So You Think You Can Measure Magnetic Fields? Ionospheric effects on polarization

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Review: Measuring Astro B

- Three standard techniques for detecting magnetic fields
  - Zeeman splitting
  - Faraday rotation
  - Synchrotron emission

- All have in common:

Polarization Measurement
Ionosphere Basics

- Region of partially ionized gas at top of Earth's atmosphere
  - Ionization via solar UV & X-ray
  - Mostly O and N atoms & molecules
- Density & temperature increase with height
  - Density peaks ~300 km
- Threaded by Earth's dipole field
  - ~35-50 μtesla
  - Wildly varying dip angle
Plasmas and EM Wave Propagation

- The index of refraction $n$ can be expressed as

$$n^2 = \frac{1}{2} + \frac{(\alpha - 1)(\beta + 2\alpha - 1) \pm \alpha \beta \sqrt{\sin^4 \theta + 4(\alpha - 1)^2 \cos^2 \theta}}{(1 - \alpha - \beta + \alpha \beta \cos^2 \theta)}$$

- Where

$$\alpha = \left(\frac{\omega_p}{\omega}\right)^2; \quad \omega_p^2 = \frac{n_e e^2}{\varepsilon_0 m_e}; \quad \text{plasma frequency}$$

$$\beta = \frac{\omega_c}{\omega}; \quad \omega_c = \frac{eB}{m_e}; \quad \text{cyclotron frequency}$$

$n_e$: electron (number) density

$B$: magnetic flux density

$\theta$: angle of the wave vector relative to the magnetic field

- Synopsis

$$n \propto f(n_e, B, \theta)$$

- Plasmas are birefringent
  - Different polarizations travel at different speeds
Alternative Formulation of $n$

- The index of refraction $n$ can also be written to highlight the superposition of right and left hand modes

$$\tan^2 \theta = \frac{(n^2 - R)(n^2 - L)}{(\frac{R+L}{2} n^2 - RL)(n^2 - P)}$$

- Where

$$R = 1 - \alpha \left(\frac{1}{1 - \beta}\right); \quad \text{Right-hand circularly polarized wave}$$

$$L = 1 - \alpha \left(\frac{1}{1 + \beta}\right); \quad \text{Left-hand circularly polarized wave}$$

$$P = 1 - \alpha; \quad \text{Isotropic wave}$$
Alternative formulation of $n$

- The index of refraction $n$ can also be written to highlight the superposition of right and left hand modes

$$n^2 = \frac{2PS + (PS + 2LR)\tan^2 \theta \pm \sqrt{(2PD \sec \theta)^2 + (PS - 2LR)^2 \tan^4 \theta}}{2(2P + S \tan^2 \theta)}$$

- Where

$$R = 1 - \alpha \left( \frac{1}{1 - \beta} \right); \quad \text{Right-hand circularly polarized wave}$$

$$L = 1 - \alpha \left( \frac{1}{1 + \beta} \right); \quad \text{Left-hand circularly polarized wave}$$

$$P = 1 - \alpha; \quad \text{Isotropic wave}$$

$$S = L + R; \quad D = L - R$$
Limiting Case: $\parallel$ to Magnetic Field

- In the limit that $\theta$ goes to $0^\circ$
  \[ n^2 \rightarrow L, R \]
  - i.e. an EM wave propagates as pure right- and left-hand modes
- Recall a linearly polarized wave is just $L+R$
  \[ \begin{align*}
    E_L & + E_R = \uparrow \\
    E_L & + E_R = \downarrow 
  \end{align*} \]
- The different $L&R$ phase speeds lead to Faraday rotation
  \[ \begin{align*}
    \alpha_{FR} & = \frac{1}{2} \int \Delta k \, ds = \frac{1}{2} \frac{\omega}{c} \int (n_R - n_L) \, ds \\
    & \approx \frac{C}{\omega^2} \int n_e \bar{B} \cdot d\bar{s}
  \end{align*} \]
Limiting Case: \( \perp \) to Magnetic Field

- In the limit that \( \theta \) goes to 90°
  \[
  n^2 \rightarrow \frac{2RL}{R + L}, P
  \]
  - i.e. two linear modes (x- and o-mode)

- Different phase speeds lead to elliptization
  - Known as Cotton-Mouton effect
  - 2\textsuperscript{nd} order approximation:
    \[
    \alpha_{CM} \approx \frac{C}{\omega^3} \int n_e (\vec{B} \times \hat{k})^2 \, ds
    \]
  - Goes as \( \omega^3, \sin^2 \theta \)
Refractive (Ray Bending) Effects

- Different polarizations take different paths
  - Significant ray bending at low frequencies
Ionospheric Effects: Faraday Rotation

- FR becomes large in ionosphere below ~100MHz
  - Especially for thick ionospheres
  - TEC: Total Electron Content (integrated density along line of sight, $\int n_e ds$)
    - [1 TECU $\equiv 10^{16}$ m$^{-2}$]
- BUT: Typical rotation measures are small: < 2rad/m$^2$
  - Small compared to astro
Ionospheric Effects: Ellipticity

- $CM$ is much weaker than Faraday rotation
  - Due to $\omega^3, \sin^2 \theta$
  - Can be important if there is a region of near-normal incidence
Ionospheric Effects: Refraction/Ray Bending

- Detected radiation in different polarizations samples different parts of the ionosphere
  - Will likely have different phases
  - 10MHz below to emphasize effect
- Leads to both polarization rotation and elliptization
Interferometers and the Ionosphere

- Since both FR and CM are relatively weak – why bother?
- Interferometers make differential measurements
- Earth’s B in Barcelona and Dwingaloo are different
  - Dip angle, $\theta_{\text{Bar}} \sim 55^\circ$, $\theta_{\text{Dwi}} \sim 67^\circ$
  - $\sim 15\%$ difference in FR
  - Can be compensate for: Earth field is known and stable

- More importantly: The ionospheric density is both non-uniform and dynamic
  - Structures of order $>100\text{m}$
  - Variations of $10\%$; $>100\%$ on large (100km) scales
  - Fluctuation time 10s of seconds to hours
Motivational Slide:

- HF/VHF arrays (VLA, LOFAR, LWA) are extremely sensitive to ΔTEC
  - VLA probes ΔTEC variations to ~100 m, ~1 min, over 20° FoV
**Ionosphere Problem @ Long Wavelengths**

- Ionospheric effects severely limit resolution & sensitivity
- Current radio interferometers are in regimes 1 & 3.
- LOFAR, etc. are regime 4
  - Spatial variations in the ionosphere across each station beam distort the image

- Each station sees a different direction-dependent blur.
Consequences for Polarization Measurements

- Density fluctuations (spatial and temporal) are primary concern

- Faraday rotation:
  - Different stations will measure different polarizations
  - Variations of 10s of degrees @ 100MHz
    - Looking for changes of 2°/kHz
  - Can change rapidly

- Elliptization:
  - Likely not a problem - unless you’re unlucky
  - In Europe, elevations around 35° elevation, towards north

- Refraction:
  - Likely only an issue below 50MHz
  - Depending on complexity of ionosphere, could destroy polarization
Compensation Approaches

- **Calibrator sources with known, strong polarization**
  - Few known at low frequencies (<300MHz)
  - Must be near source of interest

- **Model ionosphere**
  - Erickson, *et al.* used GPS satellites to measure and correct to better than 4°

- **Use telescope data**
  - Electron density is the critical unknown
  - Work being done on calibration schemes to reconstruct ionosphere
  - LWA plans to use a rapidly scanning calibrator beam
    - 100 sources in 10s
Conclusions

- The ionosphere is a hindrance to polarization (magnetic field) measurements at low frequency
- Density inhomogeneities are primary concern
- Standard correction techniques (calibrator stars) may be insufficient
  - Ionospheric reconstruction may help, if density can be measured sufficiently accurately
  - Personal bias: Someone needs to take a lead in this effort!
- Know Your Limitations!