So You Think You Can Measure Magnetic Fields?

Ionospheric effects on polarization

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UNM

Review: Measuring Astro B

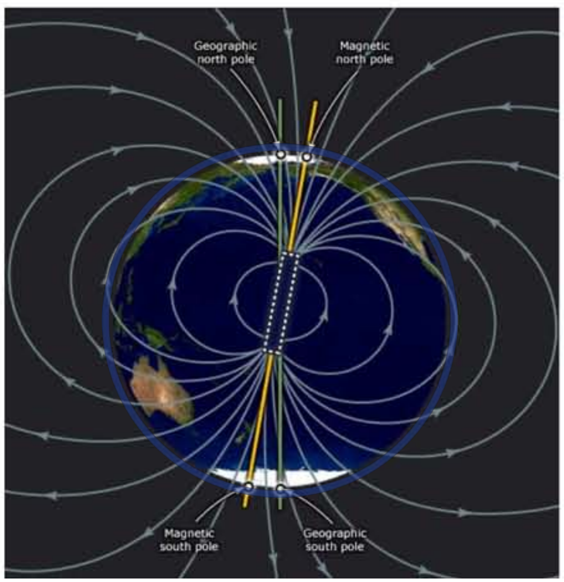
- Three standard techniques for detecting magnetic fields
  - Zeeman splitting
  - Faraday rotation
  - Synchrotron emission

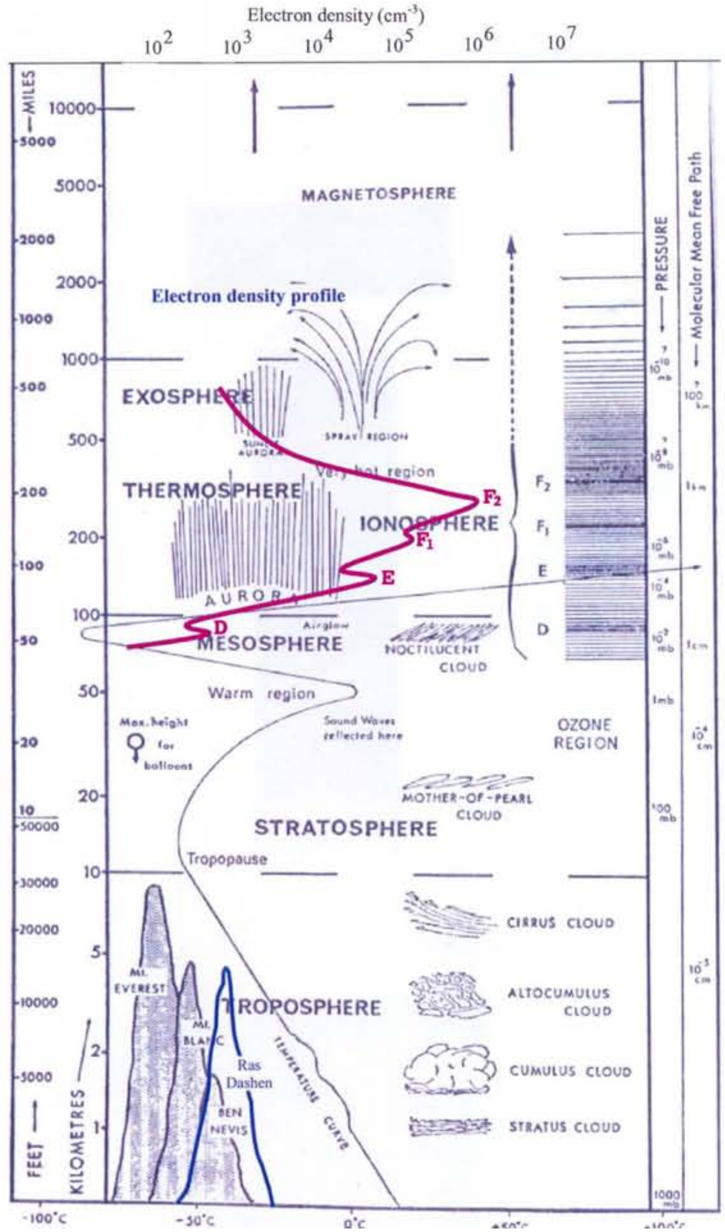
- All have in common:

Polarization Measurement

Ionosphere Basics

- Region of partially ionized gas at top of Earth's atmosphere
  - Ionization via solar UV & X-ray
  - Mostly O and N atoms & molecules
- Density & temperature increase with height
  - Density peaks ~300 km
- Threaded by Earth's dipole field
  - ~35-50 μtesla
  - Wildly varying dip angle





Plasmas and EM Wave Propagation

- The index of refraction *n* can be expressed as

$$n^2 = \frac{1}{2} + \frac{(\alpha - 1)(\beta + 2\alpha - 1) \pm \alpha\beta\sqrt{\sin^4\theta + 4(\alpha - 1)^2 \cos^2\theta / \beta}}{(1 - \alpha - \beta + \alpha\beta\cos^2\theta)}$$

  - Where

$$\alpha = \left(\frac{\omega_p}{\omega}\right)^2; \quad \omega_p^2 \equiv \frac{n_e e^2}{\varepsilon_0 m_e}; \text{ plasma frequency}$$

$$\beta = \frac{\omega_c}{\omega}; \quad \omega_c = \frac{eB}{m_e}; \quad \text{cyclotron frequency}$$

n_e : electron (number) density

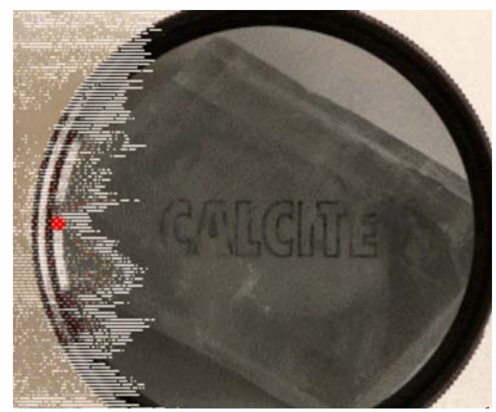
B : magnetic flux density

θ : angle of the wave vector relative to the magnetic field

- Synopsis

$$n \propto f(n_e, B, \theta)$$

  - Plasmas are birefringent
    - Different polarizations travel at different speeds

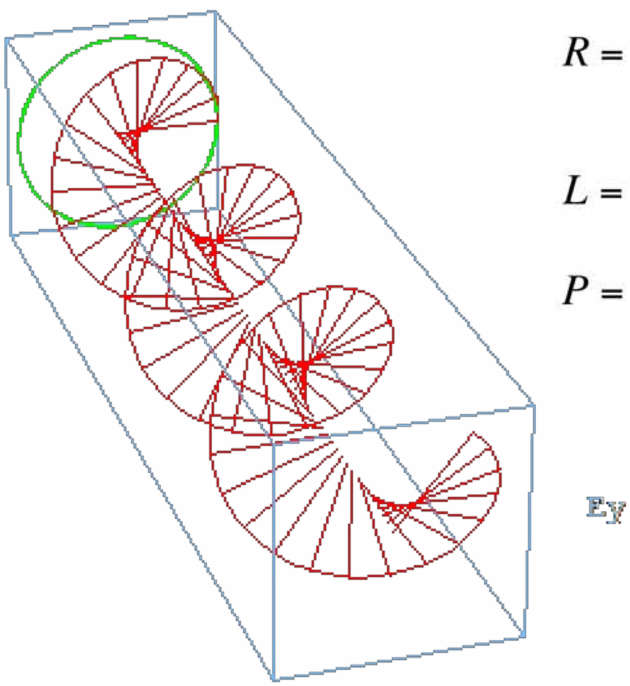


Alternative Formulation of n

- The index of refraction n can also be written to highlight the superposition of right and left hand modes

$$\tan^2\theta = \frac{(n^2 - R)(n^2 - L)}{(\frac{R+L}{2}n^2 - RL)(n^2 - P)}$$

- Where



$R = 1 - \alpha\left(\frac{1}{1-\beta}\right)$; Right-hand circularly polarized wave

$L = 1 - \alpha\left(\frac{1}{1+\beta}\right)$; Left-hand circularly polarized wave

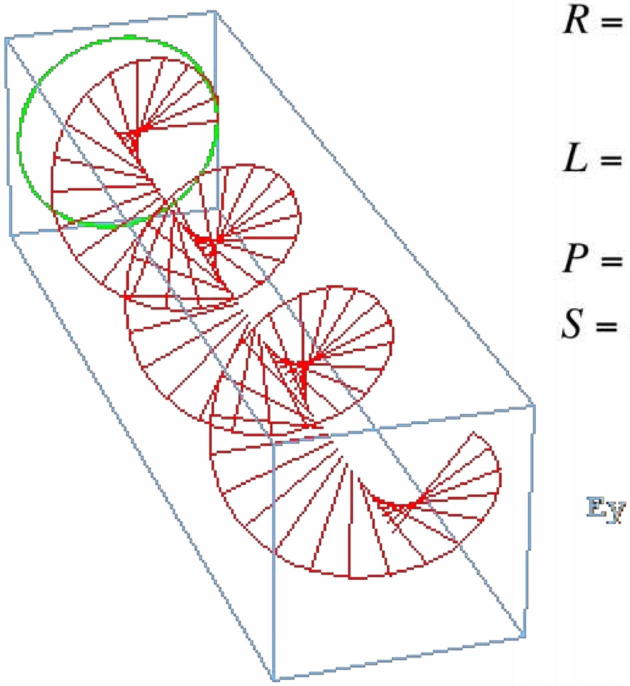
$P = 1 - \alpha$; Isotropic wave

Alternative formulation of n

- The index of refraction n can also be written to highlight the superposition of right and left hand modes

$$n^2 = \frac{2PS + (PS + 2LR)\tan^2\theta \pm \sqrt{(2PD\sec\theta)^2 + (PS - 2LR)^2\tan^4\theta}}{2(2P + S\tan^2\theta)}$$

  - Where



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$L = 1 - \alpha\left(\frac{1}{1+\beta}\right);$ Left-hand circularly polarized wave

$P = 1 - \alpha;$ Isotropic wave

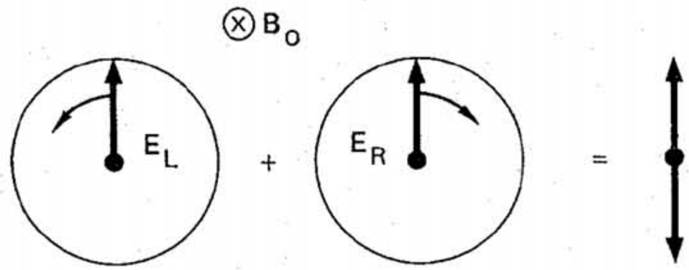
$S = L + R; \quad D = L - R$

Limiting Case: ‖ to Magnetic Field

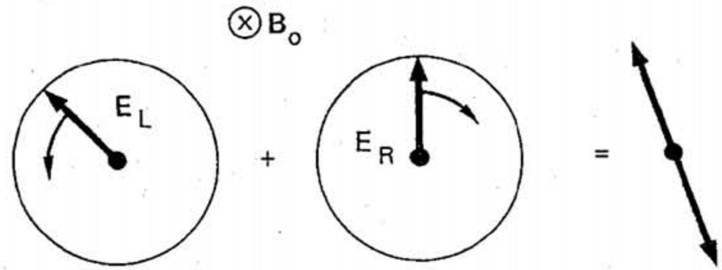
- In the limit that θ goes to 0°

$$n^2 \rightarrow L, R$$

  - i.e. an EM wave propagates as pure right- and left-hand modes
- Recall a linearly polarized wave is just L+R



- The different L&R phase speeds lead to Faraday rotation



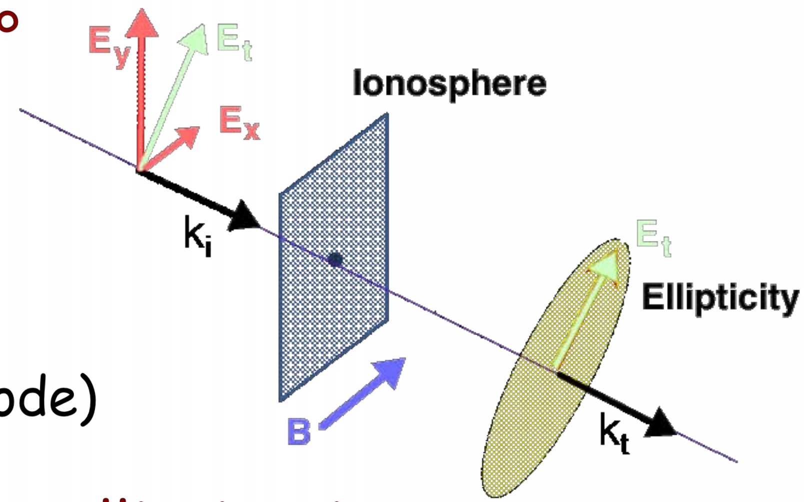
2nd order approximation

$$\alpha_{FR} = \frac{1}{2}\int \Delta k\, ds = \frac{1}{2}\frac{\omega}{c}\int (n_R - n_L)\, ds \approx \boxed{\frac{C}{\omega^2}\int n_e \vec{B}\cdot d\vec{s}}$$

Limiting Case: ⊥ to Magnetic Field

- In the limit that θ goes to 90°

$$n^2 \rightarrow \frac{2RL}{R+L}, P$$



  - i.e. two linear modes (x- and o-mode)

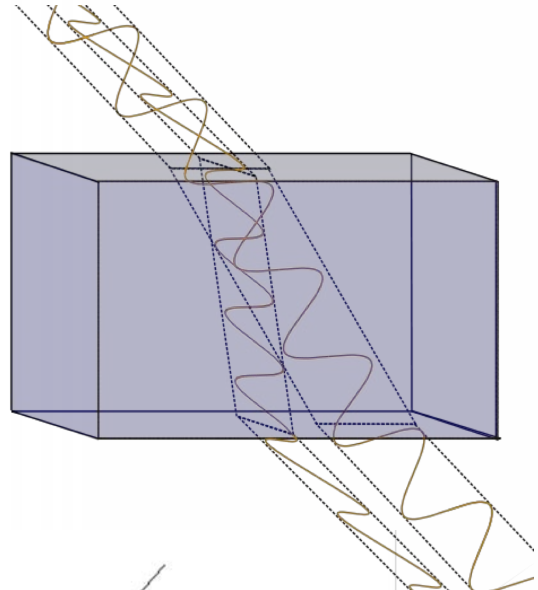
- Different phase speeds lead to elliptization
  - Known as Cotton-Mouton effect
  - 2nd order approximation:

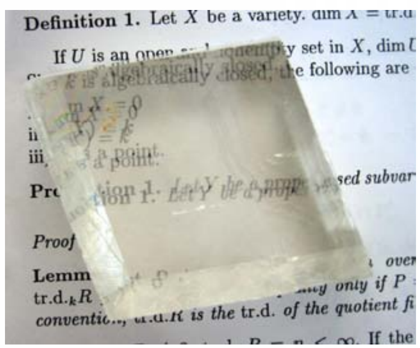
$$\alpha_{CM} \approx \frac{C}{\omega^3} \int n_e (\vec{B} \times \hat{k})^2 \, ds$$

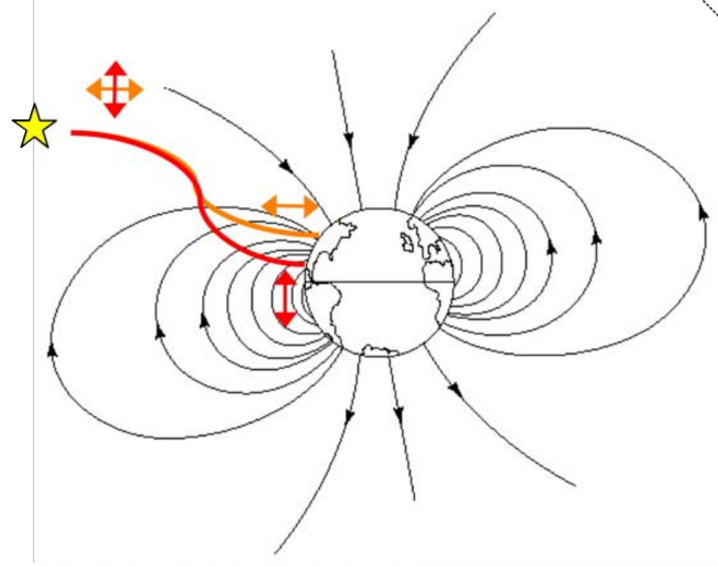
  - Goes as ω^3, $\sin^2\theta$

Refractive (Ray Bending) Effects

- Different polarizations take different paths
  - Significant ray bending at low frequencies

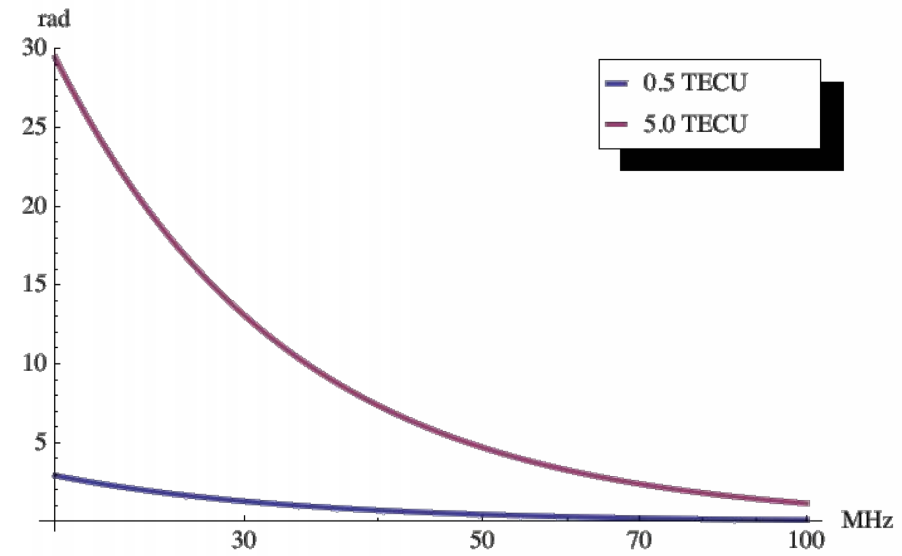


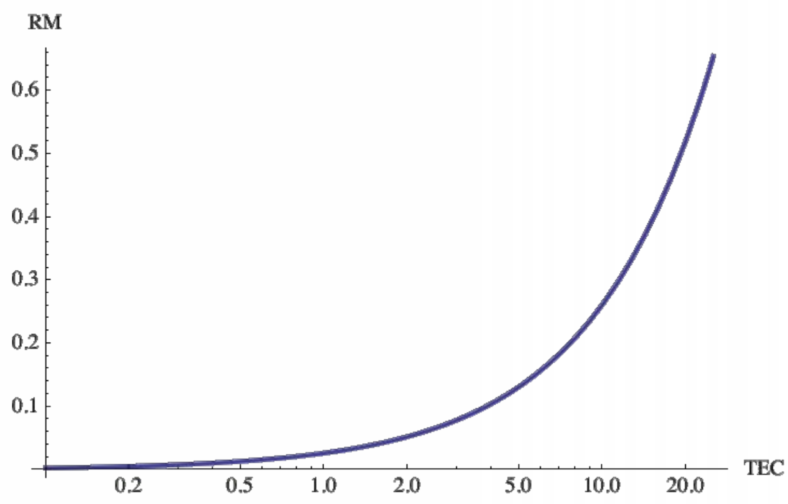


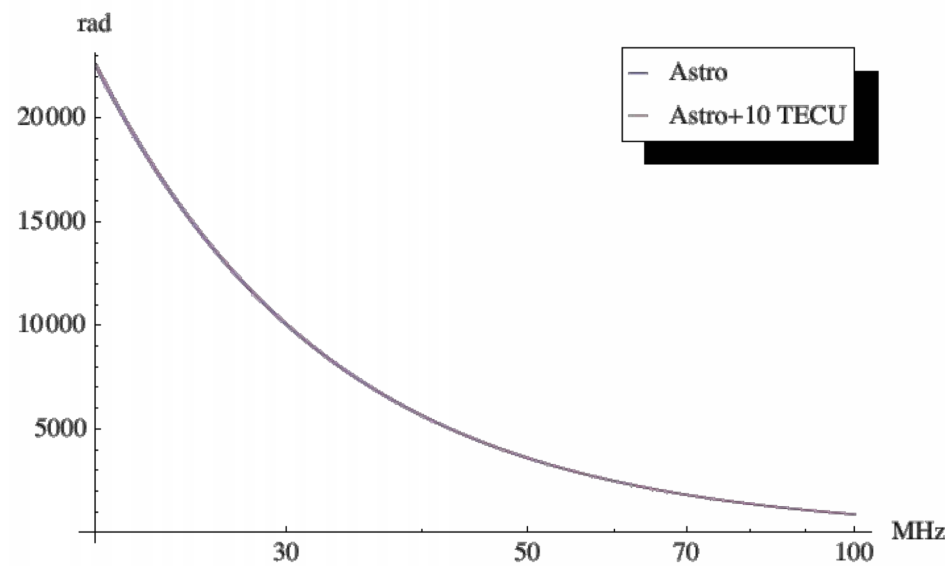


Ionospheric Effects: Faraday Rotation

- FR becomes large in ionosphere below ~100MHz
  - Especially for thick ionospheres
  - TEC: Total Electron Content (integrated density along line of sight, $\int n_e ds$)
    - [1 TECU ≡ 10^{16} m^{-2}]
- BUT: Typical rotation measures are small: < 2rad/m^2
  - Small compared to astro

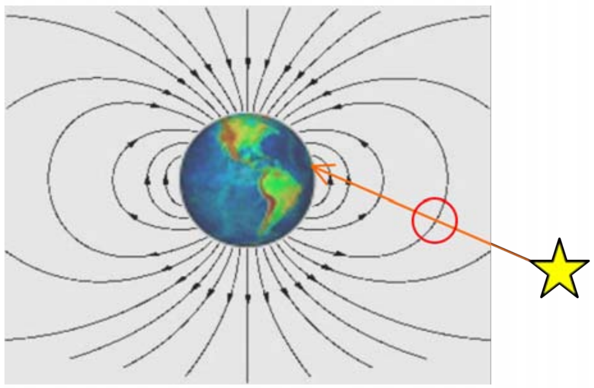






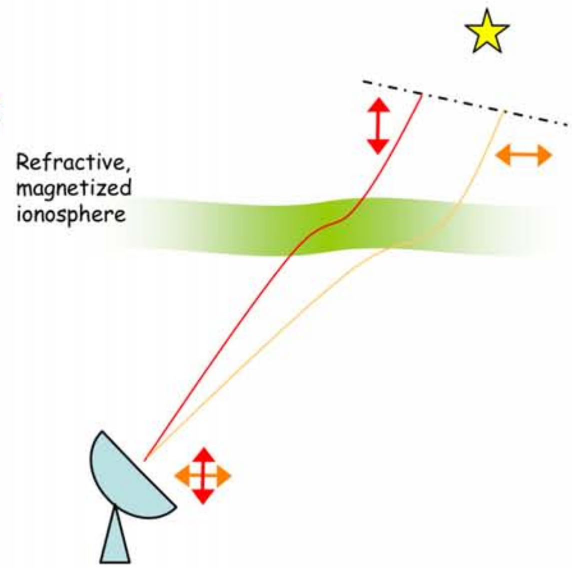
Ionospheric Effects: Ellipticity

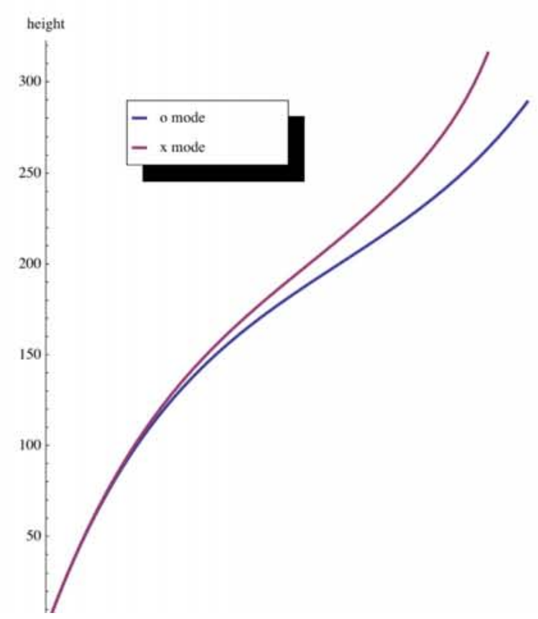
- CM is much weaker than Faraday rotation
  - Due to ω^3, $\sin^2\theta$
  - Can be important *if* there is a region of near-normal incidence



Ionospheric Effects: Refraction/Ray Bending

- Detected radiation in different polarizations samples different parts of the ionosphere
  - Will likely have different phases
  - 10MHz below to emphasize effect
- Leads to both polarization rotation and elliptization





Interferometers and the Ionosphere

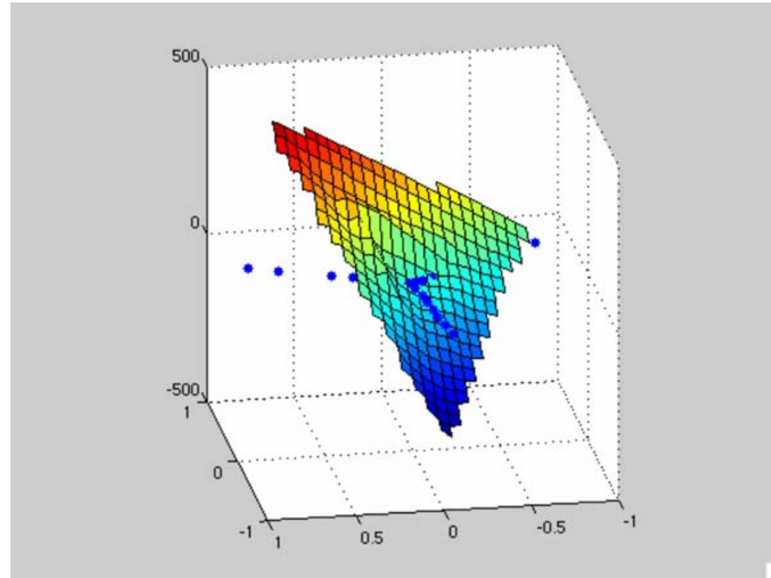
- Since both FR and CM are relatively weak – why bother?
- Interferometers make *differential* measurements
- Earth's B in Barcelona and Dwingaloo are different
  - Dip angle, θ_{Bar}~55°, θ_{Dwi}~67°
  - ~15% difference in FR
  - Can be compensate for: Earth field is known and stable

- More importantly: The ionospheric *density* is both non-uniform and dynamic
  - Structures of order >100m
  - Variations of 10%; >100% on large (100km) scales
  - Fluctuation time 10s of seconds to hours

Motivational Slide:

- HF/VHF arrays (VLA, LOFAR, LWA) are extremely sensitive to ΔTEC
  - VLA probes ΔTEC variations to ~100 m, ~1 min, over 20° FoV

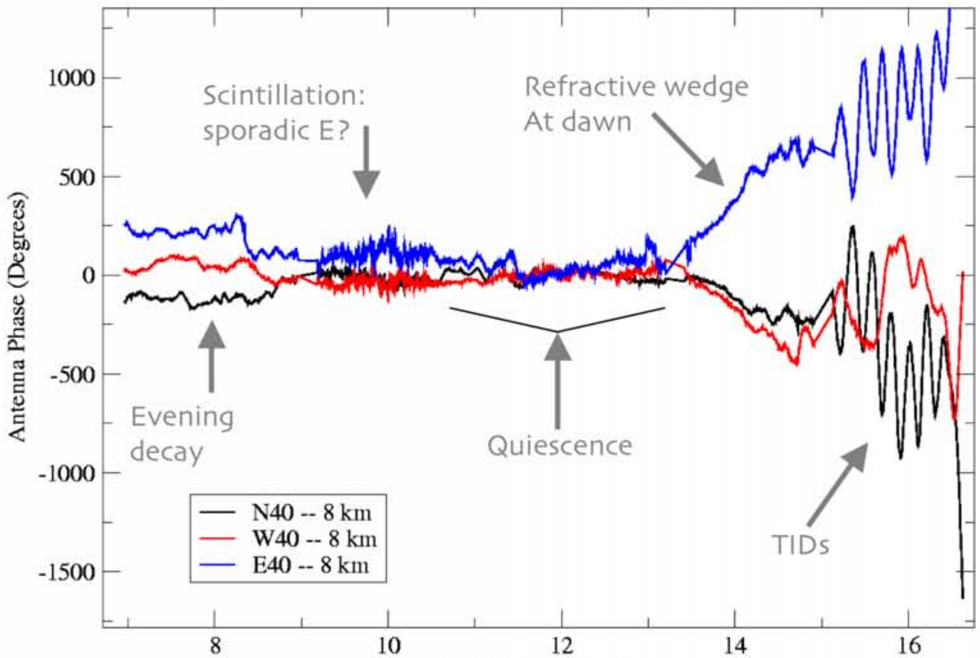
Δphase over VLA



+0.15 TECU

-0.15 TECU

19 Jan 2001
Mike Montgomery

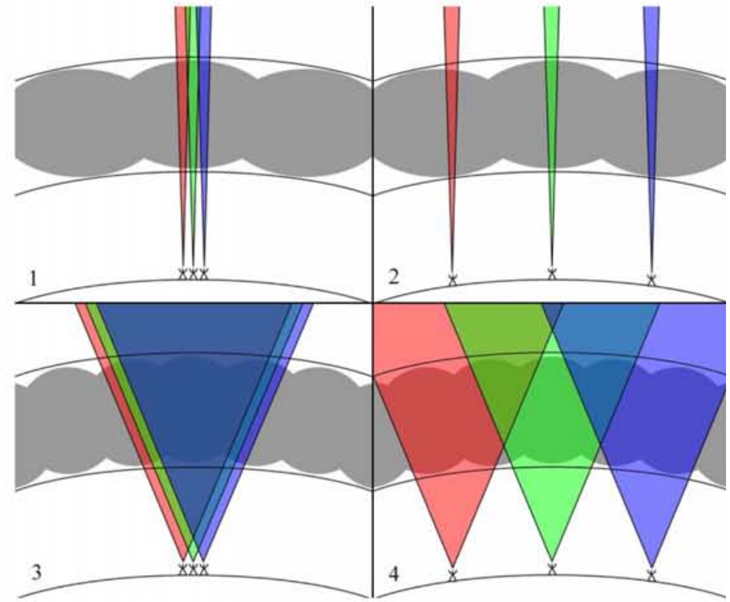


Kassim et al. 2007

NRAO

Ionosphere Problem @ Long Wavelengths

- Ionospheric effects severely limit resolution & sensitivity
- Current radio interferometers are in regimes 1 & 3.
- LOFAR, etc. are regime 4
  - Spatial variations in the ionosphere across each station beam distort the image



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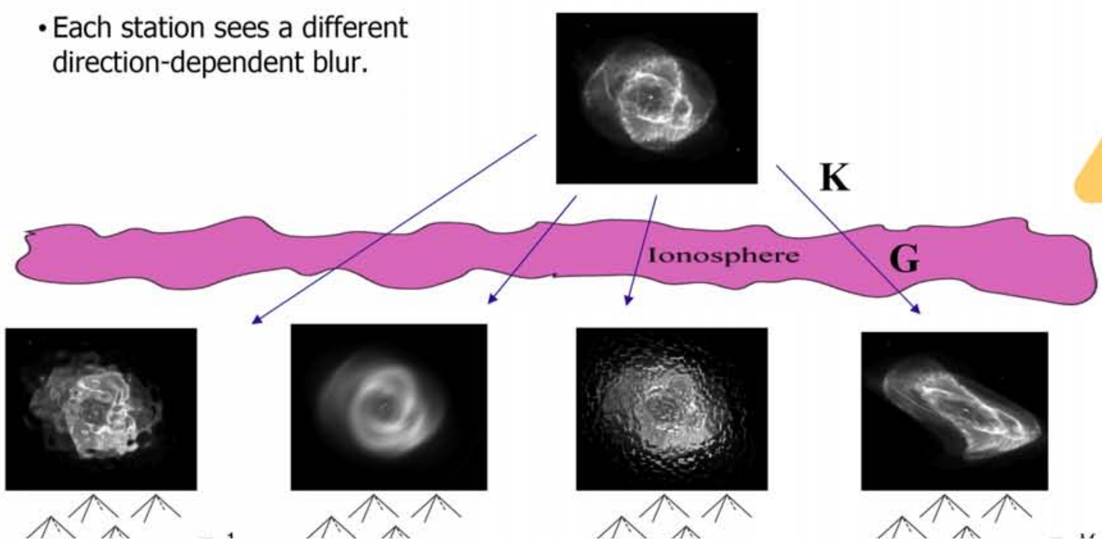
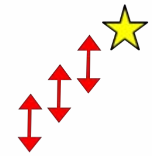


Consequences for Polarization Measurements

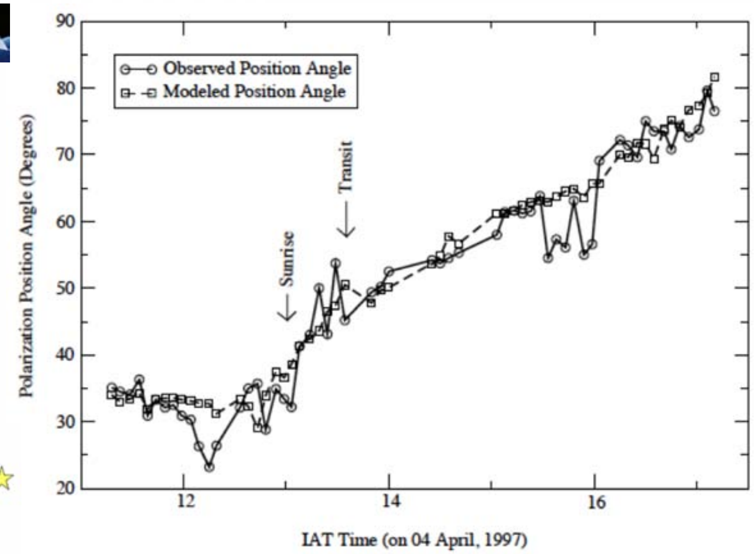
- Density fluctuations (spatial and temporal) are primary concern
- Faraday rotation:
  - Different stations will measure different polarizations
  - Variations of 10s of degrees @ 100MHz
    - Looking for changes of 2°/kHz
  - Can change rapidly
- Elliptization:
  - Likely not a problem – unless you're unlucky
  - In Europe, elevations around 35° elevation, towards north
- Refraction:
  - Likely only an issue below 50MHz
  - Depending on complexity of ionosphere, could destroy polarizaiton

Compensation Approaches

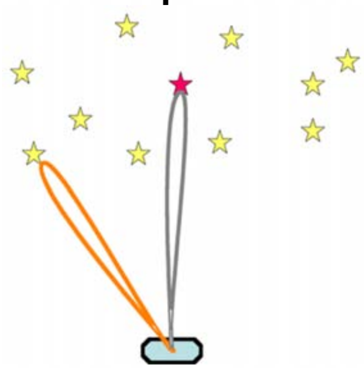
- Calibrator sources with known, strong polarization
  - Few known at low frequencies (<300MHz)
  - Must be near source of interest



- Model ionosphere
  - Erickson, *et al.* used GPS satellites to measure and correct to better than 4°



- Use telescope data
  - Electron density is the critical unknown
  - Work being done on calibration schemes to reconstruct ionosphere
  - LWA plans to use a rapidly scanning calibrator beam
    - 100 sources in 10s



Conclusions

- The ionosphere is a hindrance to polarization (magnetic field) measurements at low frequency
- Density inhomogeneities are primary concern
- Standard correction techniques (calibrator stars) may be insufficient
  - Ionospheric reconstruction may help, if density can be measured sufficiently accurately
  - *Personal bias: Someone needs to take a lead in this effort!*
- Know Your Limitations!