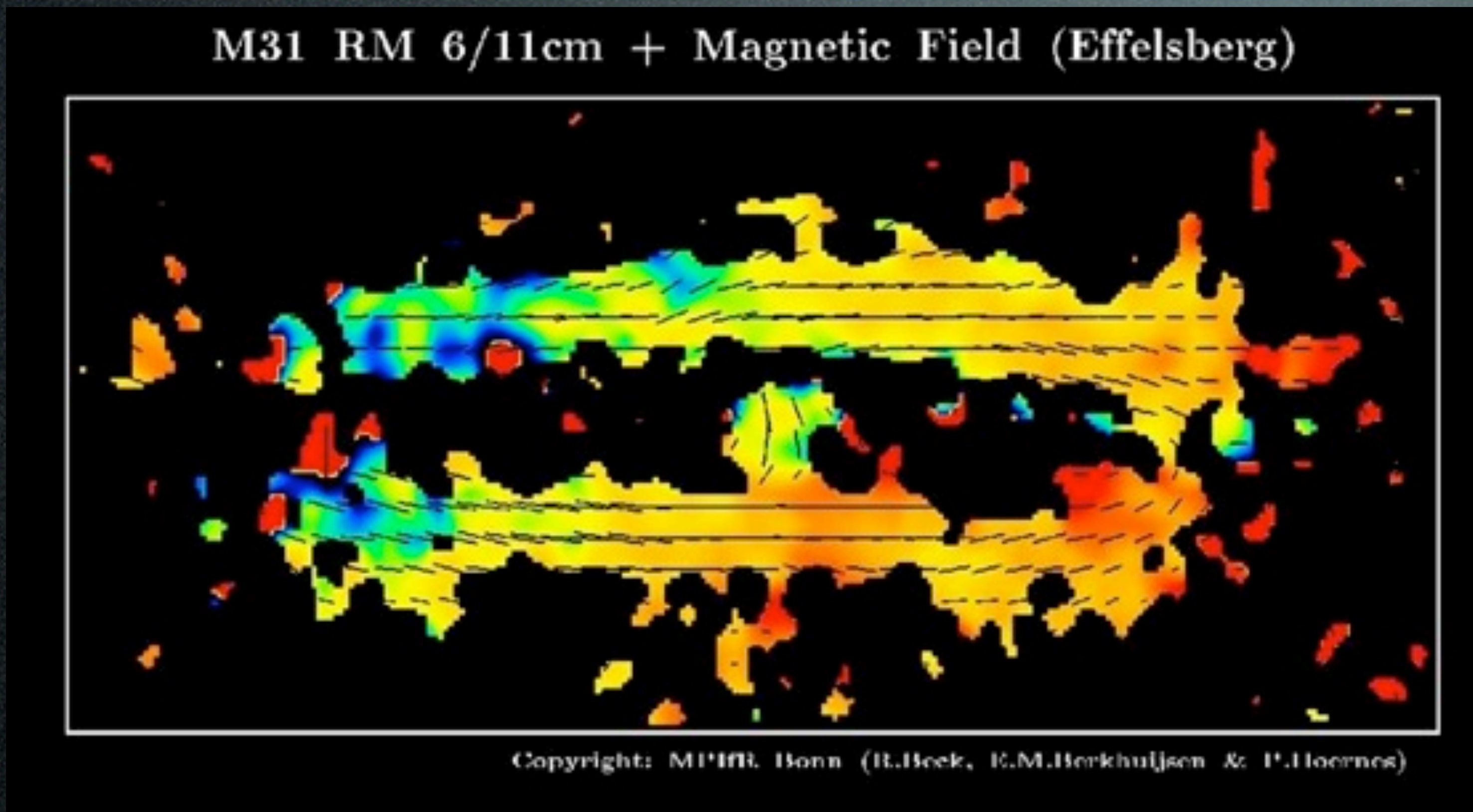
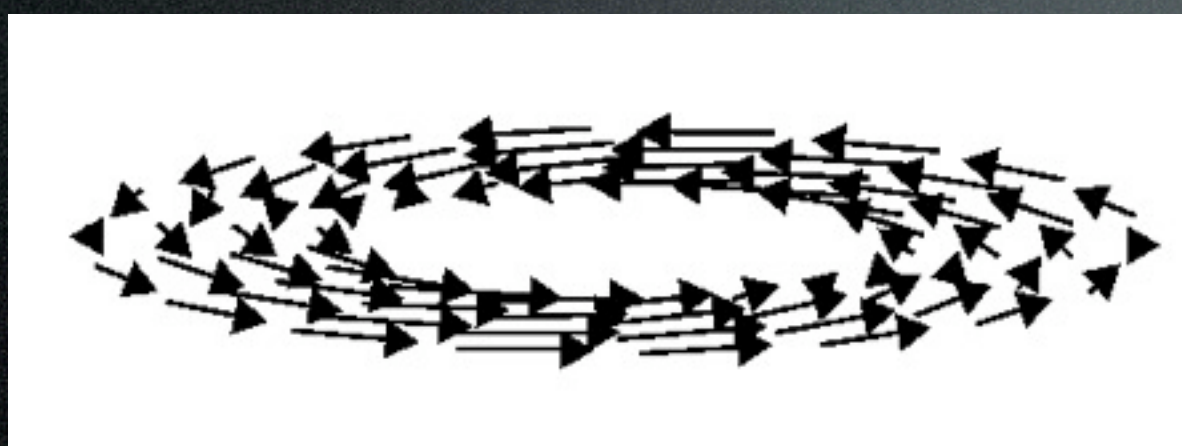
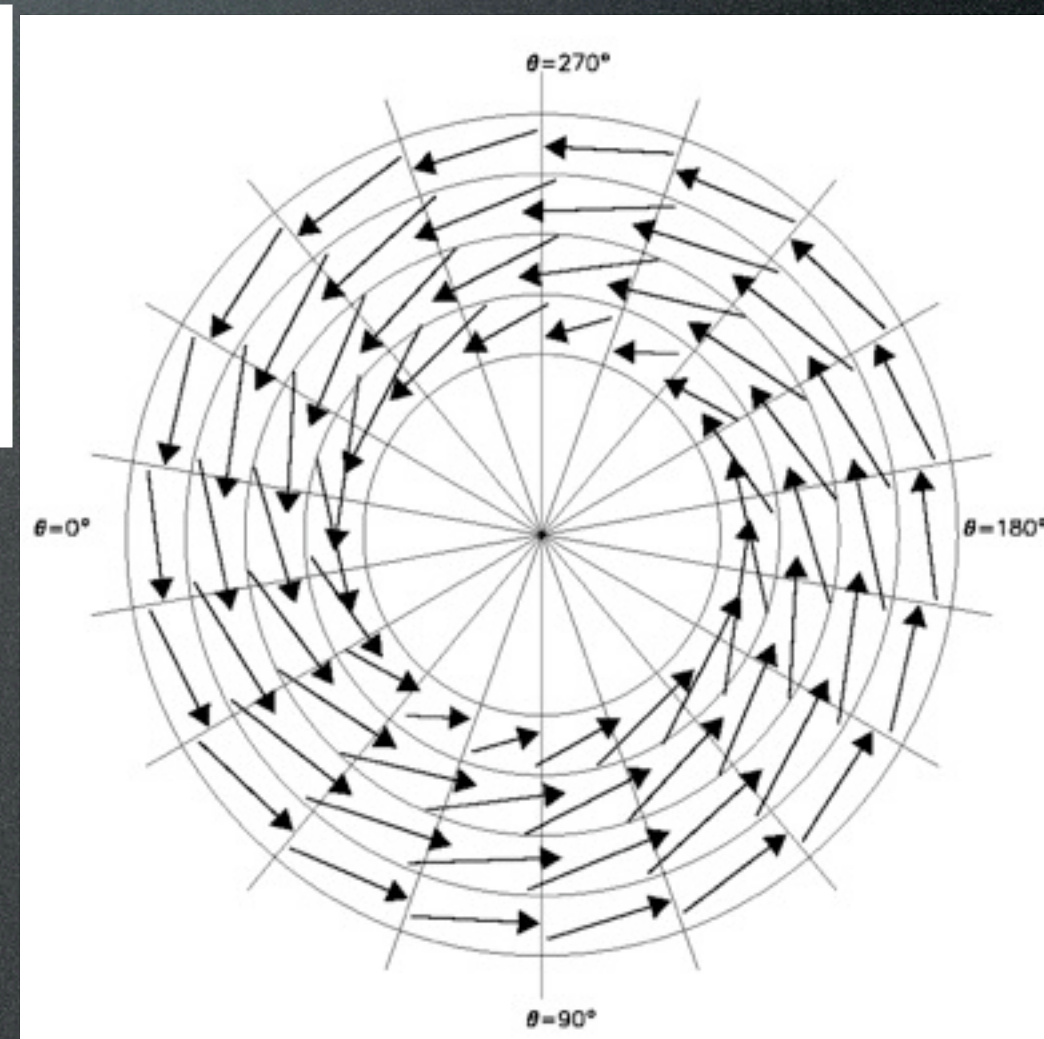


	Units	Radial range (kpc)			
		6–8	8–10	10–12	12–14
RM_{fg}	rad m^{-2}	−93 ±5	−99 ±5	−93 ±5	−89 ±4
R_0	rad m^{-2}	+83 ±7	+96 ±9	+115 ±9	+99 ±6
p_0	deg	−13 ±4	−19 ±3	−11 ±3	−8 ±3
R_2	rad m^{-2}	+45 ±10			
p_2	deg	−2 ±12			
β_2	deg	−43 ±7			
S		58	59	62	62
χ^2		63	63	65	65

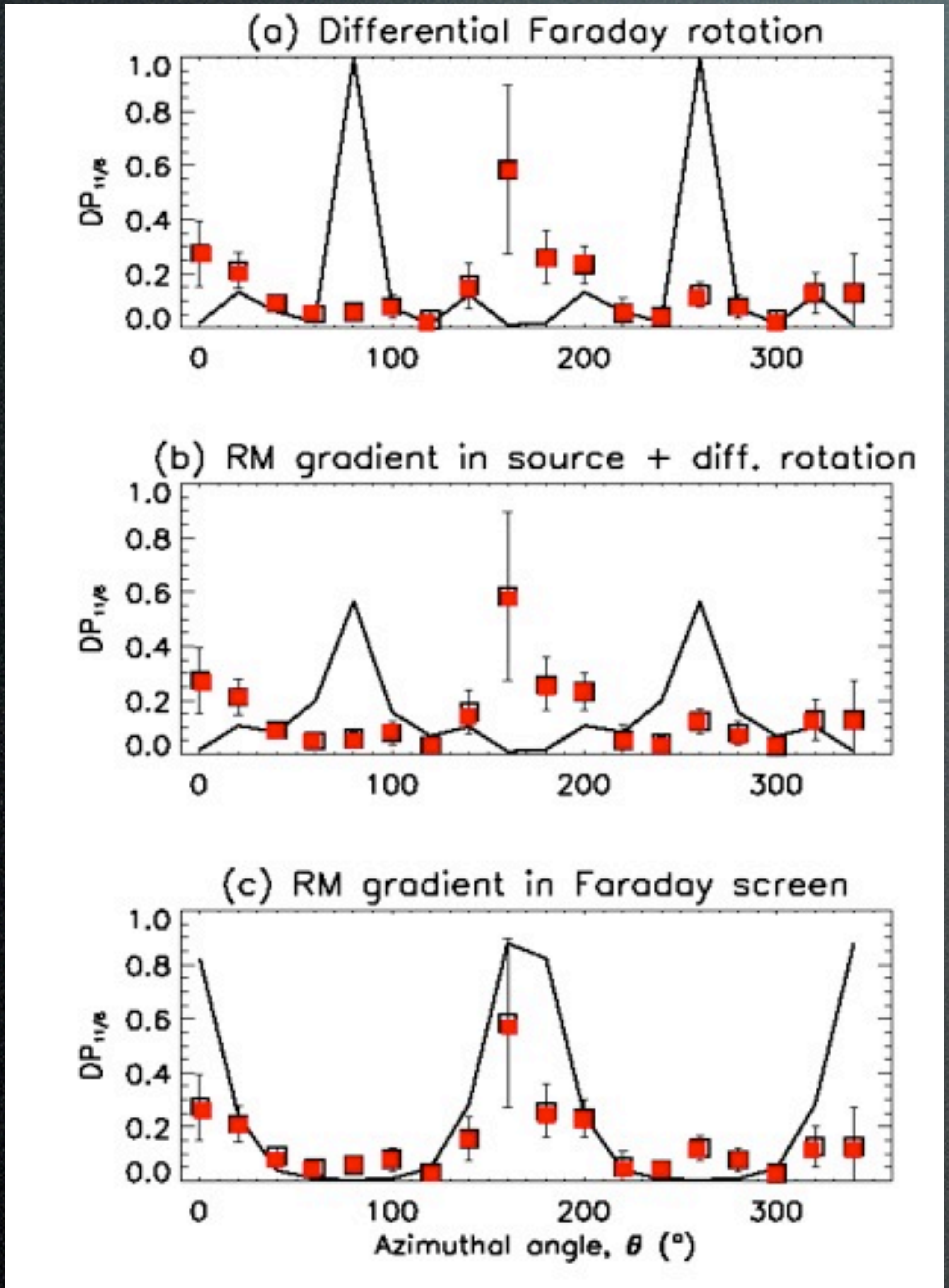
$$B_r = B_0 \sin(p_0) + B_1 \sin(p_1) \cos(\theta - \beta_1)$$

Fletcher et al. 2004





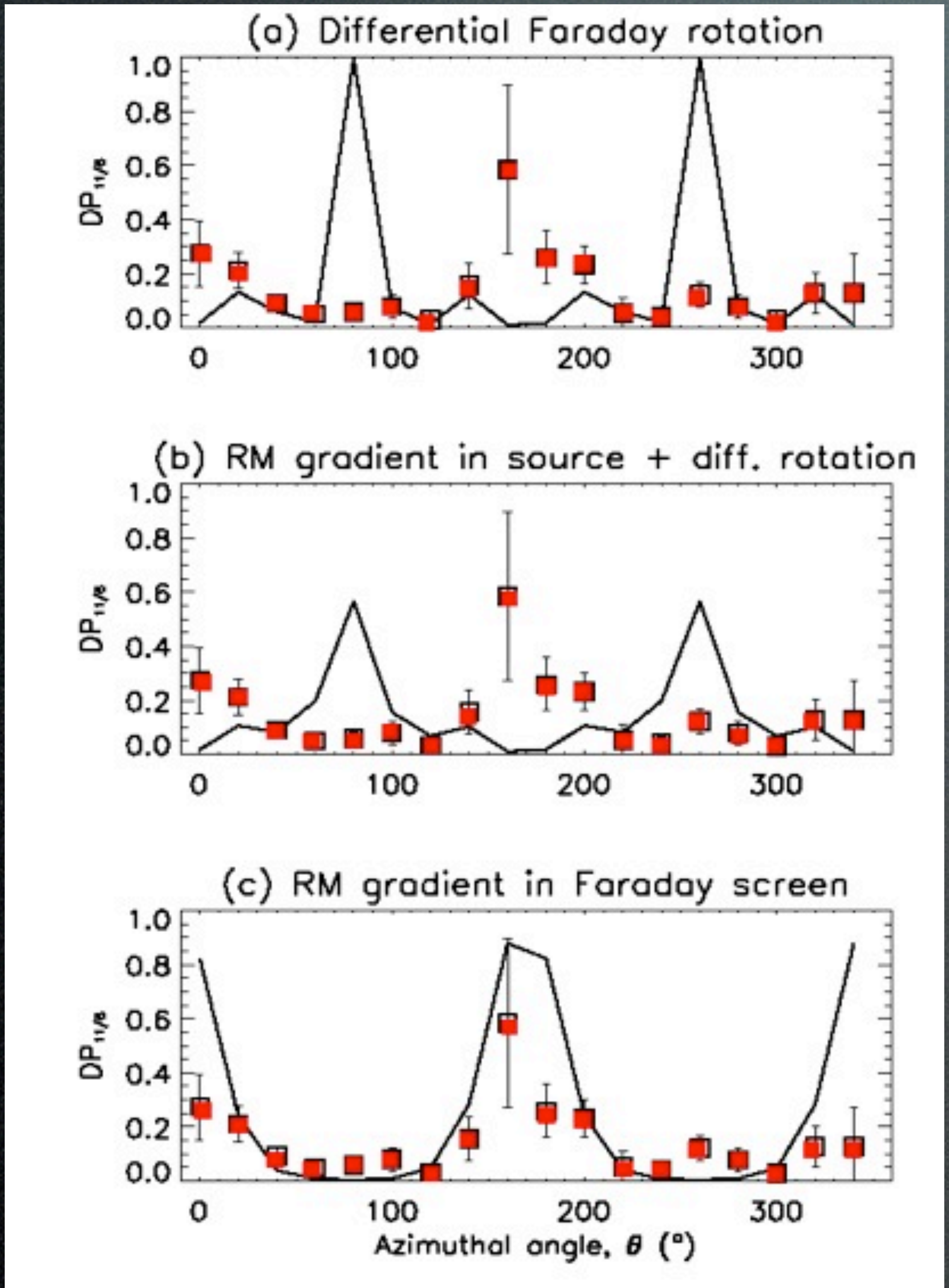
Even possible to fit a model to observed Faraday depolarization, consistent with modelled B field.

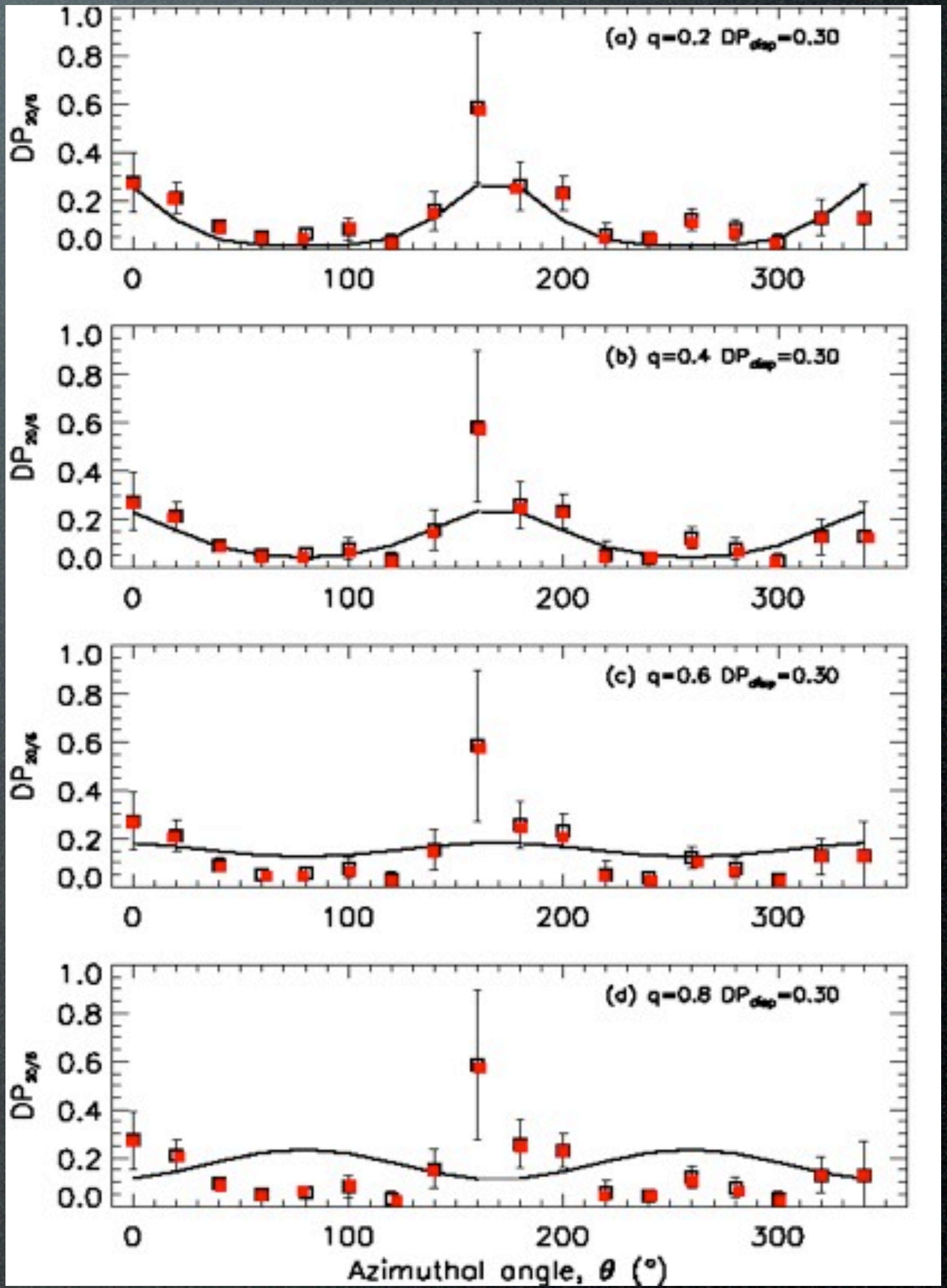


Fletcher et al. 2004

Even possible to fit a model to observed Faraday depolarization, consistent with modelled B field.

Fletcher et al. 2004



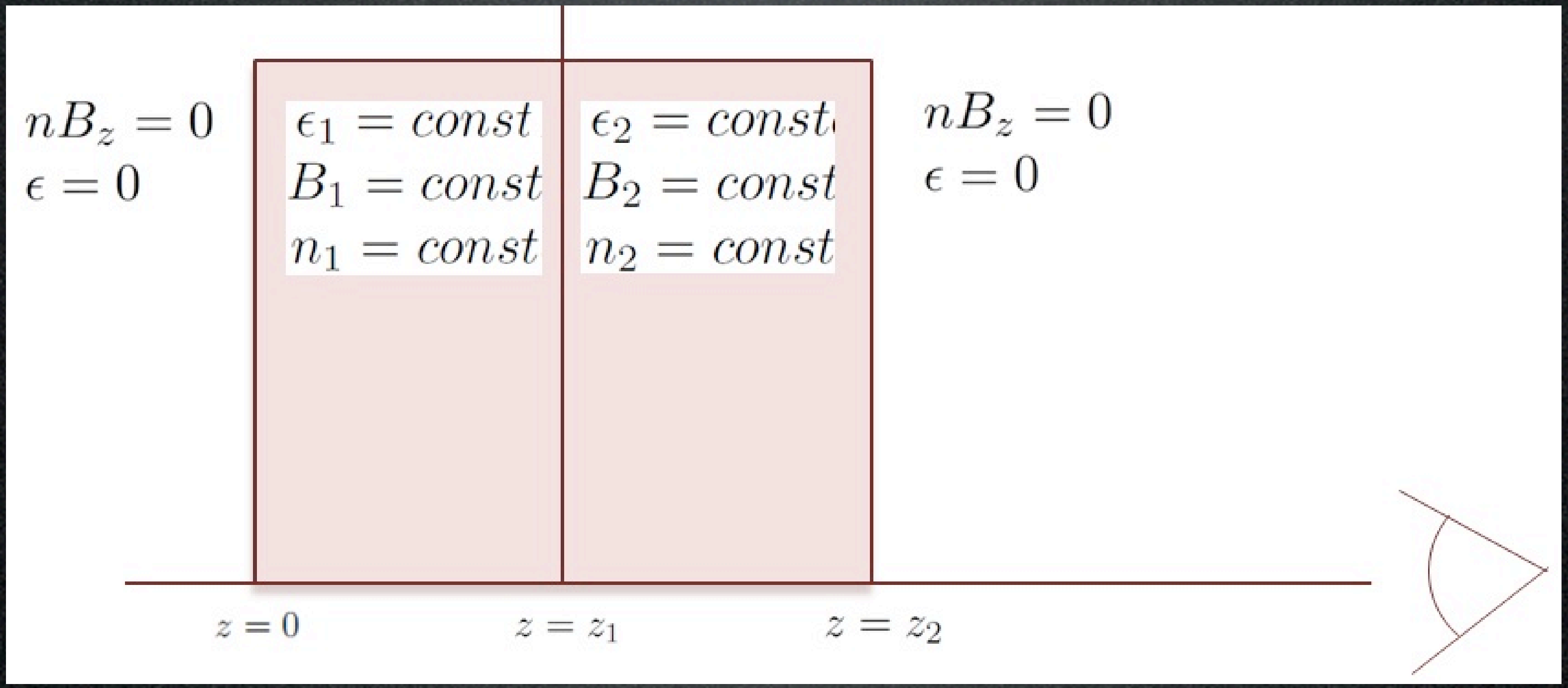


Components of a model

Faraday screens: modulation of background polarized emission in a Faraday rotating layer.

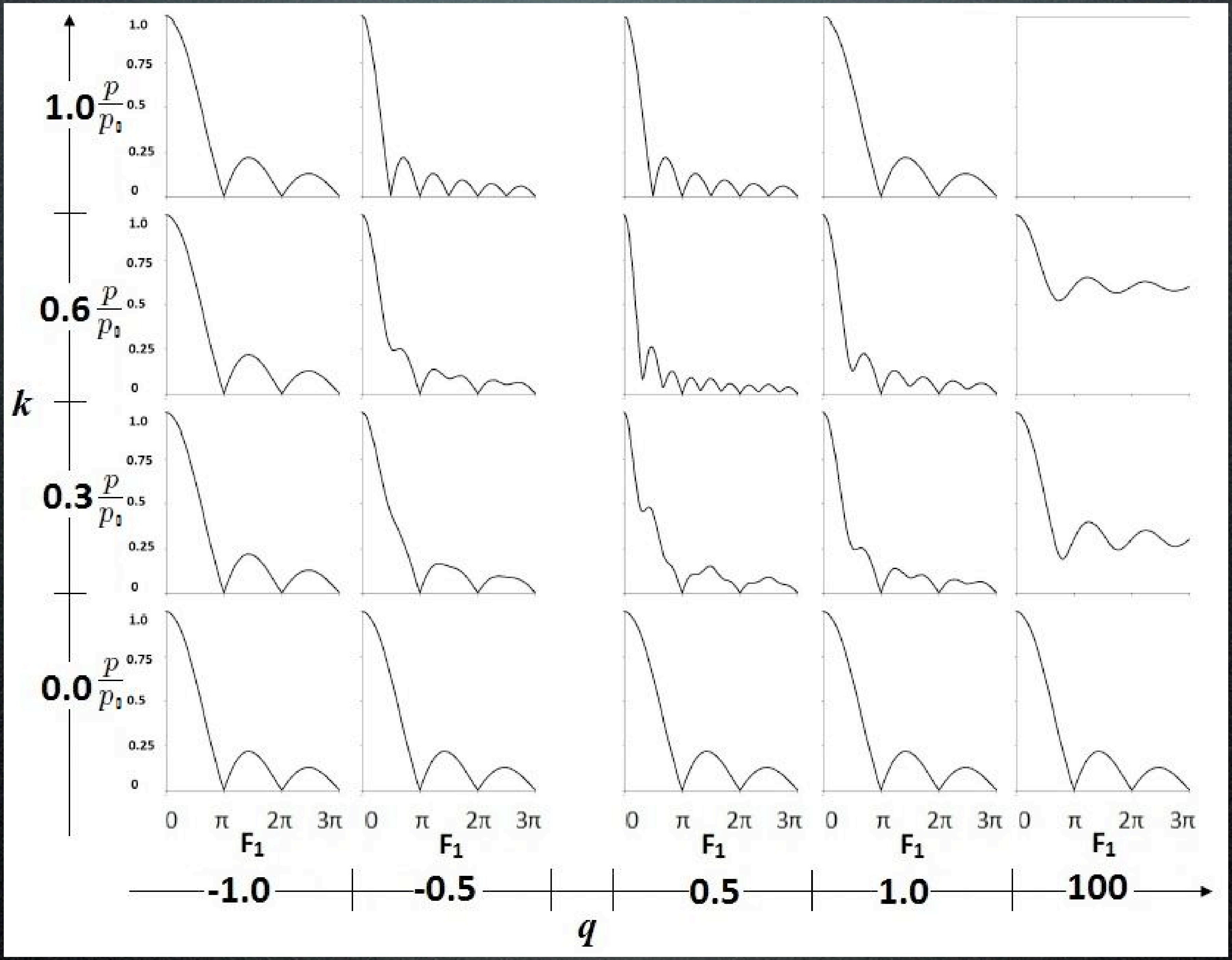
Q. Cosmic ray electron distribution inhomogeneous?
Q. What exactly is a Faraday screen?

Consider 2 uniform layers



% polarization as function of Faraday depth

varying synchrotron emissivity ratio



varying Faraday depth ratio of layers

Chadderton MMath dissertation 2011

Summary

1. Processing the data

    - resolution: best not high
    - filtering: averaging, wavelets

2. Analysis & interpretation

    a. take care with data selection
    b. resist "by eye" interpretation
    c. if practical fit a model to data