Impact of a priori gradients on VLBI- and GNSS-derived reference frames

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IERS Conventions

\[ D_L(e) = D_z \cdot m(e) = D_{zh} \cdot m_h(e) + D_{zw} \cdot m_w(e) \]

\[ + m_g \cdot [G_N \cdot \cos(a) + G_E \cdot \sin(a)] \]

gradients
Gradient mapping function $m_g$

- MacMillan 1995
  - goes back to Davis et al. 1993 ("wet refractivity")
  - $\cot(e) \cdot mf_h(e)$ (← singularity at horizon)

- Chen and Herring 1997
  - $\frac{1}{\tan(e) \cdot \sin(e) + C}$  \quad C = 0.0032

<table>
<thead>
<tr>
<th>Hydrostatic</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
</tr>
<tr>
<td>H</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wet</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
</tr>
<tr>
<td>H</td>
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</tbody>
</table>
“Conventional” approach

• Comparison with ray-traced delays shows no clear preference of one type
• Impact on station coordinates is small (< 1mm)
• We recommend to use the model by Chen and Herring (1997) with the coefficient $C = 0.0032$.
  – There is no singularity at the horizon.
  – Easier to implement.
  – Allows the comparability of different solutions.
A priori gradients

- VLBI Analysis Centers use mean a priori gradients determined from data of the Goddard Data Assimilation Office (DAO) by integration of vertical refractivity gradients
- DAO gradients are available at VLBI sites
- IGS ACs expressed interest in global model
A Priori Gradient model APG

- ECMWF 40 Years Re-Analysis monthly mean pressure level data
  - horizontal resolution of 5°
- Asymmetric delays towards north/east at e=5°
  - determined by ray-tracing
- North and east gradients
  - using Chen and Herring with C = 0.0032
- Average over all 12 months
East gradients from the ECMWF averaged over 12 months, 5° x 5° resolution
North gradients from the ECMWF averaged over 12 months, 5° x 5° resolution
Spherical harmonics expansion
up to degree and order 9
Residual north gradients
ray-traced gradients minus model
GPS analysis by CODE

• Bernese network solution from 2007 to 2008
• Orbits/EOPs/station coordinates estimated together
• 3° cutoff elevation angle, down-weighting with $\cos^2z$
• No constraints on 24 h piecewise linear gradients
APG versus GPS-derived mean north gradients

GPS (C = 0.0032)

APG
APG versus GPS-derived mean north gradients

GPS (C = 0.0032)

APG

smoothed
GPS: mean coordinate differences
With / without estimation of gradients

<table>
<thead>
<tr>
<th>Sol.</th>
<th>A priori gradients</th>
<th>Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>II.</td>
<td>no</td>
<td>Chen&amp;Herring (C = 0.0032)</td>
</tr>
</tbody>
</table>

Δ north in mm

Δ up in mm
## GPS: mean coordinate differences

Does APG help?

<table>
<thead>
<tr>
<th>A priori gradients</th>
<th>Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. APG</td>
<td>no</td>
</tr>
<tr>
<td>II. no</td>
<td>Chen&amp;Herring (C = 0.0032)</td>
</tr>
</tbody>
</table>

'Overcorrection' at some latitudes.
• **APG** are mostly larger than GPS-derived north gradients.

• Possible reasons:
  – $C = 0.0032$ is too large
    • (0.0007 helps only a bit, makes the gradients more “wet”)
  – Other effects on GPS gradients? Cutoff angle or down-weighting?
  – Error in NWM or ray-tracer?
APG vs. DAO

MacMillan and Ma, 1997

vertical integration of refractivity gradient

ray-trace at 5° elevation and spherials 9/9
APG versus DAO north gradients

DAO (determined locally from vertical integration)
APG (spherical harmonics expansion up to degree 9)
## VLBI global solutions with VieVS

<table>
<thead>
<tr>
<th>Sol.</th>
<th>a priori</th>
<th>estimated</th>
<th>absolute constraint</th>
<th>relative constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>zero</td>
<td>6 hours</td>
<td>no</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>APG fix</td>
<td>APG</td>
<td>no</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>APG est</td>
<td>APG</td>
<td>6 hours</td>
<td>0.5 mm</td>
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<td>DAO fix</td>
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</tbody>
</table>
North components w.r.t. reference solution

0.2 cm

DAO fix  APG fix
Up components w.r.t. reference solution

0.2 cm

DAO fix  APG fix
Up components w.r.t. reference solution

0.2 cm

DAO est  APG est
Summary

• APG larger than GPS-estimated gradients.
• DAO gradients agree better with VLBI-analysis than APG.
• A priori gradients are only of importance if constraints are applied.
Recommendations

• We recommend to use
  – the gradient mapping function by Chen and Herring with $C = 0.0032$ (for the sake of consistency)
  – DAO gradients for VLBI analysis
Thanks for your attention.
A priori and estimated gradients
(1990-2010, more than 20 sessions)
Contents

• Gradient mapping function
• A Priori Gradient model APG
• Comparison with DAO gradients
• Influence on terrestrial reference frame determined with GPS and VLBI
• Conclusions
Examples with ray-traced delays

Wettzell, 1 January 2008
azimuth = 90°
Examples with ray-traced delays

Wettzell, 1 January 2008
azimuth = 90°
Examples with ray-traced delays

Wettzell, 1 January 2008
azimuth = 90°
Examples with ray-traced delays

Wettzell, 1 January 2008

azimuth = 90°

C = 0.0060 to follow the observations
Examples with ray-traced delays

Wettzell, 1 January 2008
azimuth = 90°

C = 0.0060

Tsukuba, 12 August 2008
azimuth = 270°

C = −0.0030