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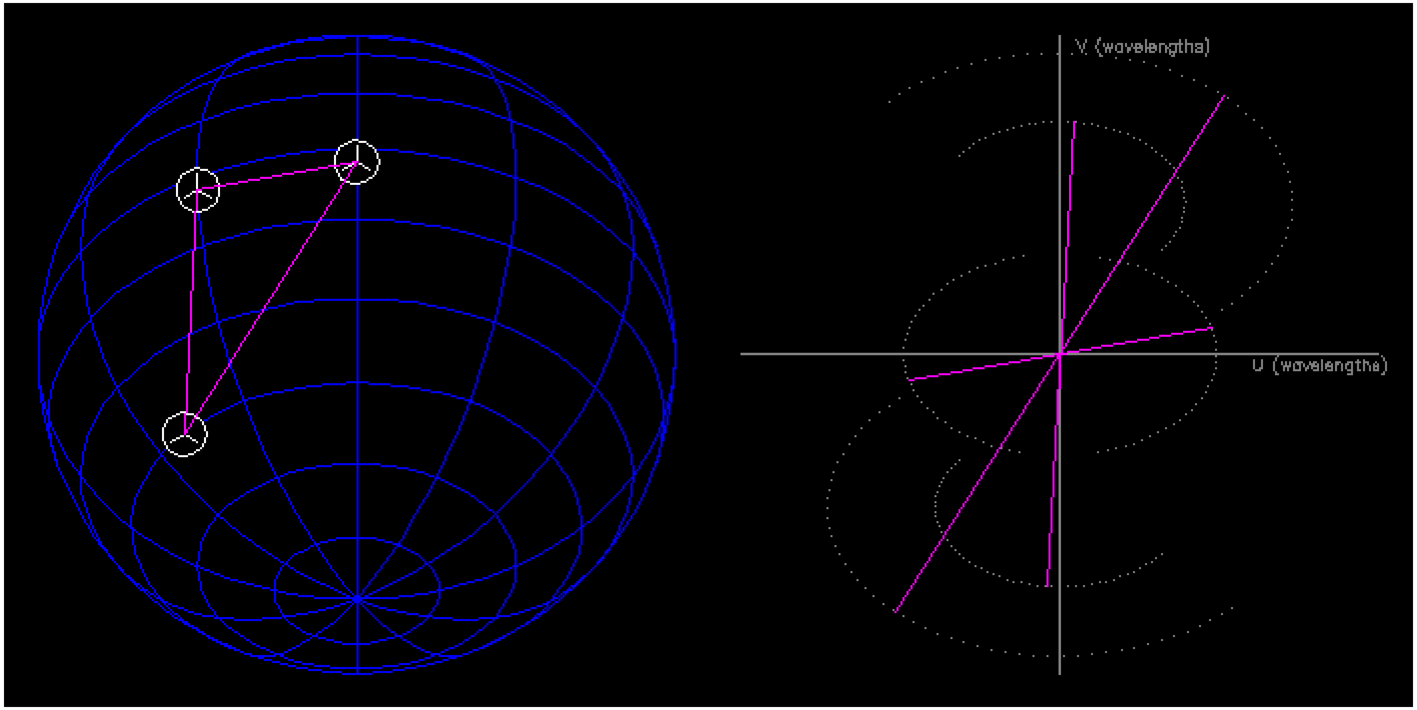
Wide-Band Imaging
or
Multi-Frequency Synthesis.

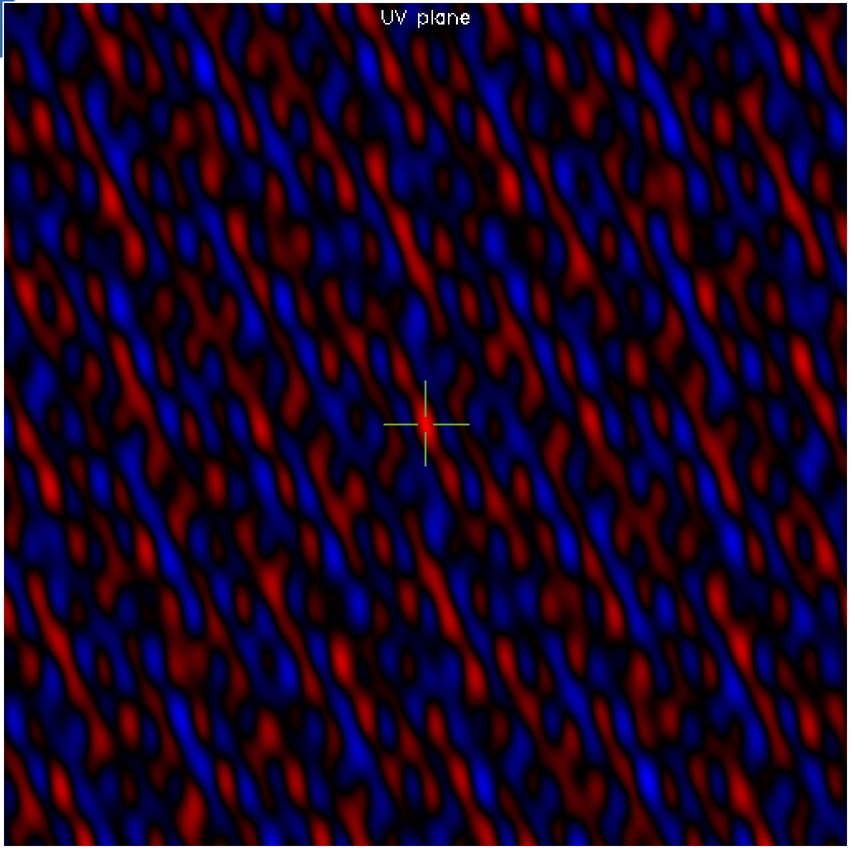
1. Basic Fourier relations (equation-free!!)
2. The ideal world vs real life.
3. Wide band interferometry:
   - Advantages:
     - greater aperture filling, thus cleaner dirty beam.
     - more data -> better SNR.
   - Disadvantages:
     - breakdown of assumption of monochromaticity -> ‘spectral artifacts’.
     - huge datasets.
     - others…
4. Weighting schemes.
5. How to clean wide-band data.

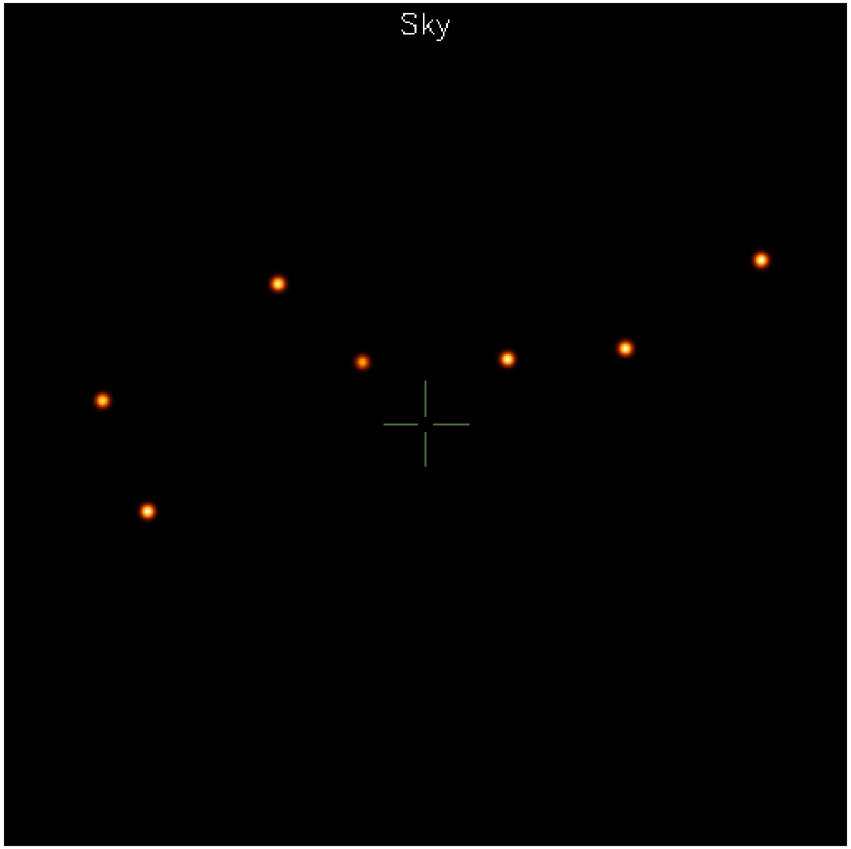
View from a (southern hemisphere) quasar…



1. Basic Fourier Relations.

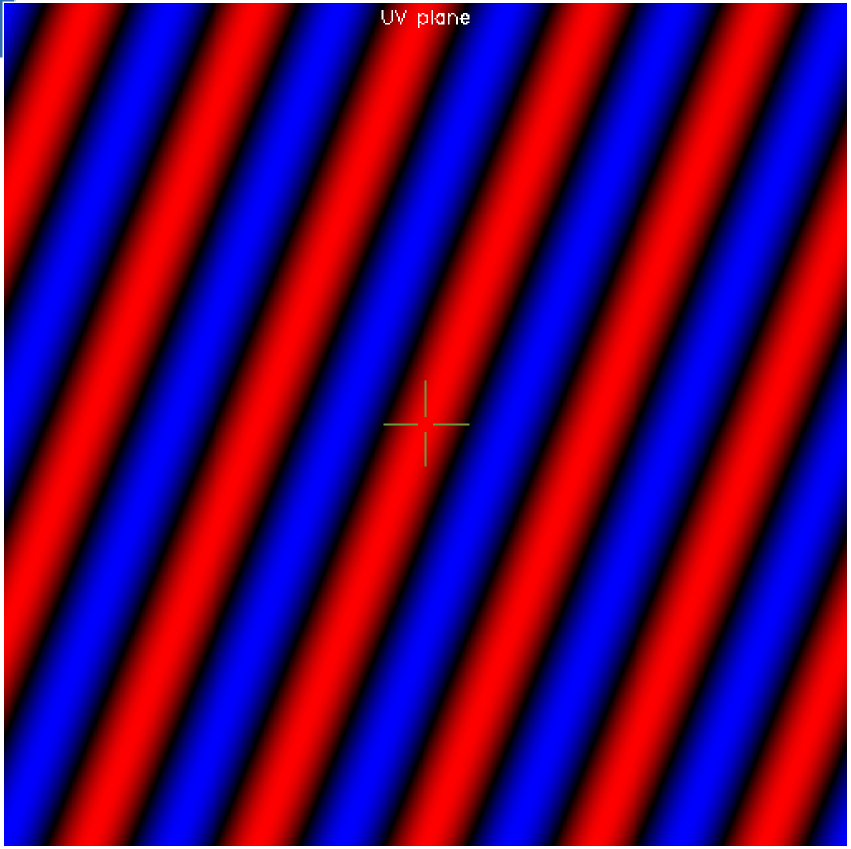
The UV plane is the 'Fourier dual' of the real sky.

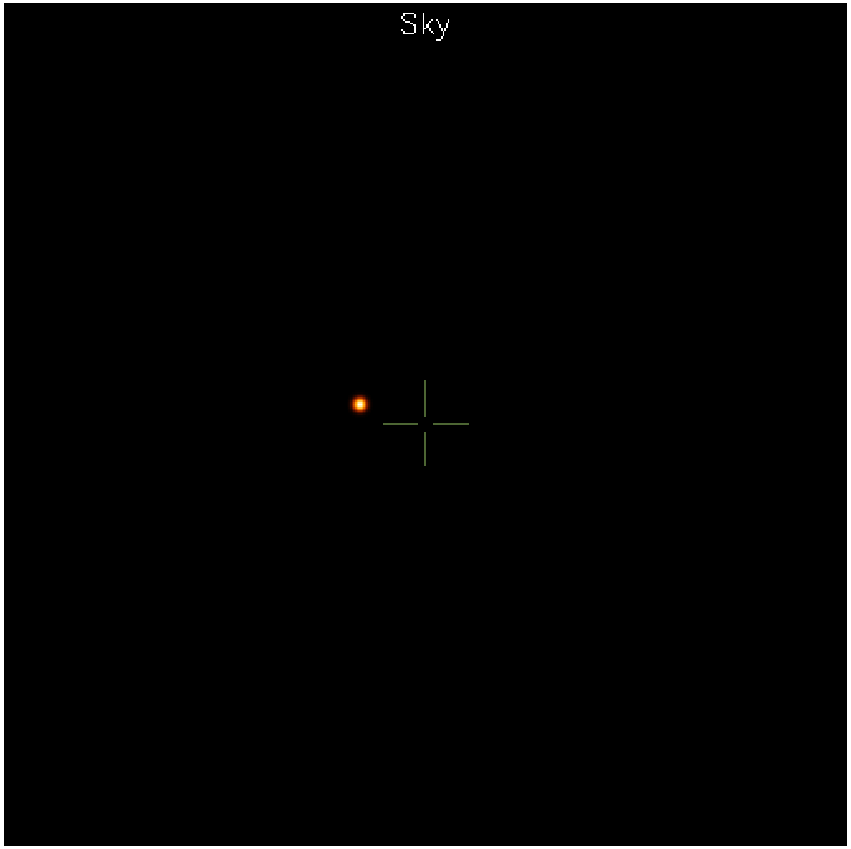




Features of the FT:

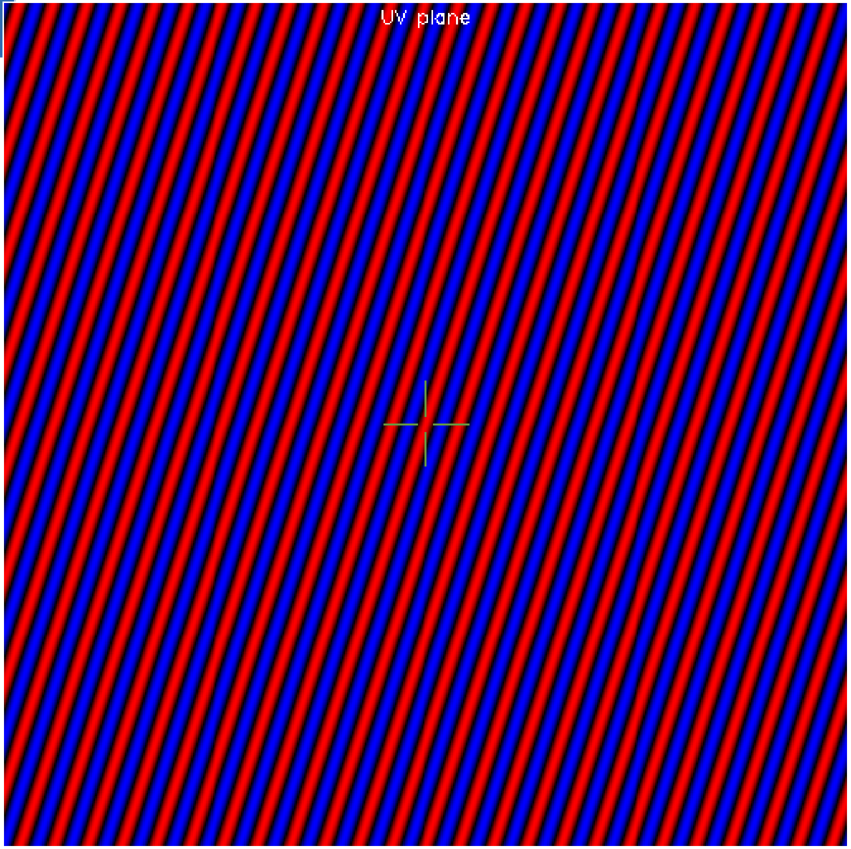
fringes ⇔ point (delta function).

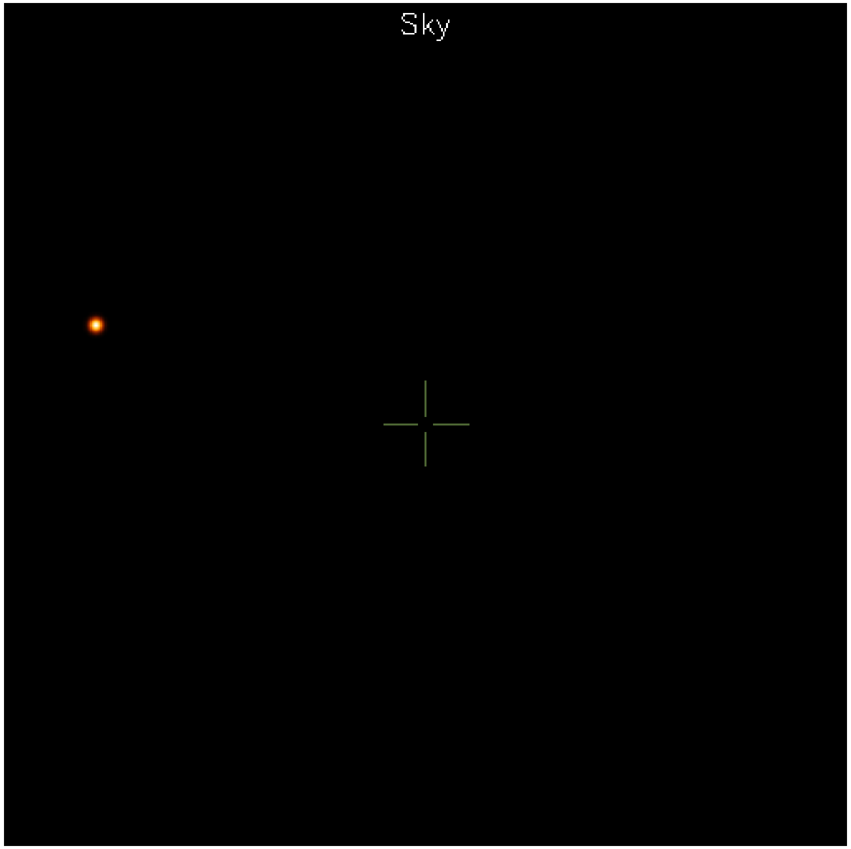




Features of the FT:

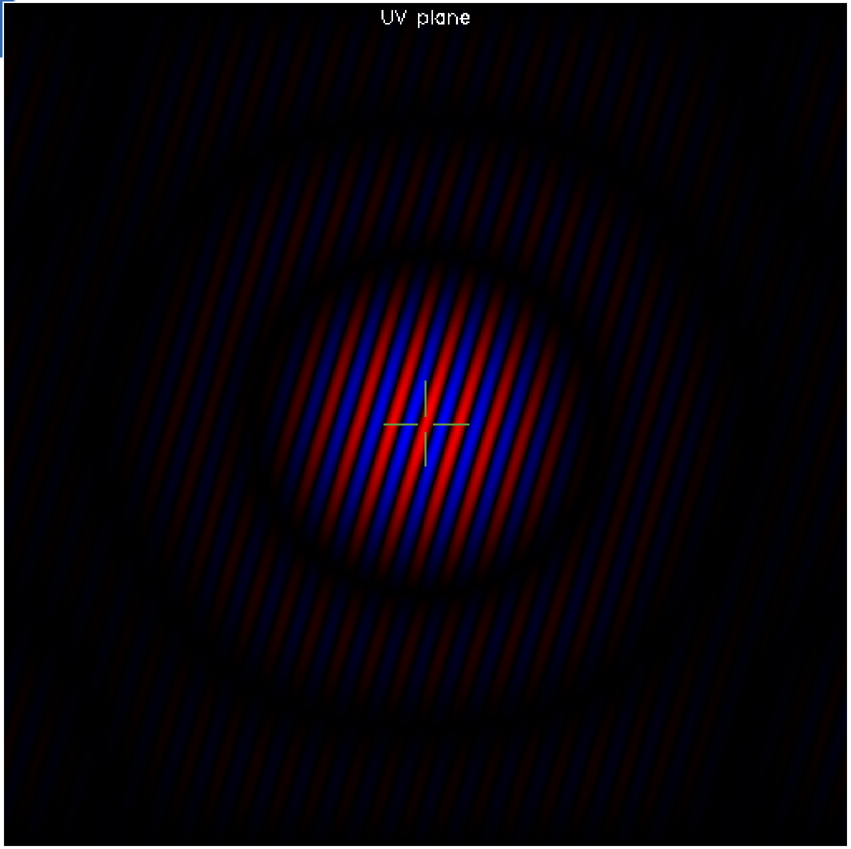
higher spatial frequency ⇔ further from the origin.

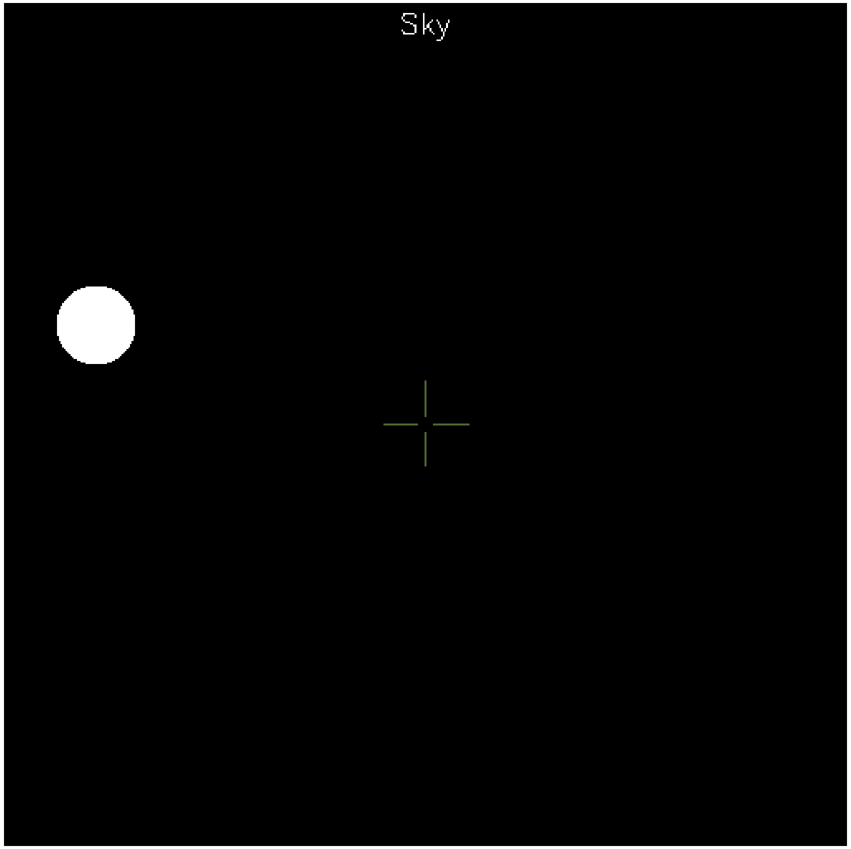




Features of the FT:

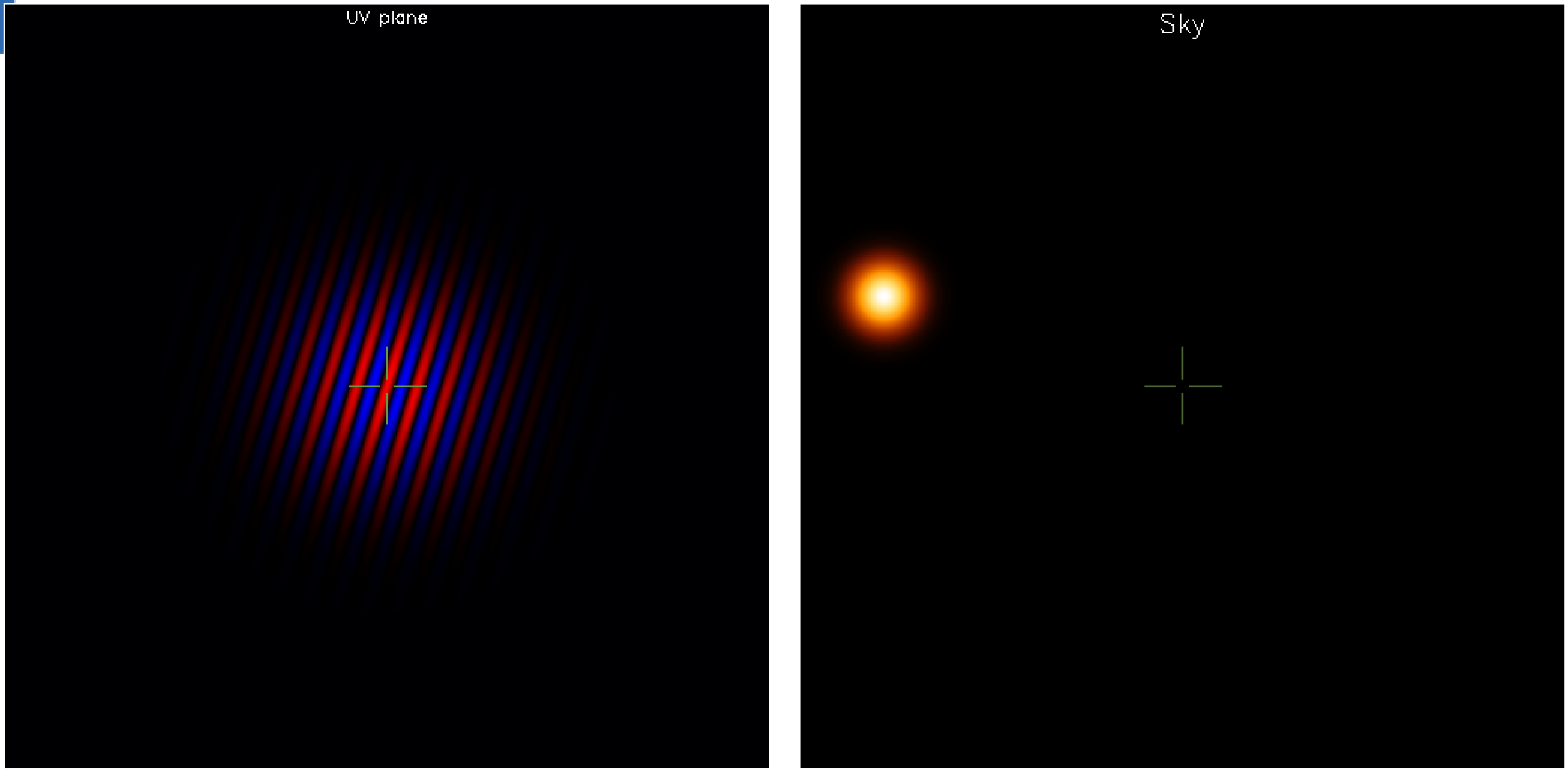
multiplication ⇔ convolution.





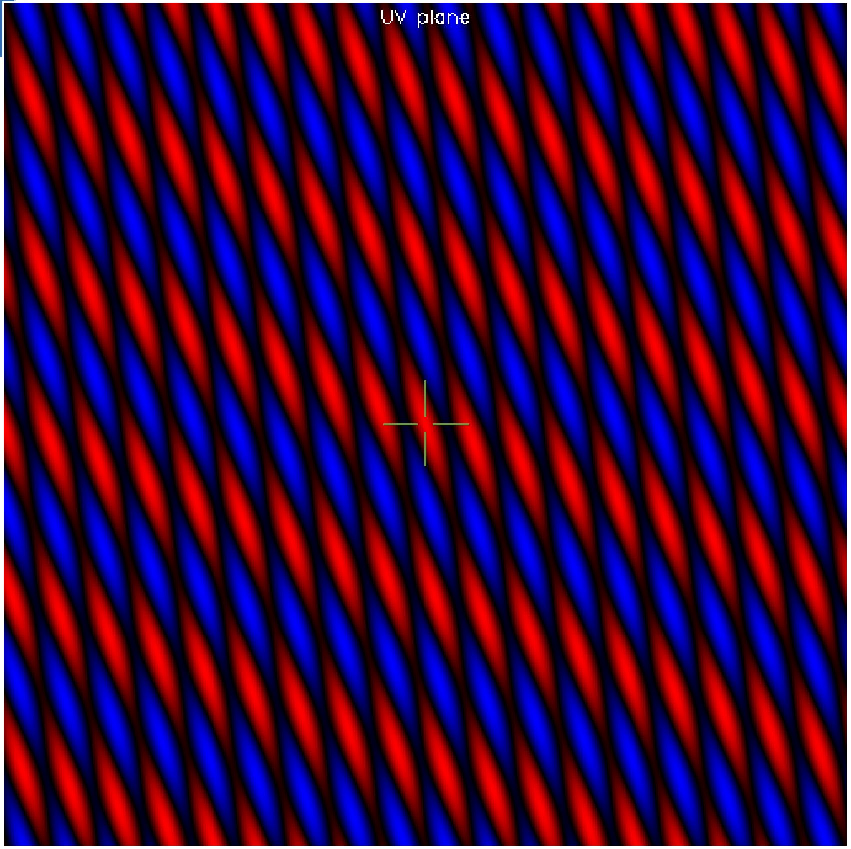
Features of the FT:

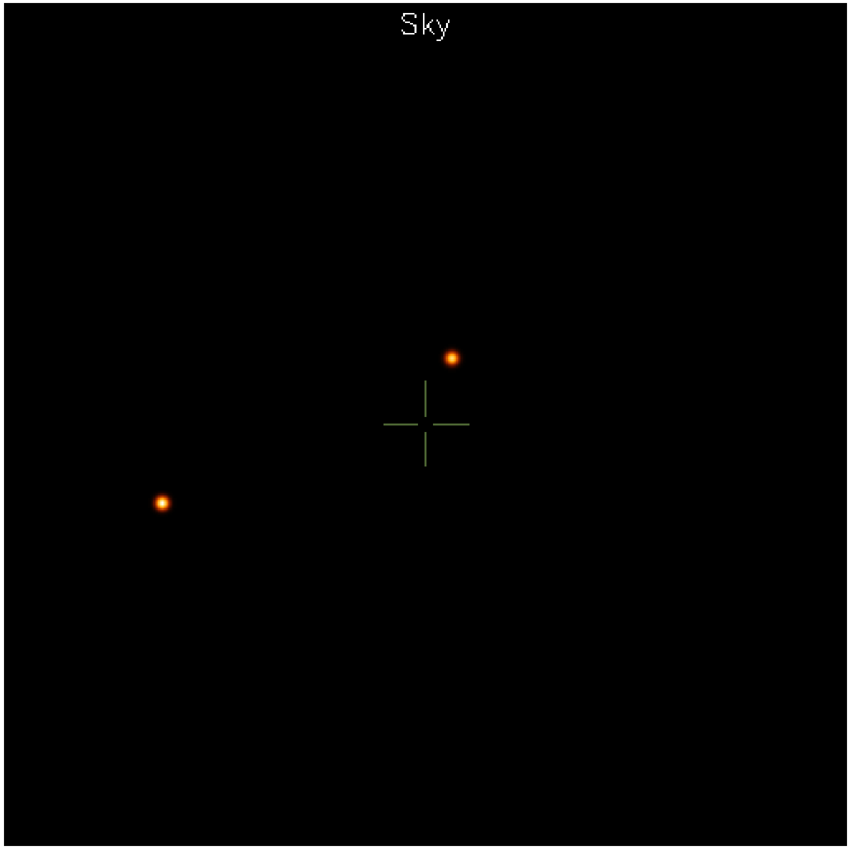
gaussian ⇔ gaussian.



2. The ideal world…

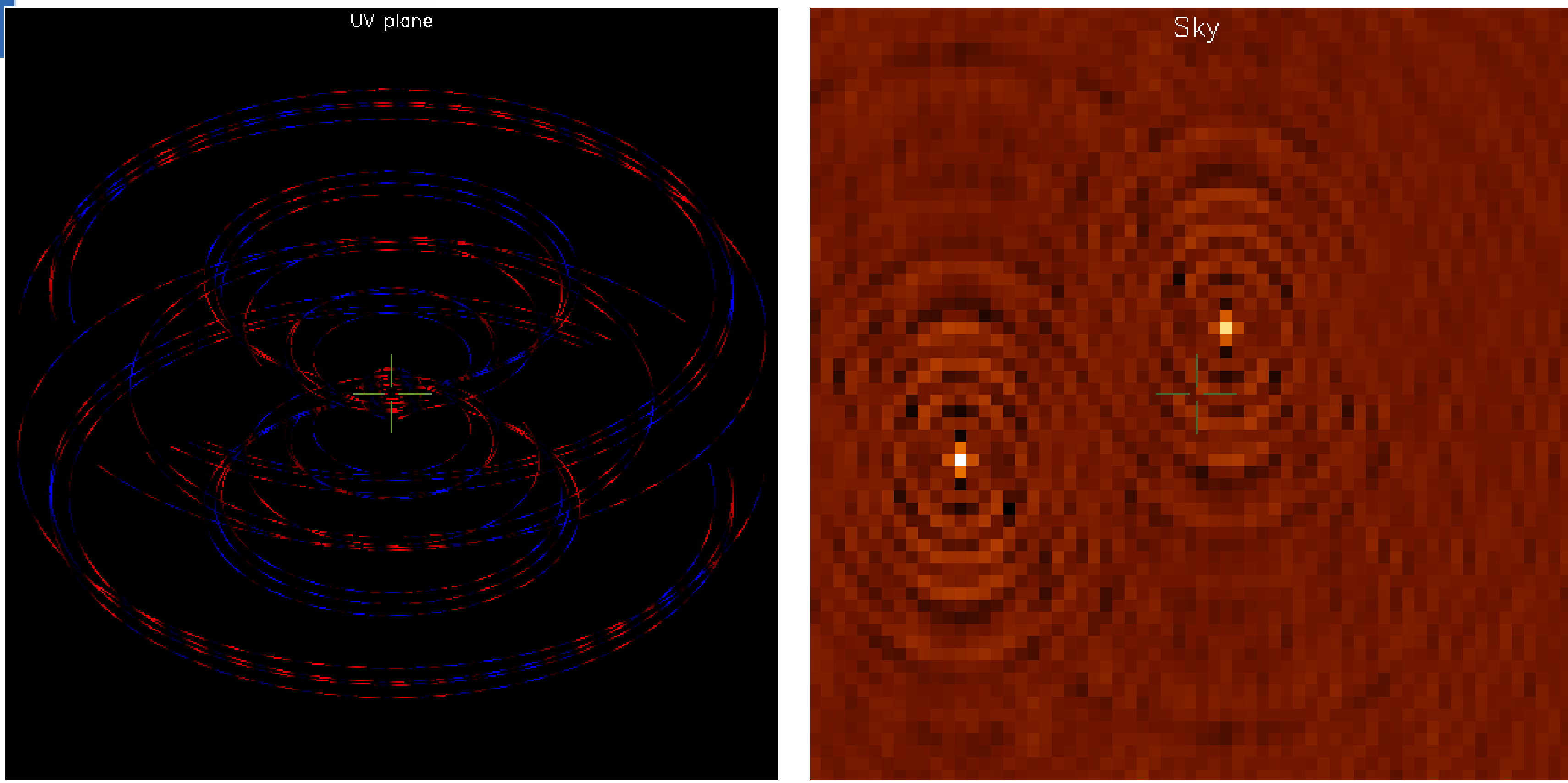
The simplest sky object which could be of interest: 2 point sources.





...vs real life.

Sparse sampling of the UV plane => 'dirty beam'.



Visibilities as measured by Merlin, δ=+35°, 16 x 1 MHz channels.

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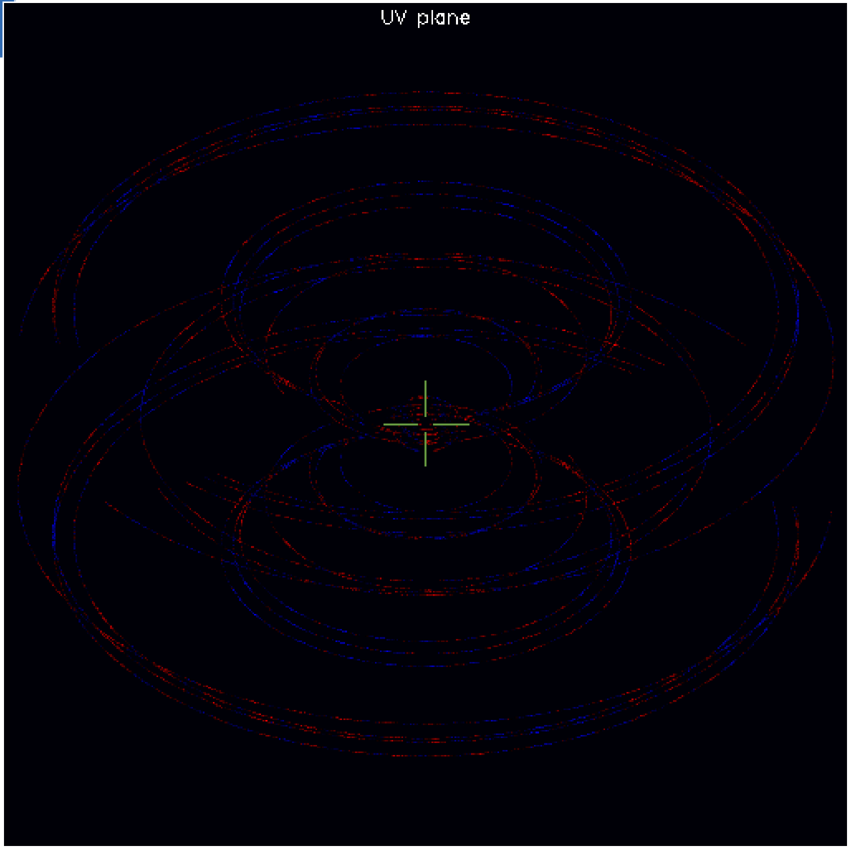
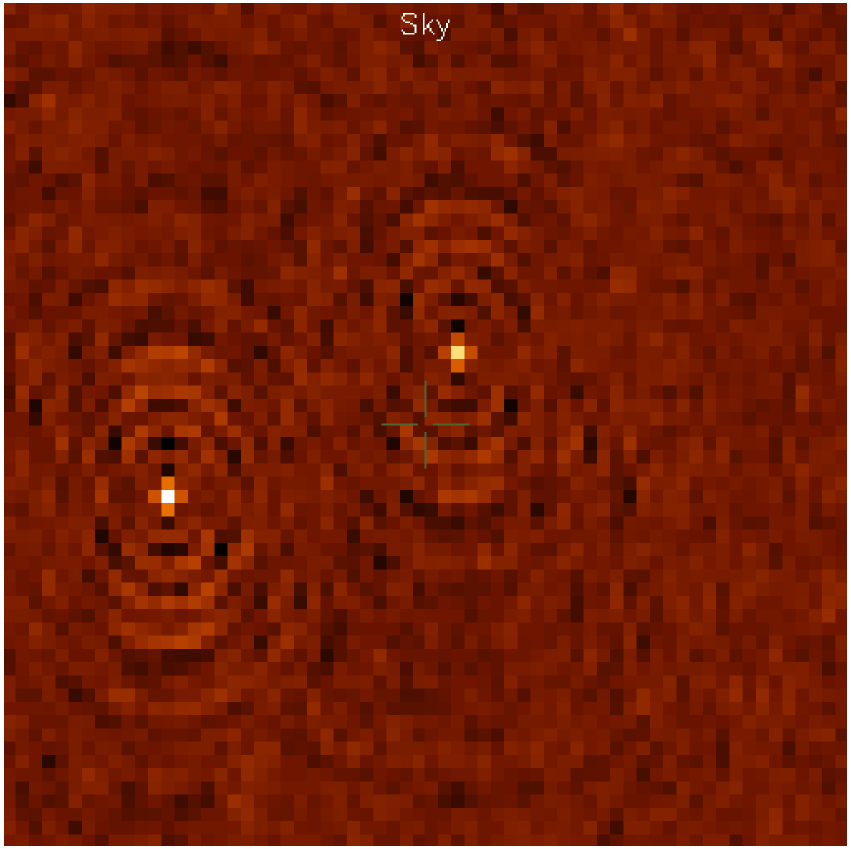
Even realer life!

Alas, every measurement includes noise…



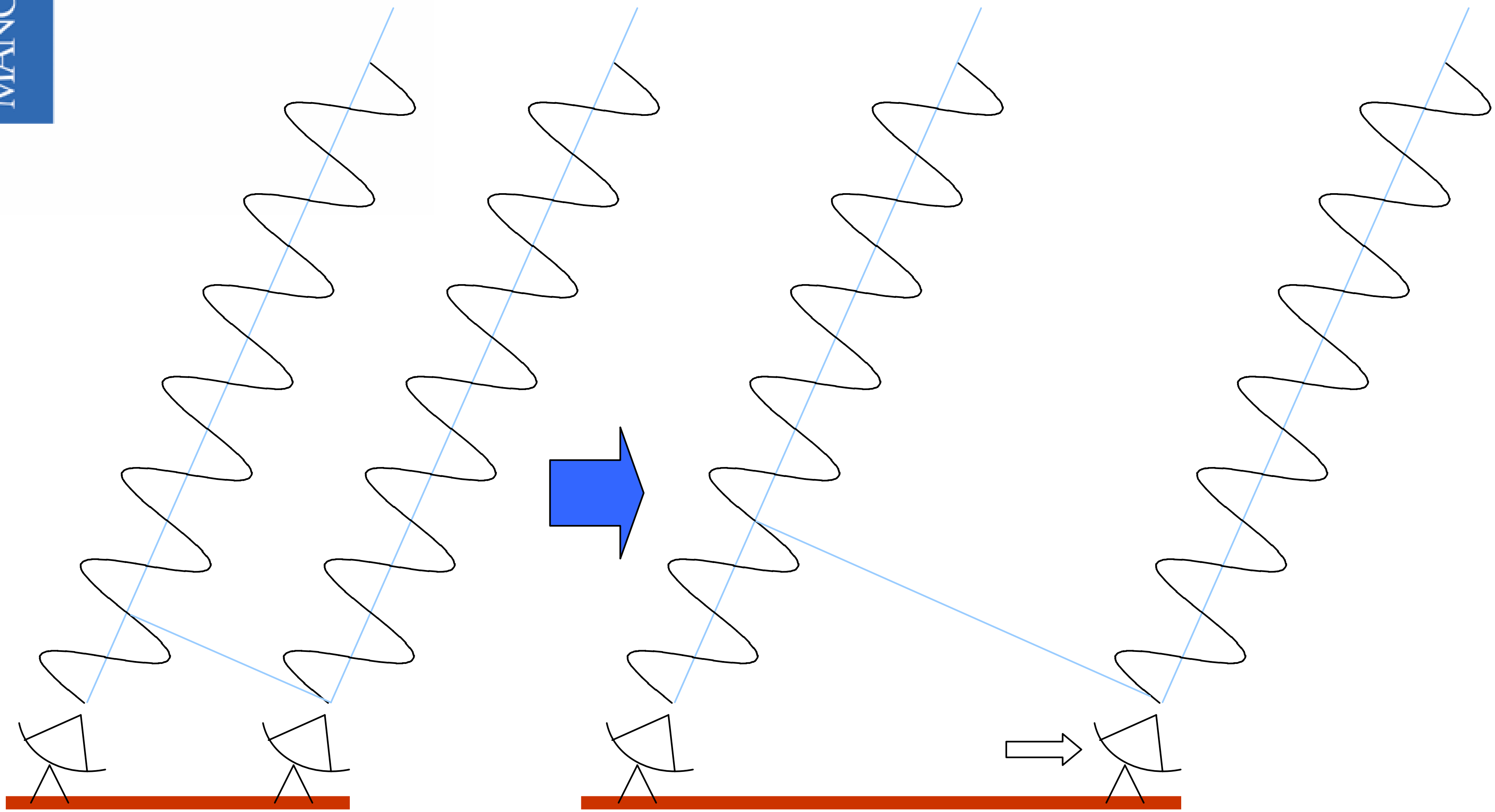


SNR of each visibility = 15%.

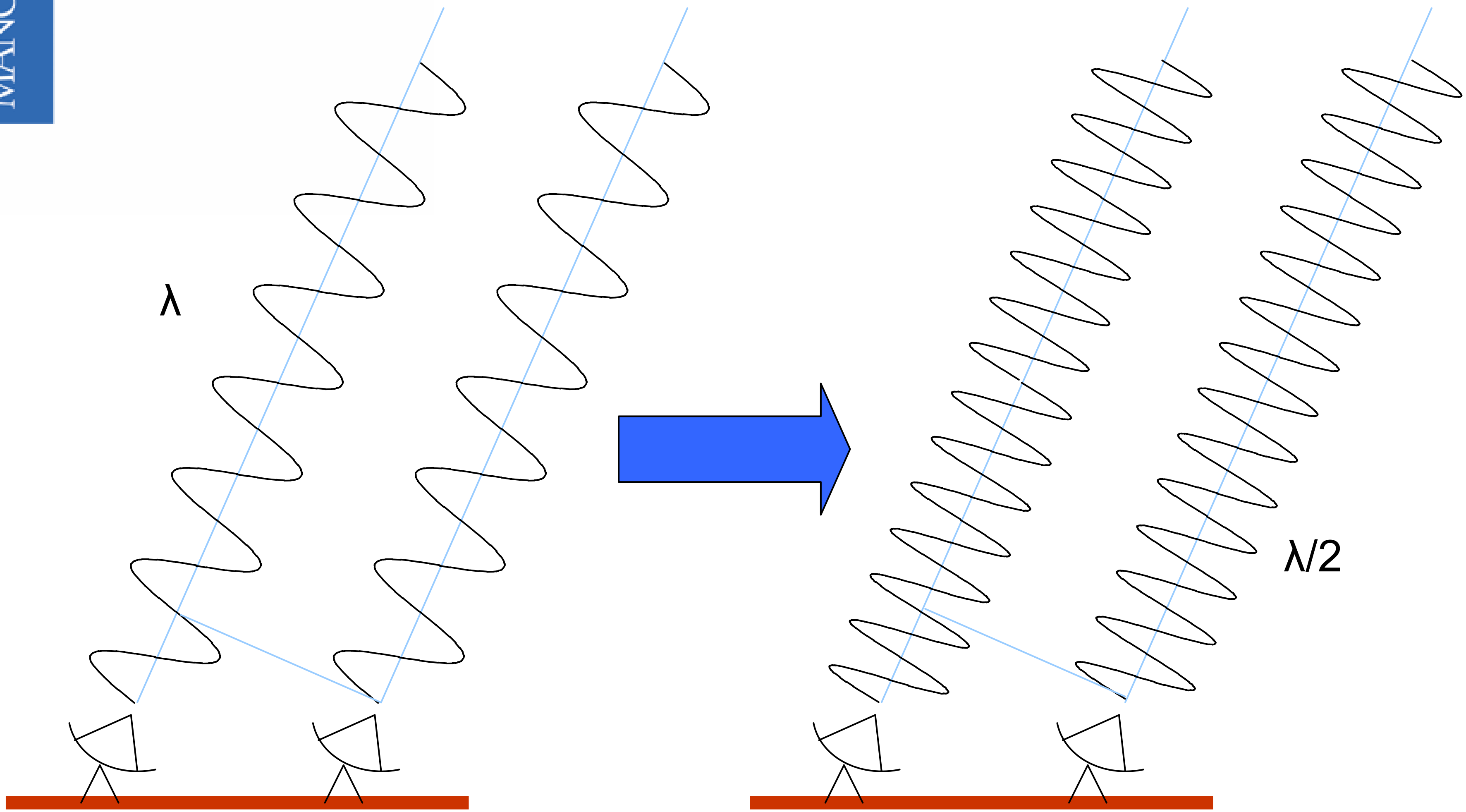
3. Wide Band Interferometry.

How to increase UV coverage?

…could get more baselines if we moved the antennas!



…but simpler to change the observing wavelength.

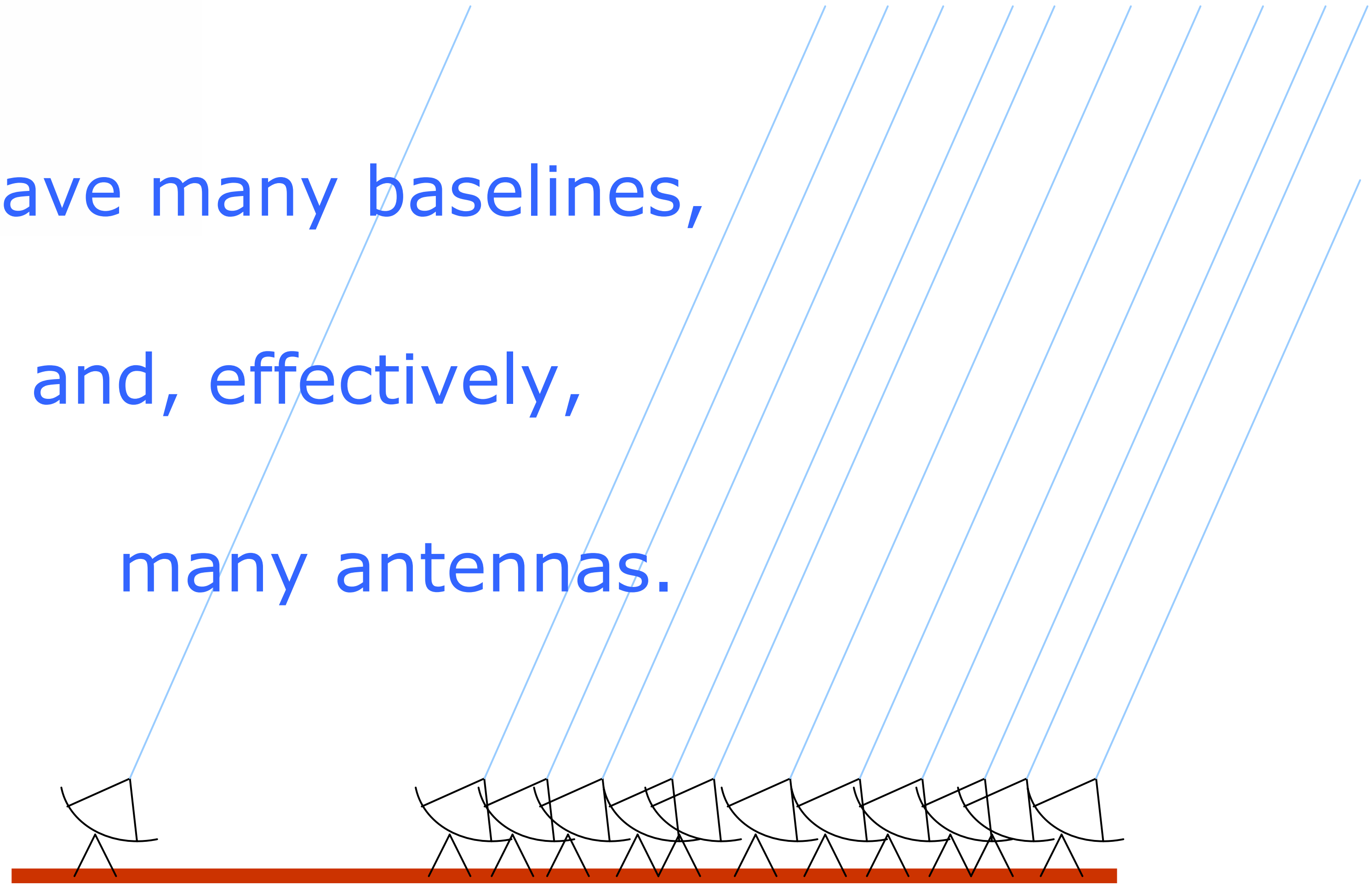


With *many* wavelengths…

…we have many baselines,

and, effectively,

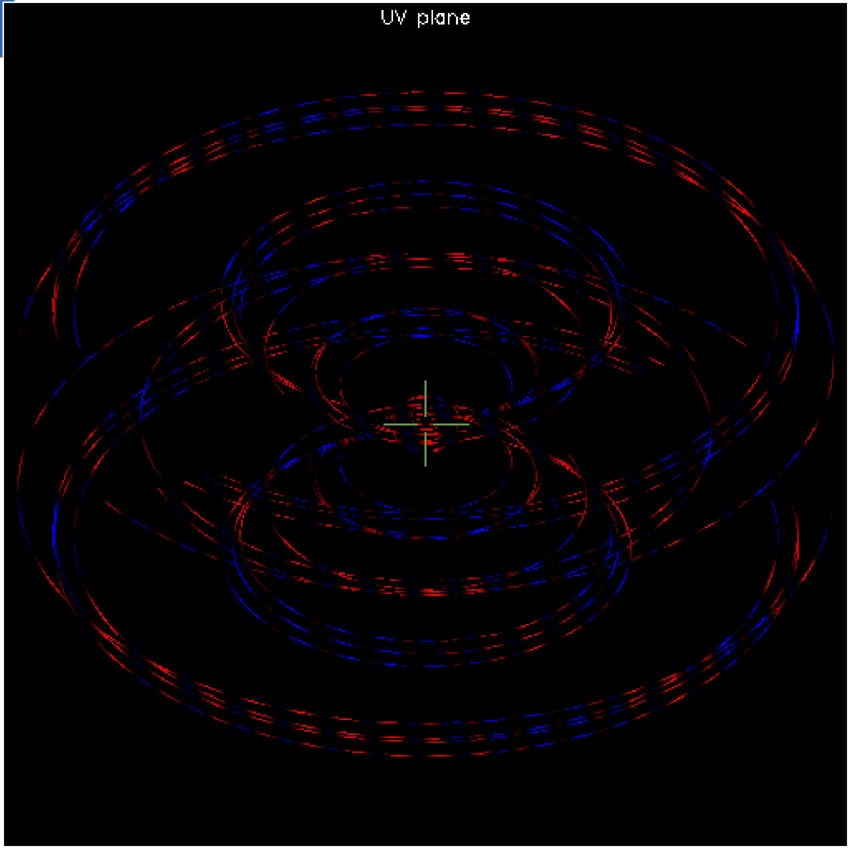
many antennas.



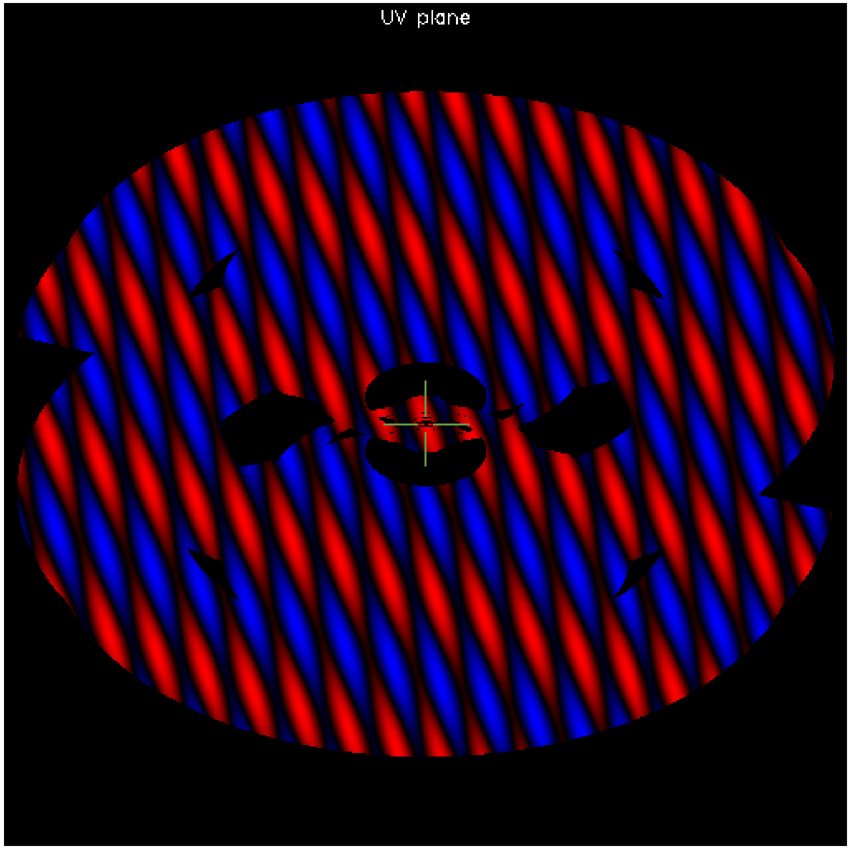
Narrow vs broad-band: UV coverage

16 x 1 MHz

500 x 4 MHz



Merlin, δ=+35°



eMerlin, δ=+35°

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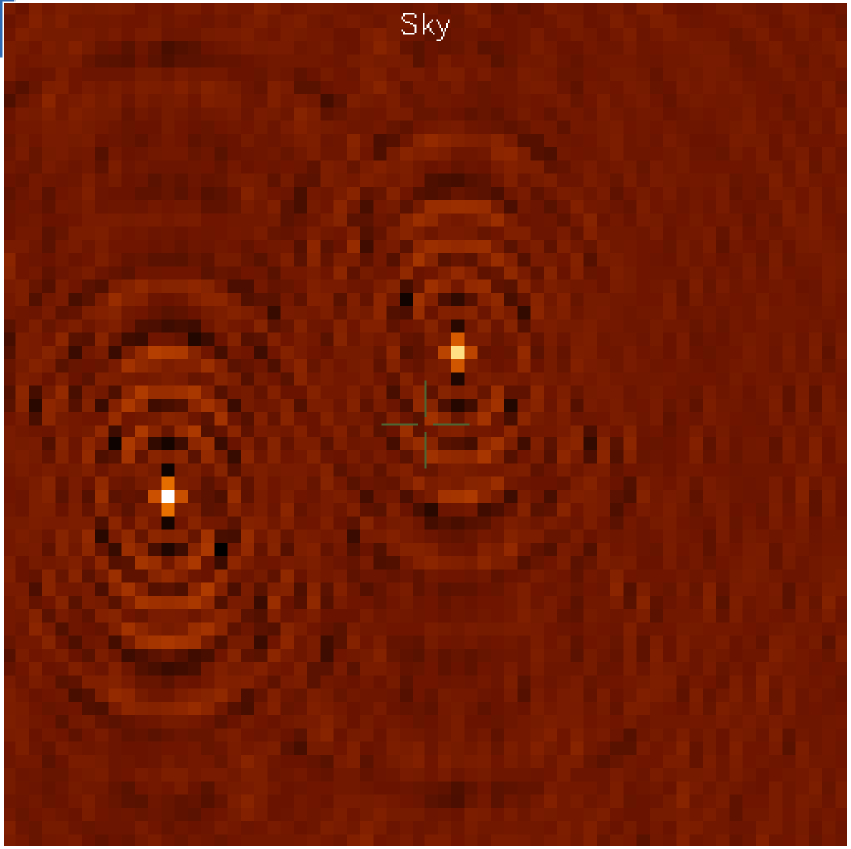
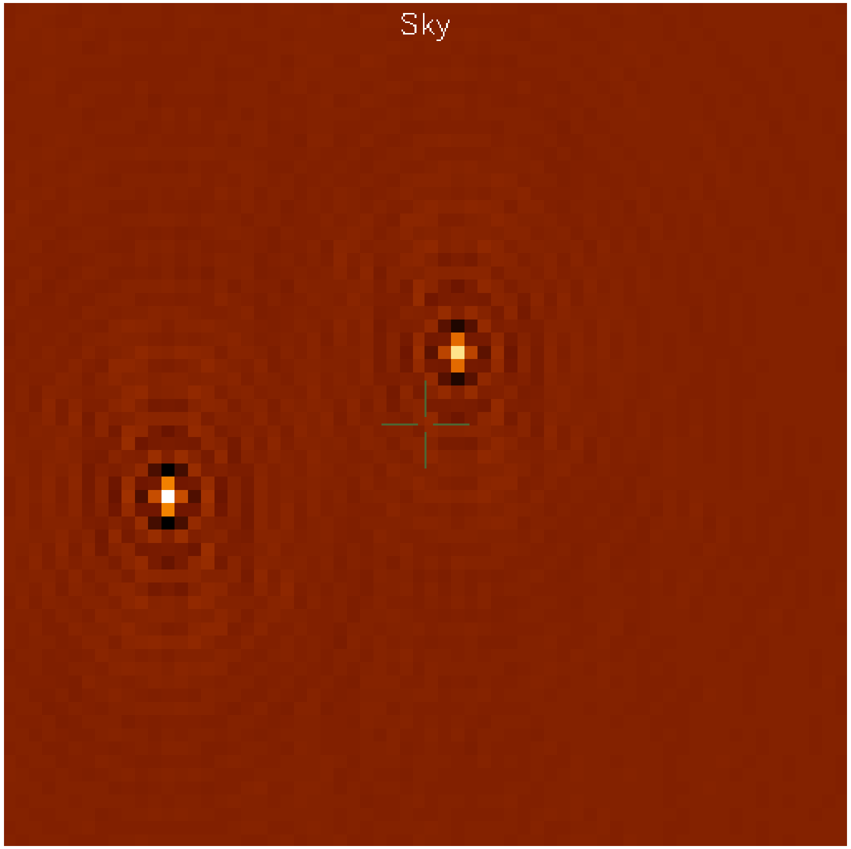
Narrow vs broad-band - without noise:

16 x 1 MHz

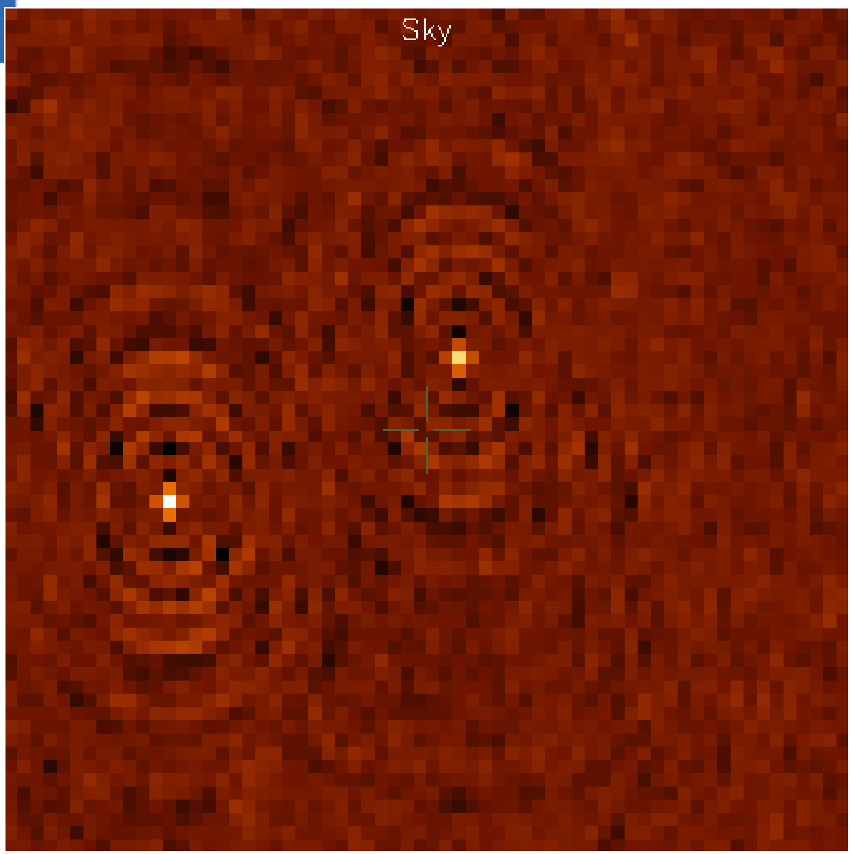


500 x 4 MHz

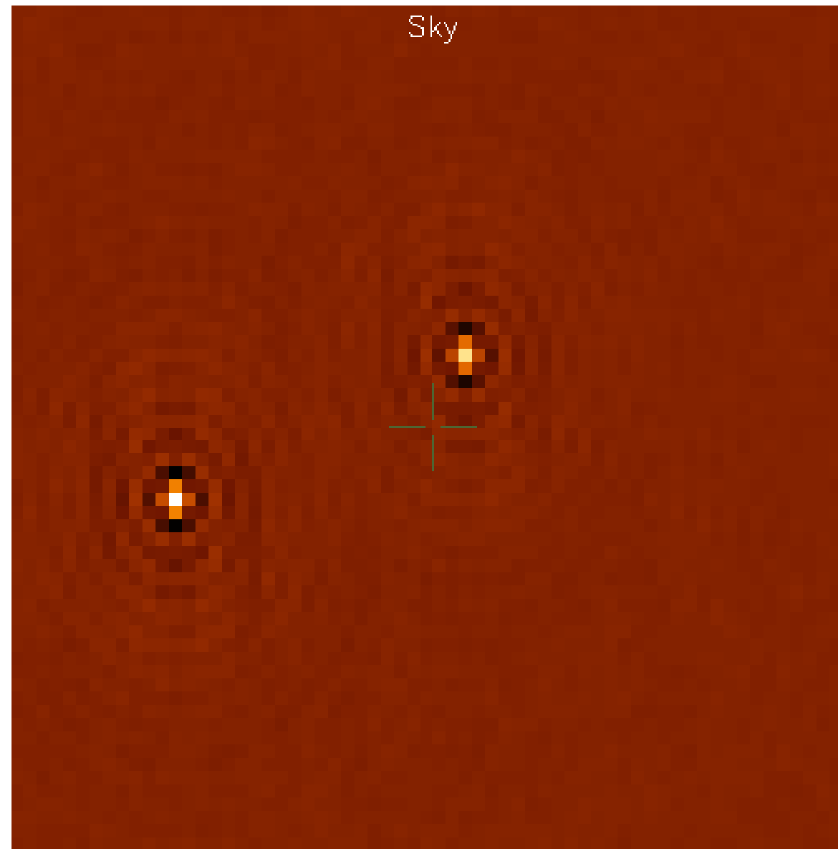


Narrow vs broad-band - with noise:

16 x 1 MHz



500 x 4 MHz



SNR of each visibility = 15%.

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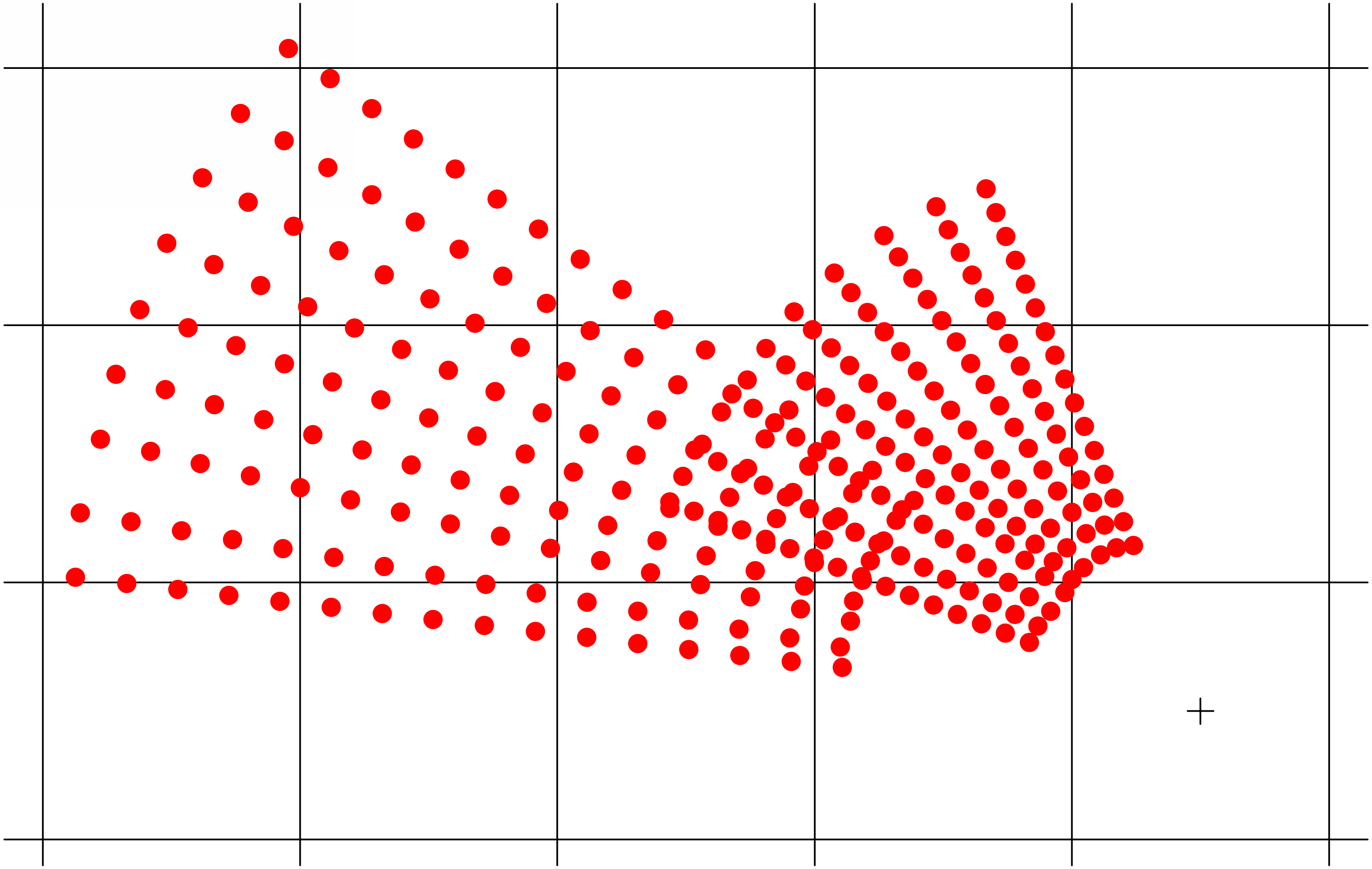
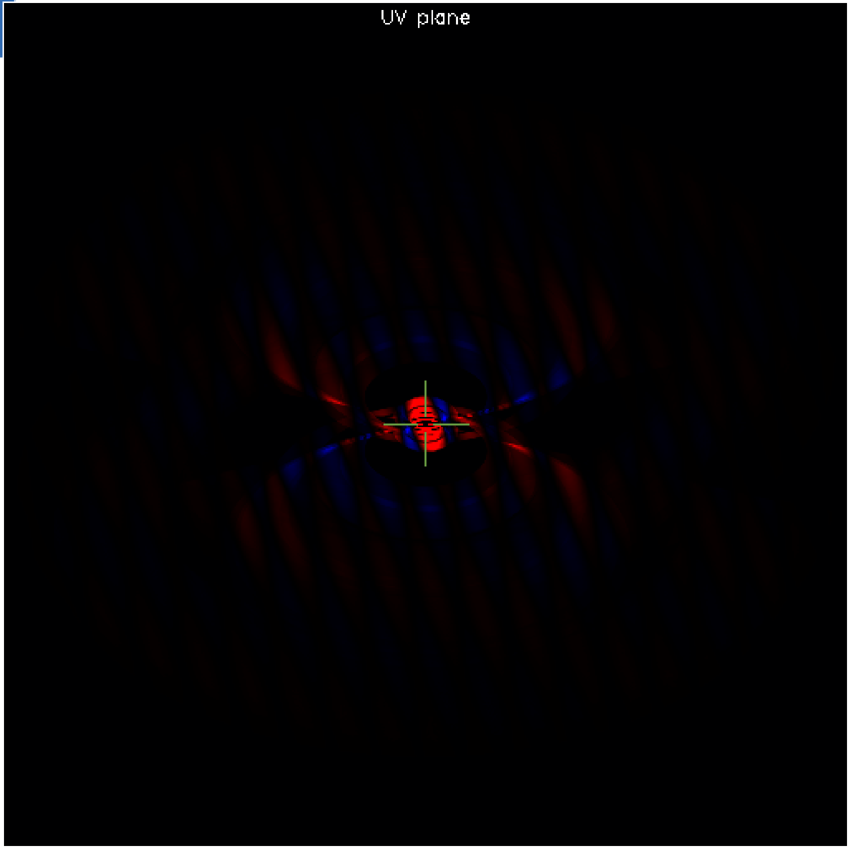
4. Weighting Schemes.

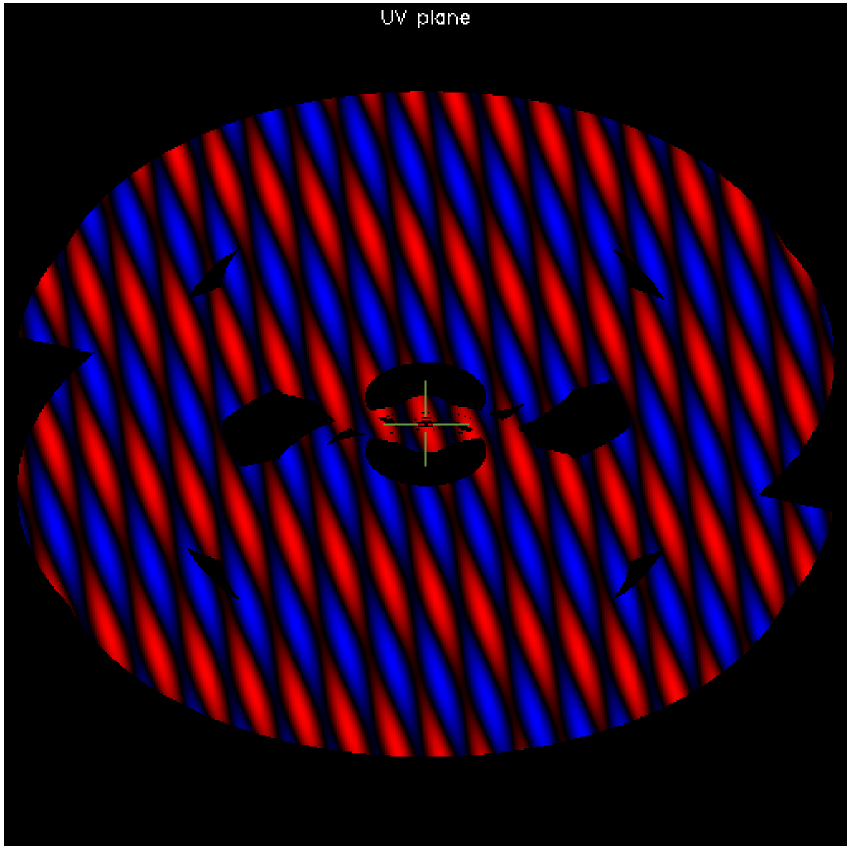


Natural vs uniform:
weighted visibilities

Natural weighting

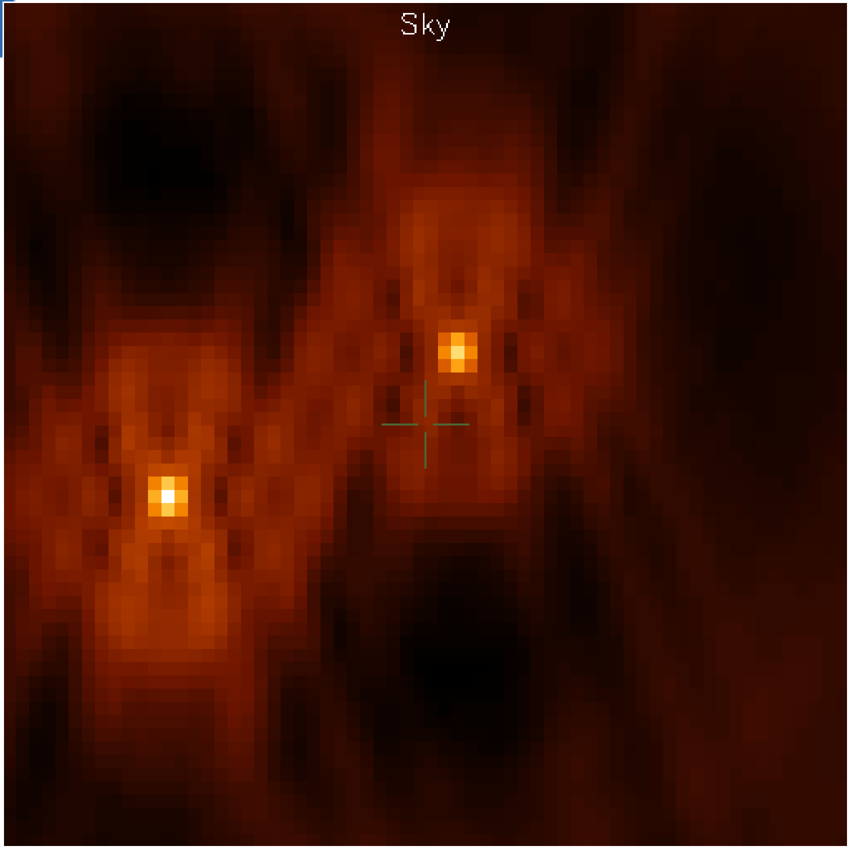


Uniform weighting

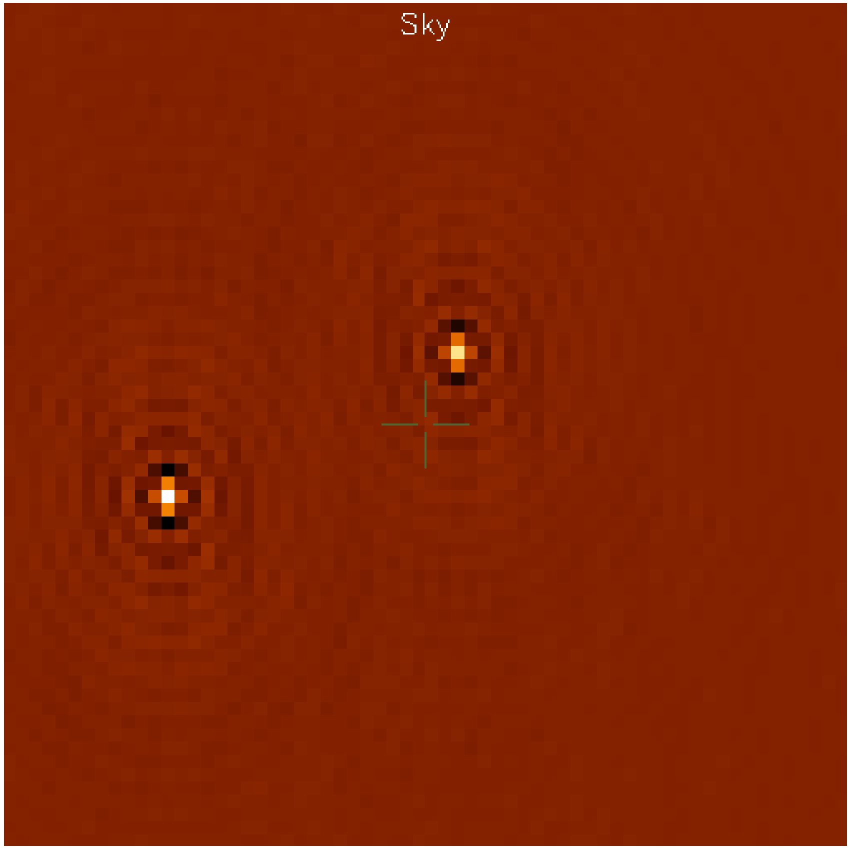


Natural vs uniform:
without measurement noise

Natural weighting

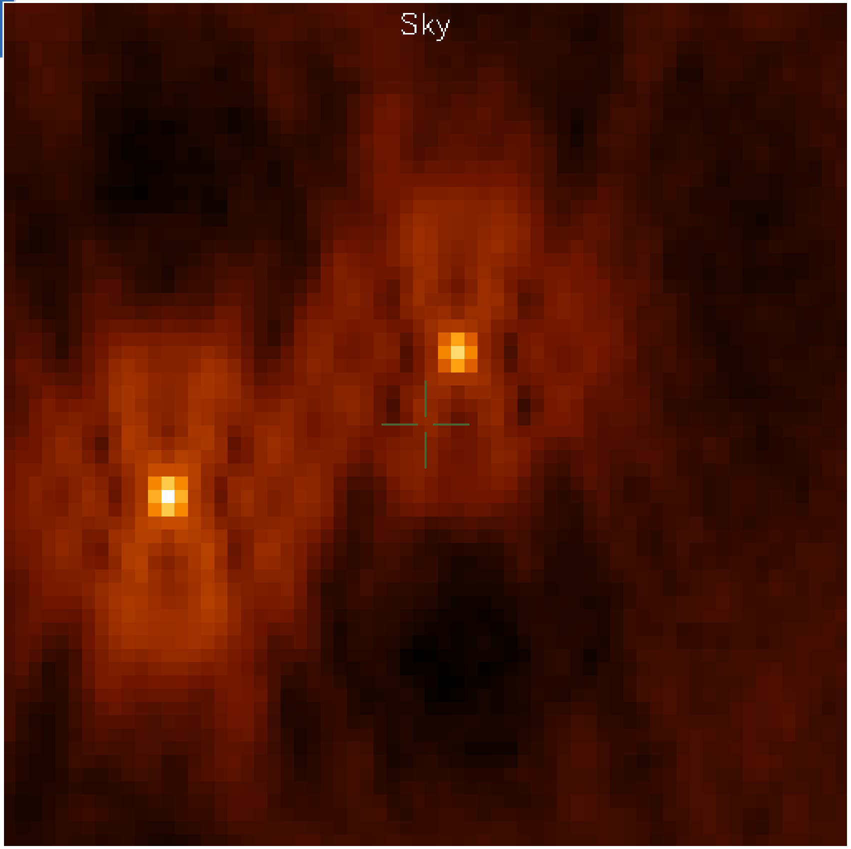


Uniform weighting

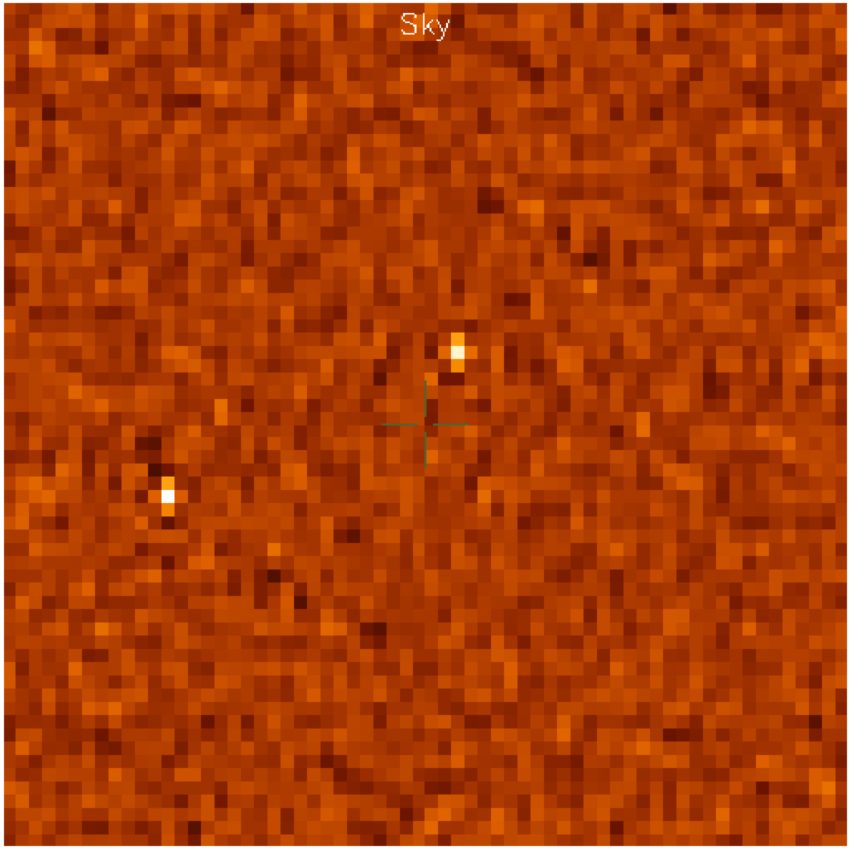


Natural vs uniform:
now with added noise.

Natural weighting



Uniform weighting



SNR of each visibility = 0.7%.

Weight optimisation: a little theory.

Let's work in 1 dimension for simplicity. The dirty beam *B* is related to weights W_j as follows:

$$B_k = \Sigma_{j=0}^{N-1} W_j \, S_j \exp(2\pi i jk/N).$$

The *V* term is neglected because all visibilities are equal to 1. *S* here is the 'coverage function' and is either 0 or 1. We have to include it to prevent us from trying to find weights for grid points at which there are no data.

Least squares theory says we should try to minimize a sum of squared residuals, given by:

$$SSR = \Sigma_{k=0}^{N-1} M_k \, [(B_k - B_k^{ideal})^2 + \sigma_k^2].$$

We probably want to choose a gaussian for B_k^{ideal}.

But what is M_k?

Weight optimisation: a little theory.

M_k is a masking function which allows us to ignore part of the beam and fit to the rest if we wish.

Setting all $\partial SSR/\partial W_j$ to zero (and making use of the fact that W must be Hermitian) gives the 'normal' equations:

$$\mathbf{Aw} = \mathbf{ß}$$

where

$$A_{jl} = \Sigma_{k=0}^{N-1} M_k \exp(2\pi i k[j-l]/N),\ w_l \text{ actually } = W_l S_l,$$

and

$$ß_j = \Sigma_{k=0}^{N-1} M_k B_k \exp(2\pi i jk/N),$$

provided $\sigma_k = 0\ \forall\ k$ (otherwise equations nonlinear).

Weight optimisation: a little theory.

If M_k=1 for all k, the solution is trivial: **A** turns into the identity matrix, so the optimum W is just the Fourier inversion of the ideal beam. If this is gaussian, so will W be. This then is just the standard tapering function.

Matters become more interesting if we set M_k to 0 for some k, eg within some radius of the phase centre.

So – linear equations in W? Let's solve them…

Weight optimisation: a little theory.

…but 8000 time samples x 15 baselines x 2000 frequency channels, gives 2.4e8 unknowns. Ulp.

But:

- As the cognoscenti know, normal equations are often ill-conditioned. So we didn't really want to solve them directly anyway.
- We can't include noise and keep linear NE.

The way around this computing impasse is to make use of the power of the FFT in an iterative solution. A single pass of the iteration does as follows:

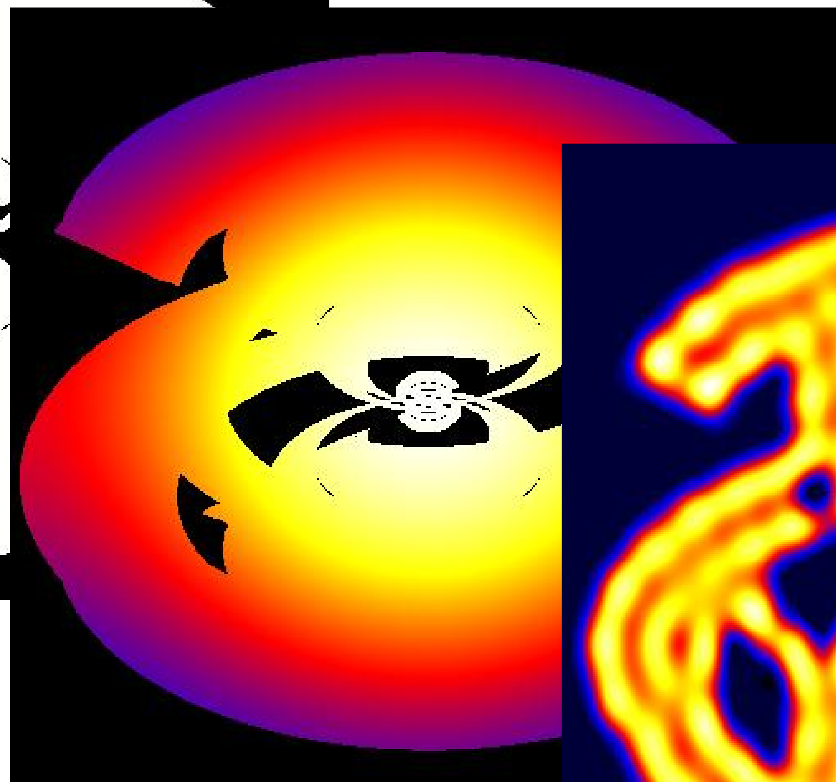
1. $\mathrm{FFT}^{-1}(W \times S) \rightarrow B$
2. $B_{\mathrm{resid}} = (B - B_{\mathrm{ideal}}) \times M$
3. $\mathrm{FFT}(B_{\mathrm{resid}}) \rightarrow W_{\mathrm{resid}}$
4. $W = W - \lambda W_{\mathrm{resid}}$

λ here is the loop gain.

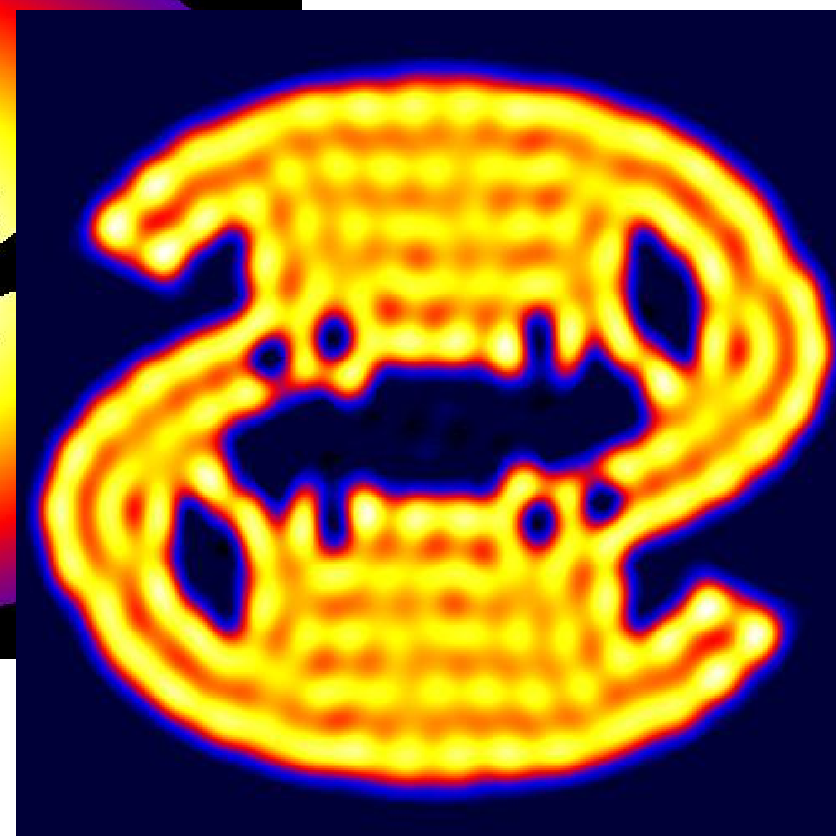
Simulated e-Merlin data.
401 x 5 MHz channels;
ν_{av} = 6 GHz;
t_{int} = 10 s;
δ = +30°

Weighting schemes:

Uniform

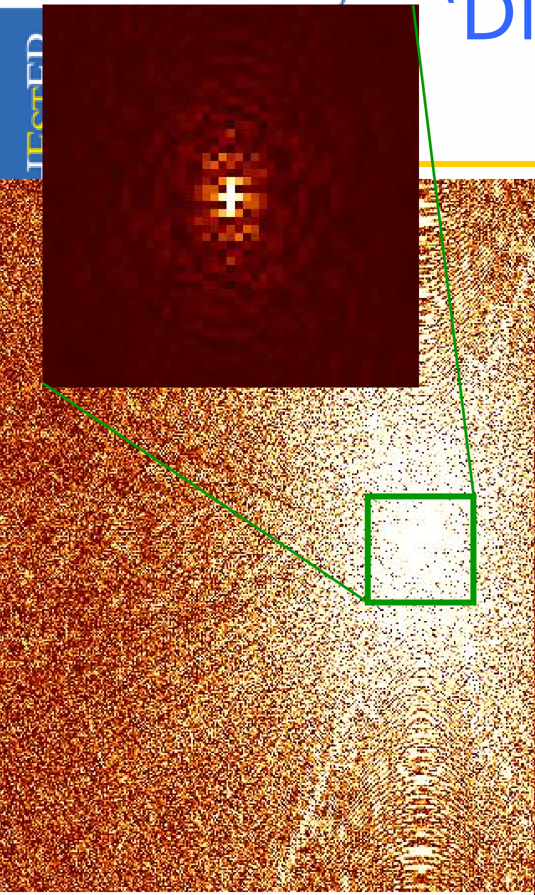


Tapered

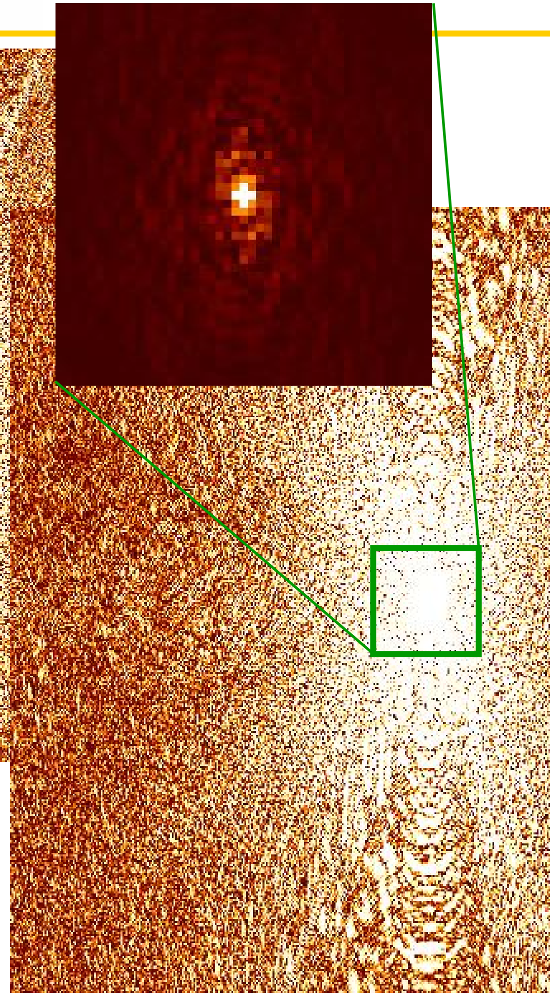


Best fit outside 20-pixel radius

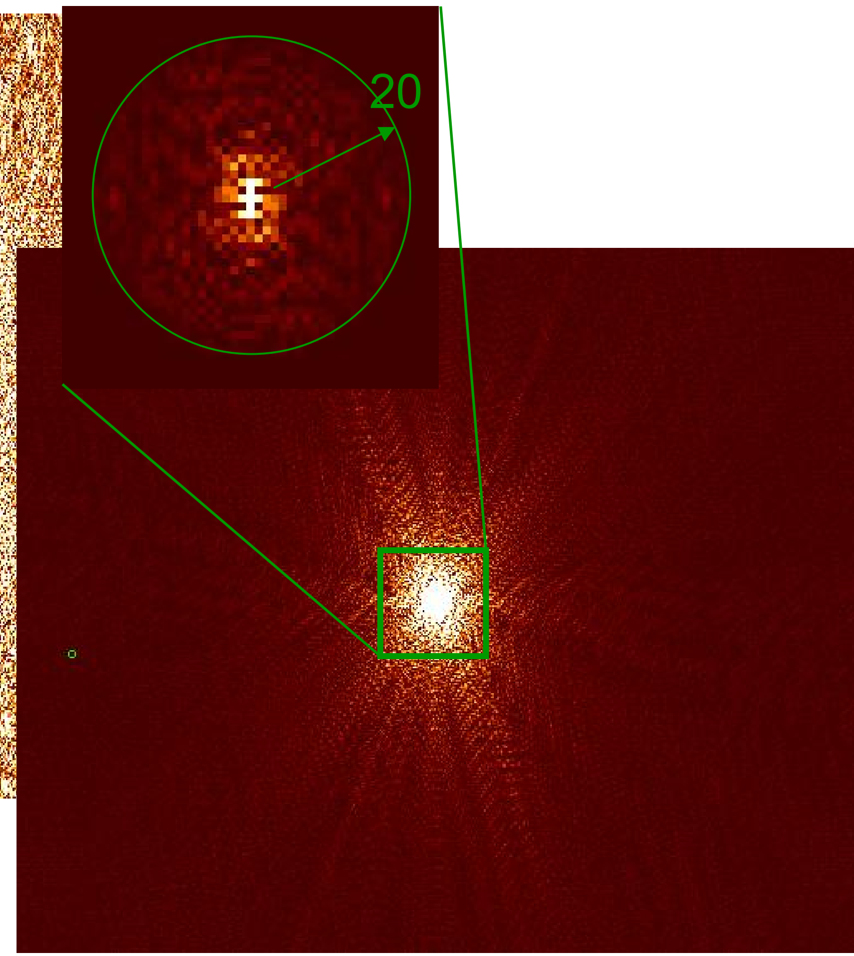
‘Dirty beam’ images (absolute values).



Uniform

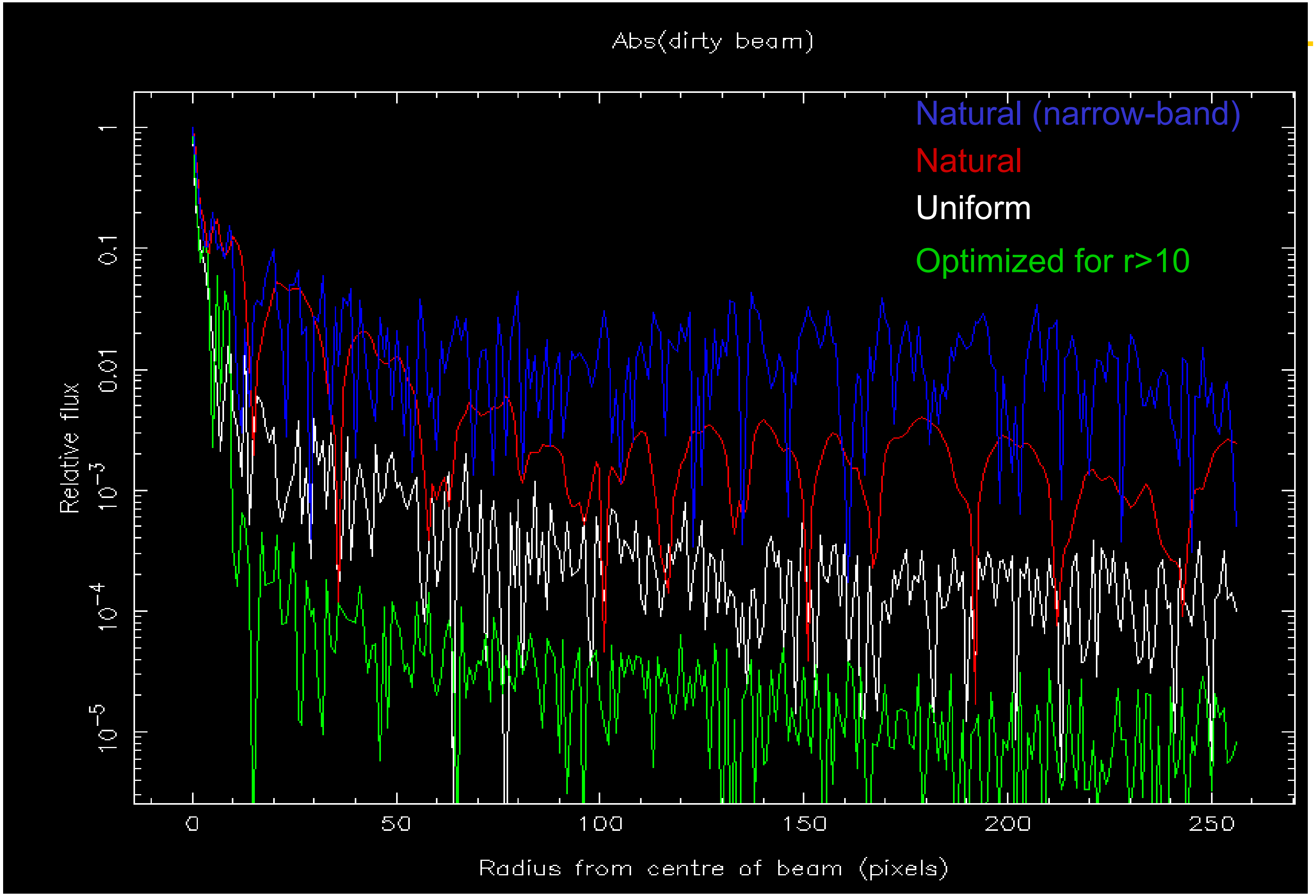


Tapered

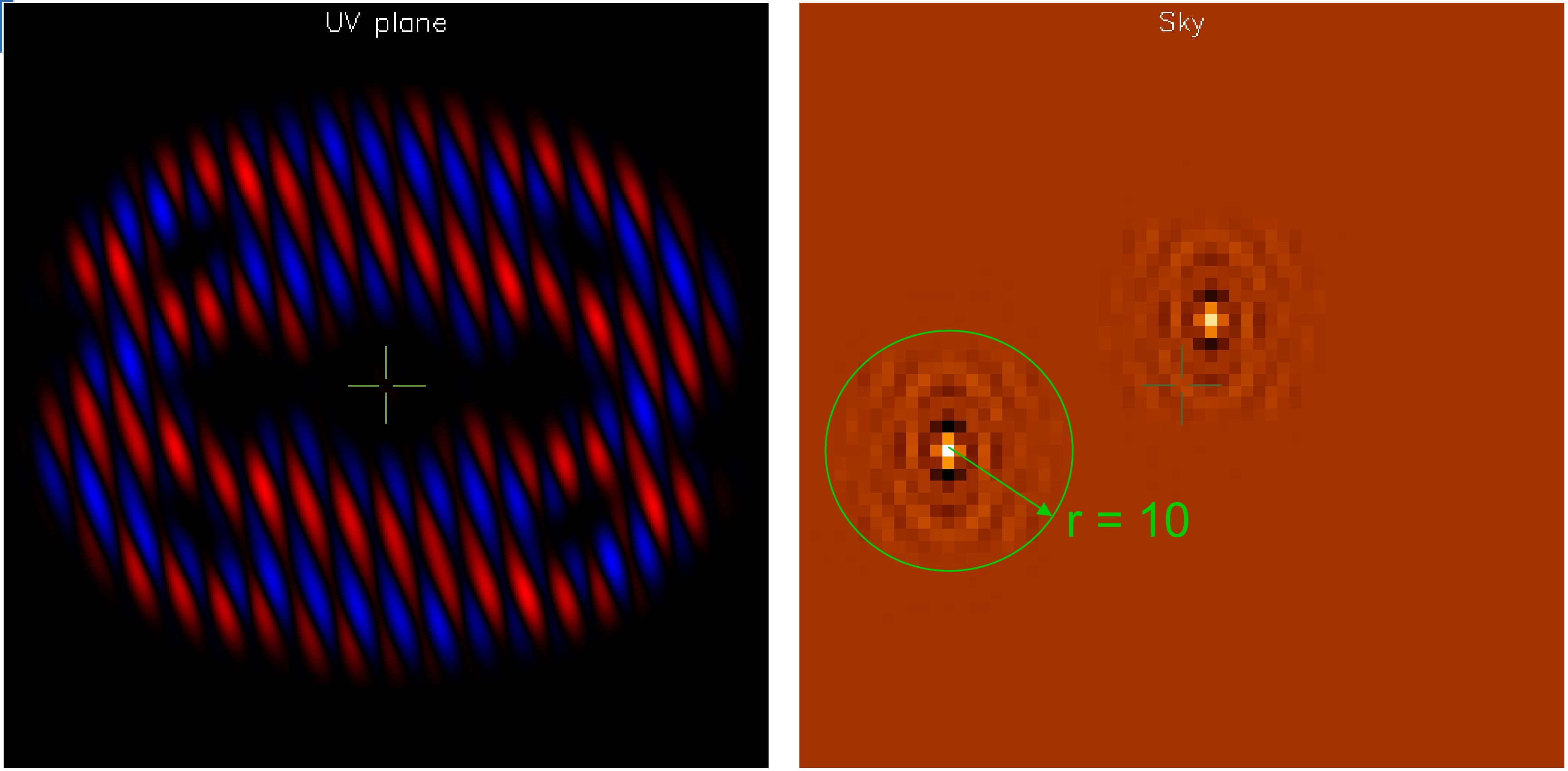


Best fit outside 20-pixel radius

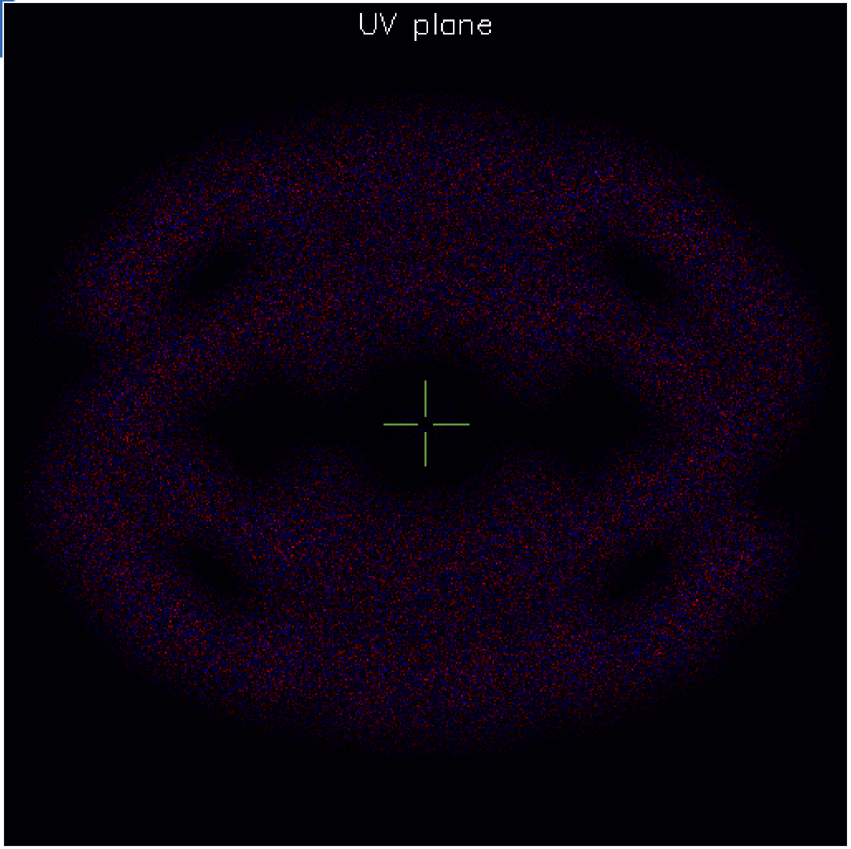
Comparison of different weighting schemes:

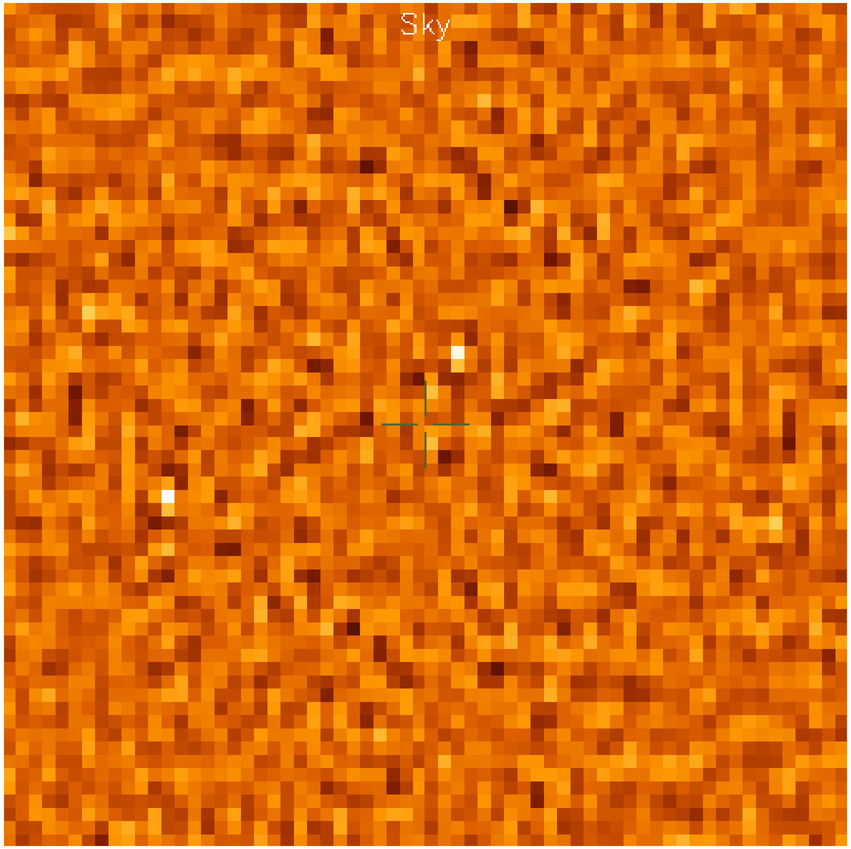


Weights optimized to remove far-field beam ripples:



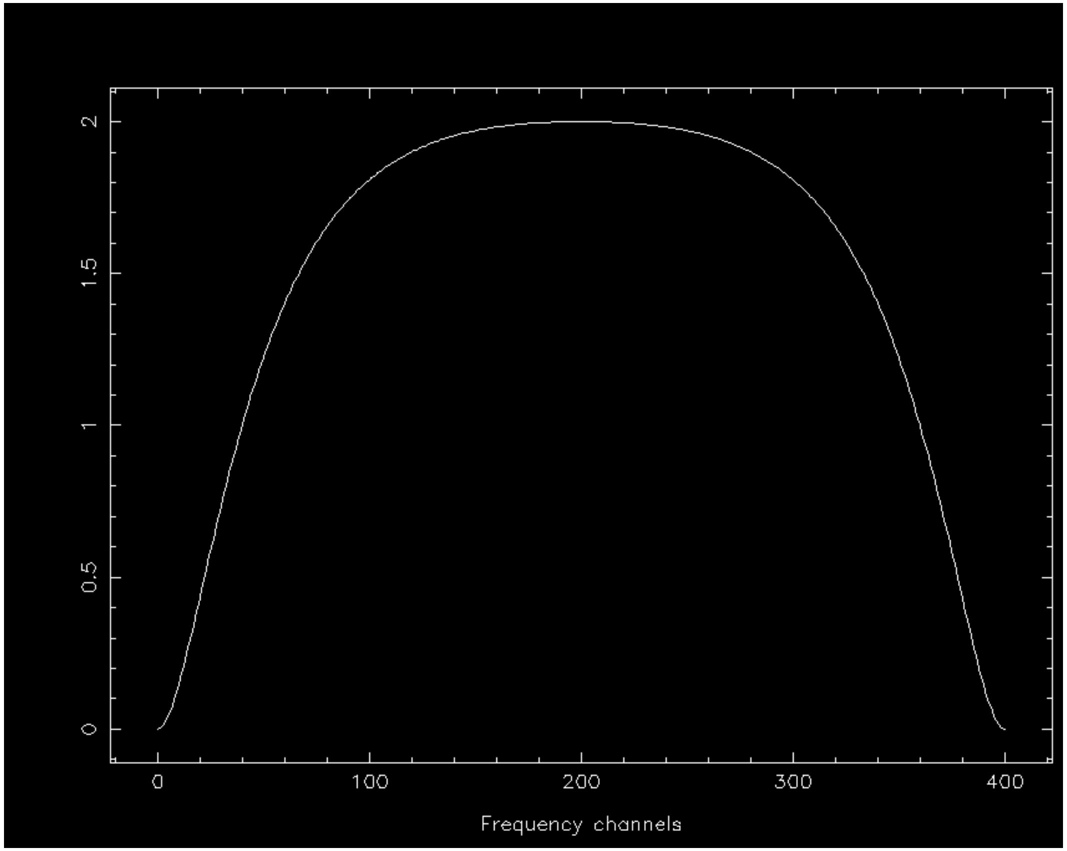
But real data is noisy…





SNR of each visibility = 5.

Other ways to achieve super-uniform weighting:



1. Multiply visibilities with a vignetting function of time and frequency, eg

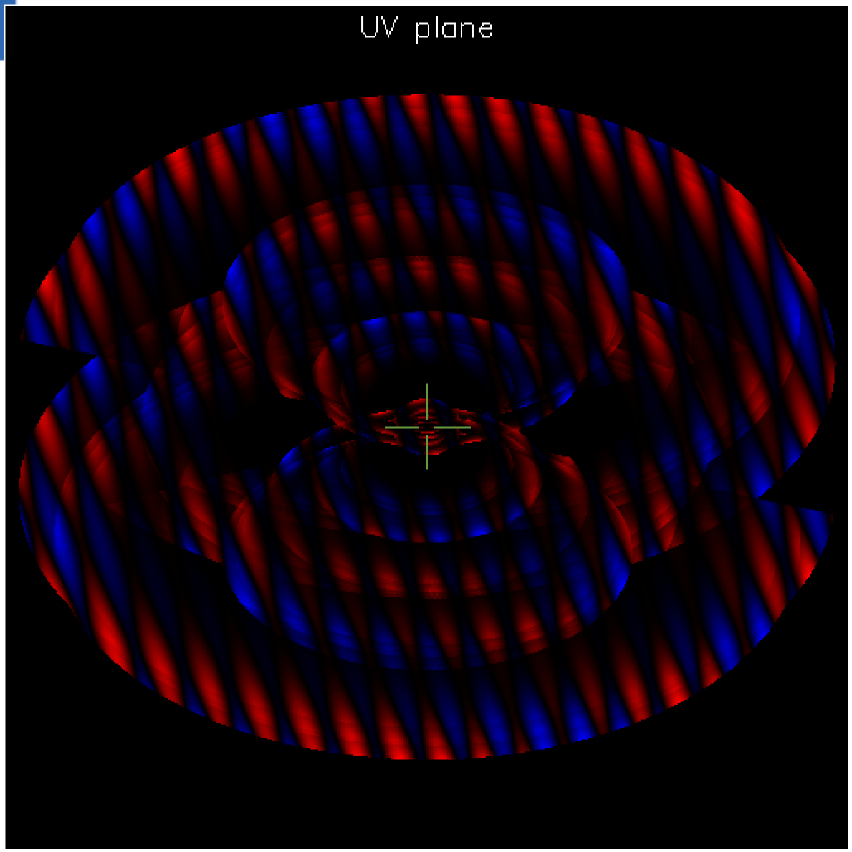
2. Aips task IMAGR parameter UVBOX: effectively smooths the weight function.

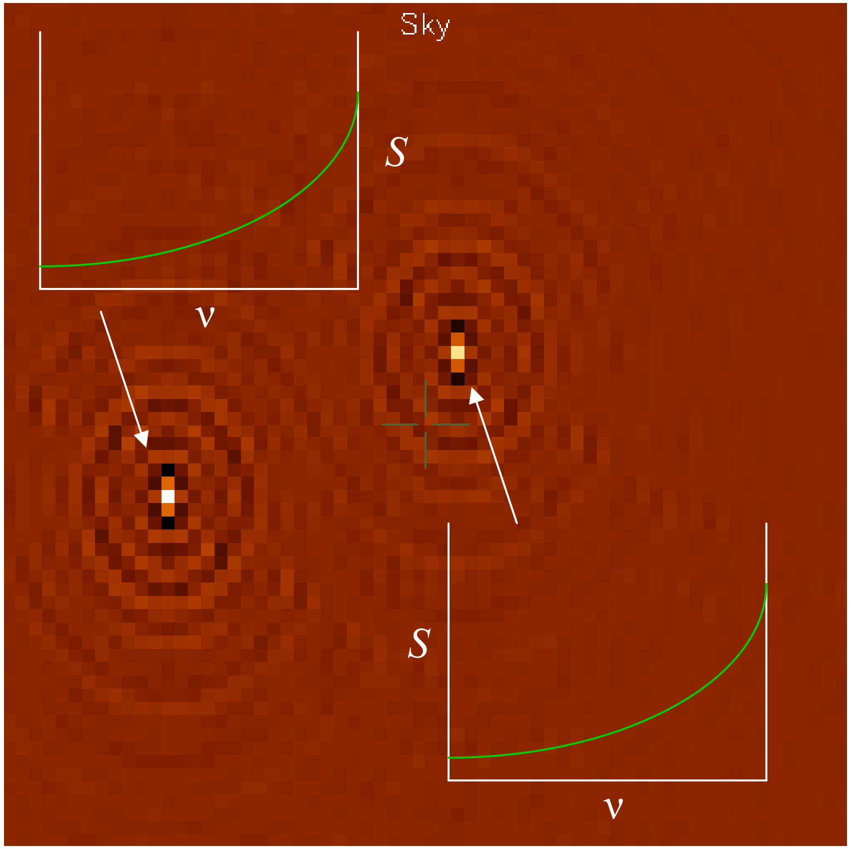
5. How to Clean Wide-Band Data.

Drawbacks of wide-band:

real objects often have non-flat spectra.

Where both point sources have identical spectra:

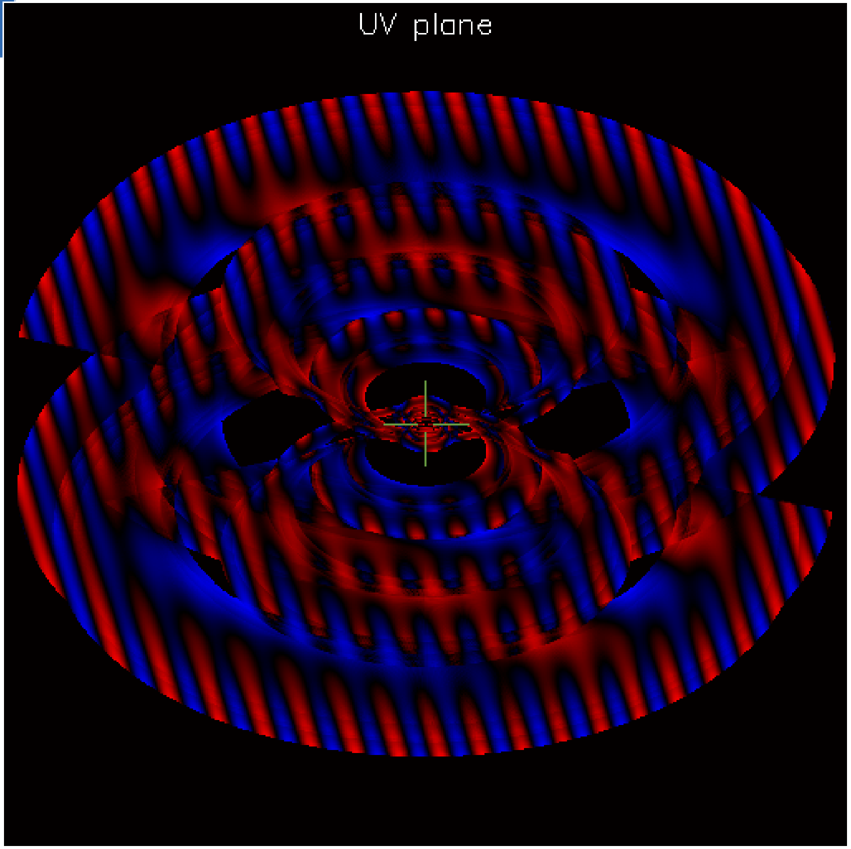


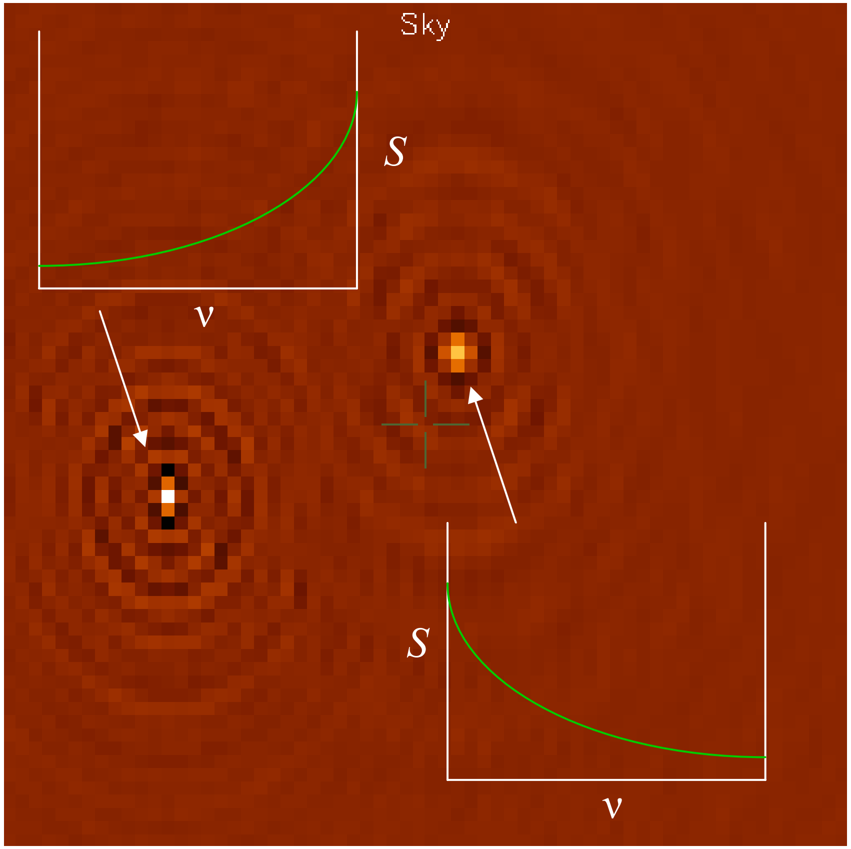


Spectral indices both +10.0 (!!!)

Drawbacks of wide-band:
real objects often have non-flat spectra.

More realistic: different spectra:





This will **not** clean away.

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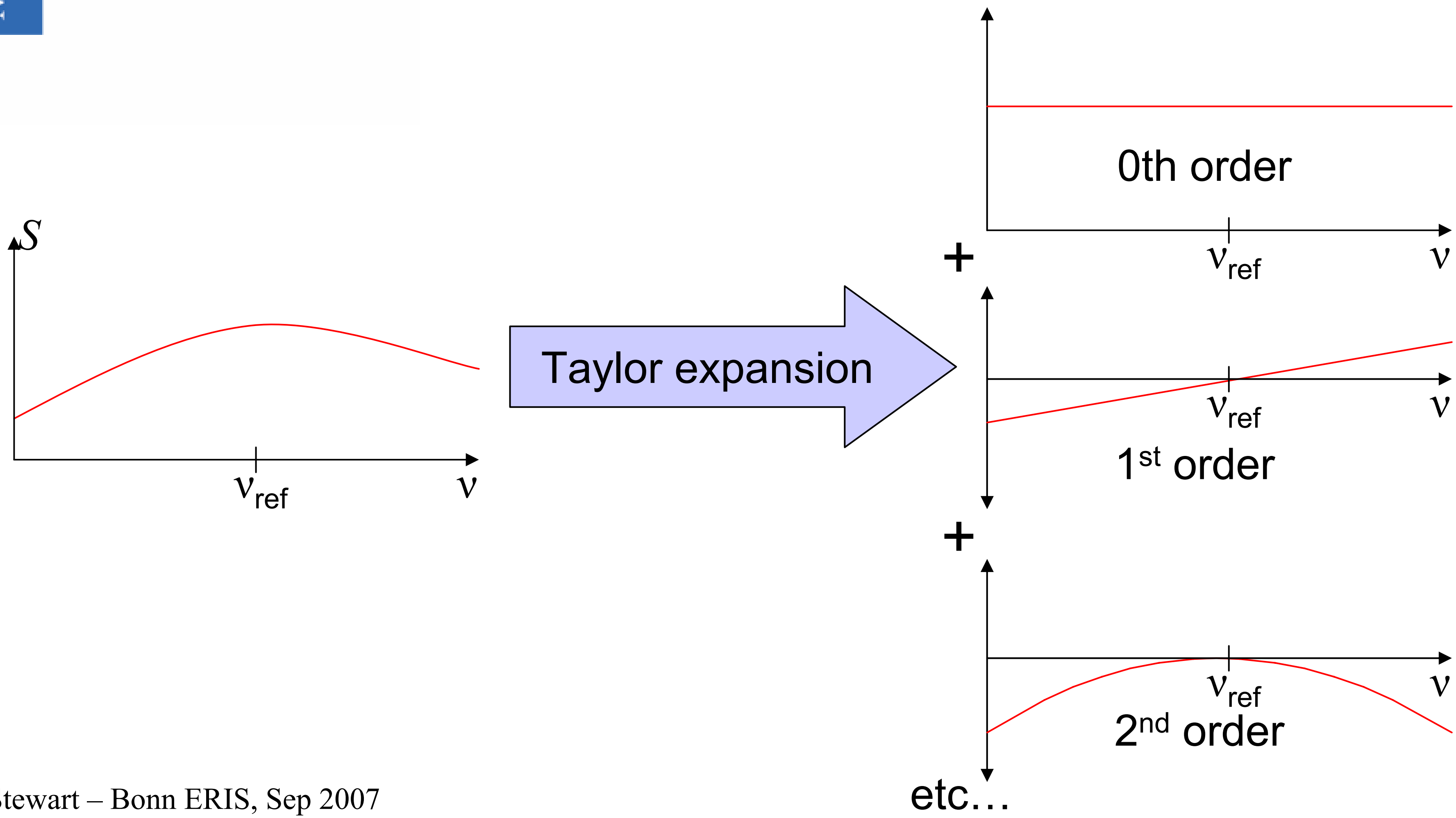
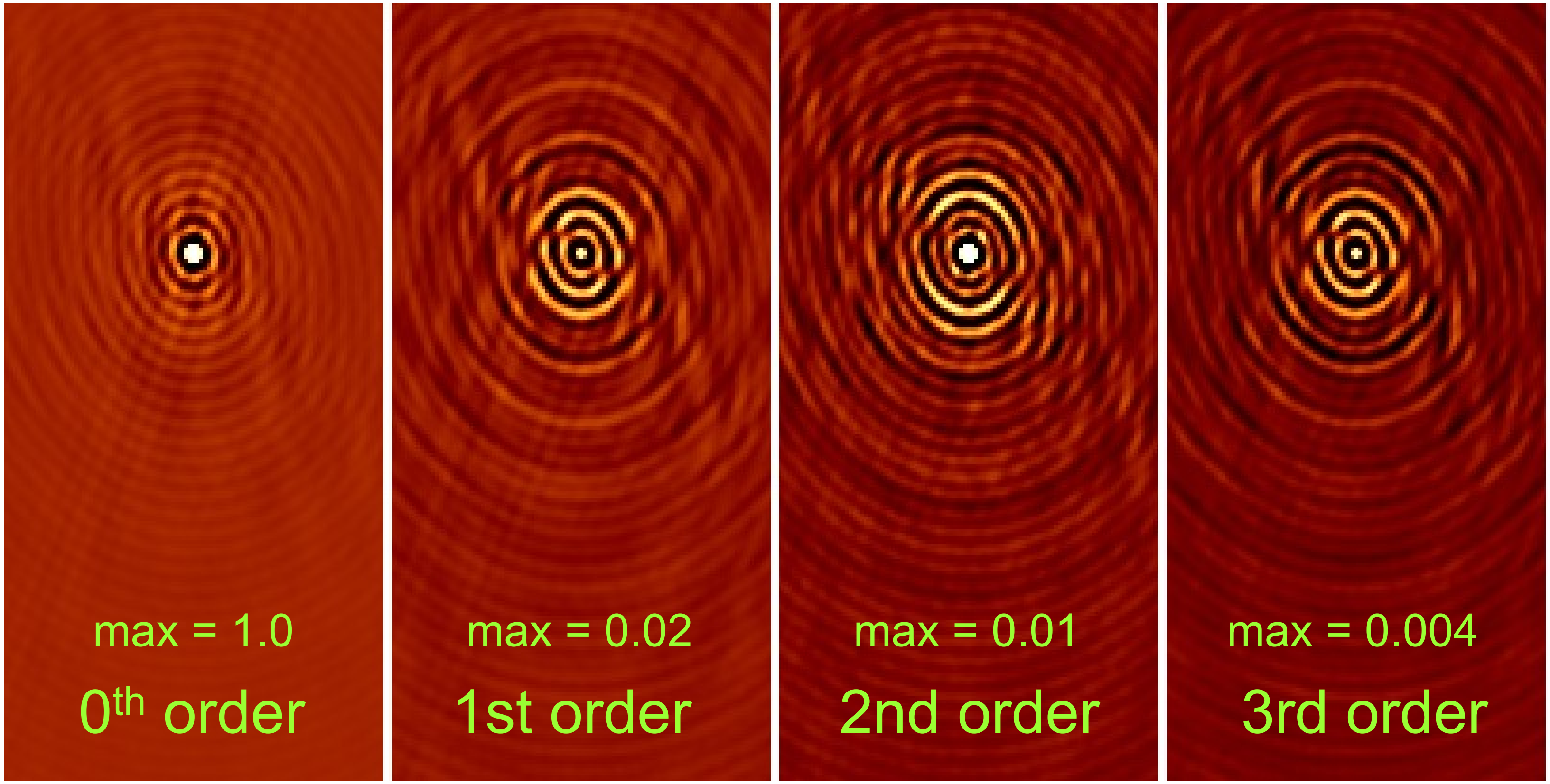
Sault-Wieringa algorithm: a generalized CLEAN.

Sault R J & Wieringa M H: A&A Suppl. Ser. 108, 585 (1994)



Taylor-term beams



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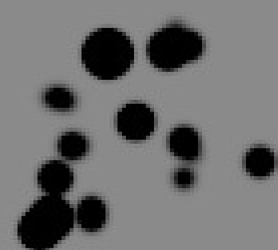
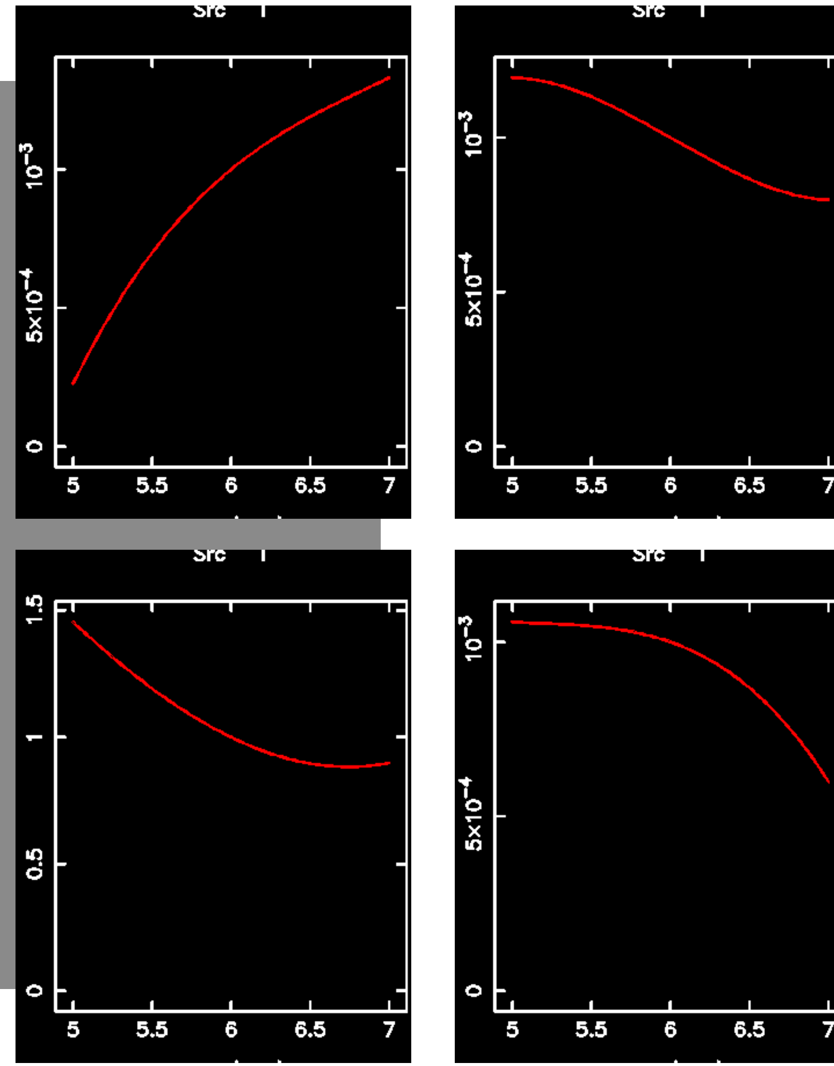
Testing the S-W algorithm: the input simulation

19 point sources
from 0.001 to 1 Jy



Spectra: cubics, with
random coefficients.

eg

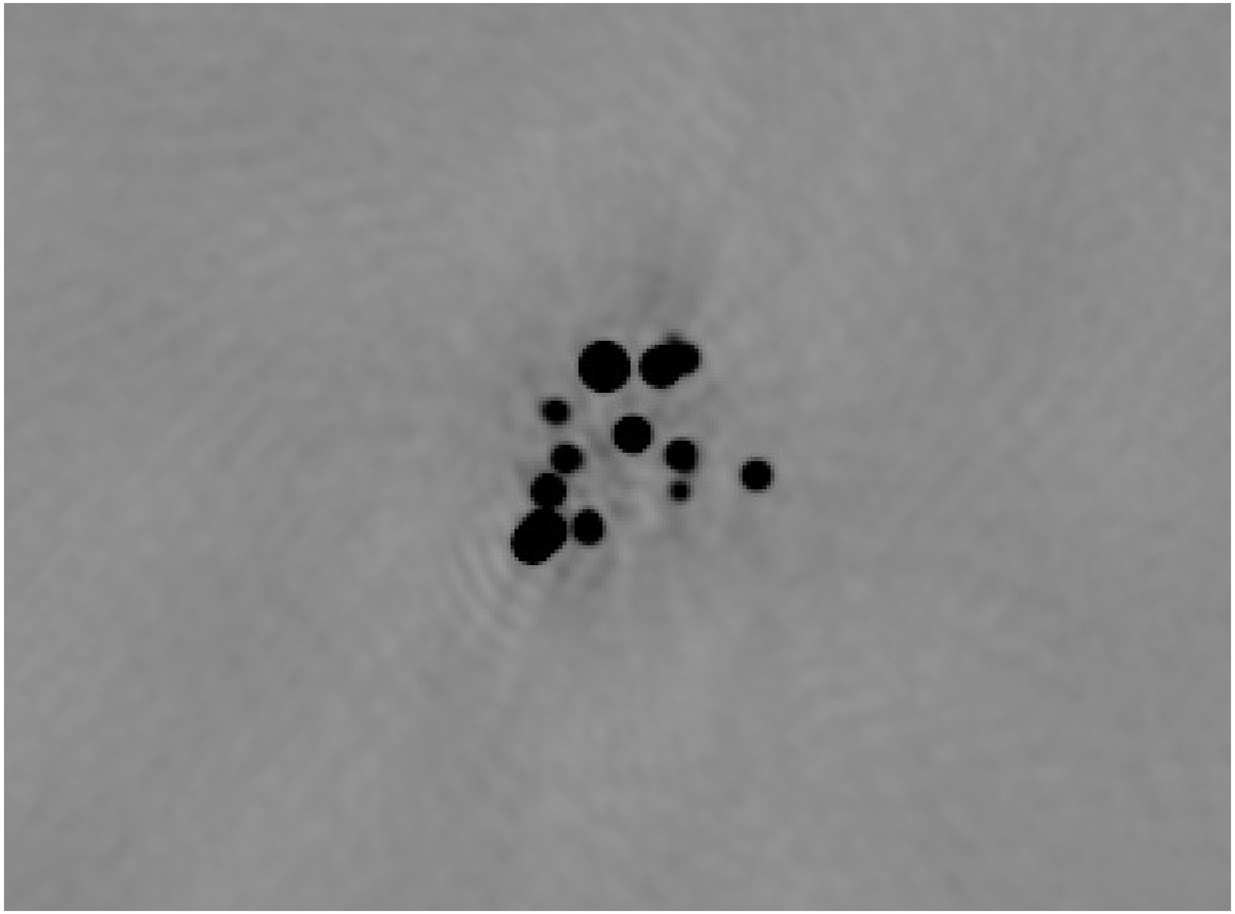


ν (GHz)

Alternate cleaning:
(i) 1000 Clark clean cycles (IMAGR)

…not good.

Alternate cleaning:
(ii) each chan cleaned, then co-added.

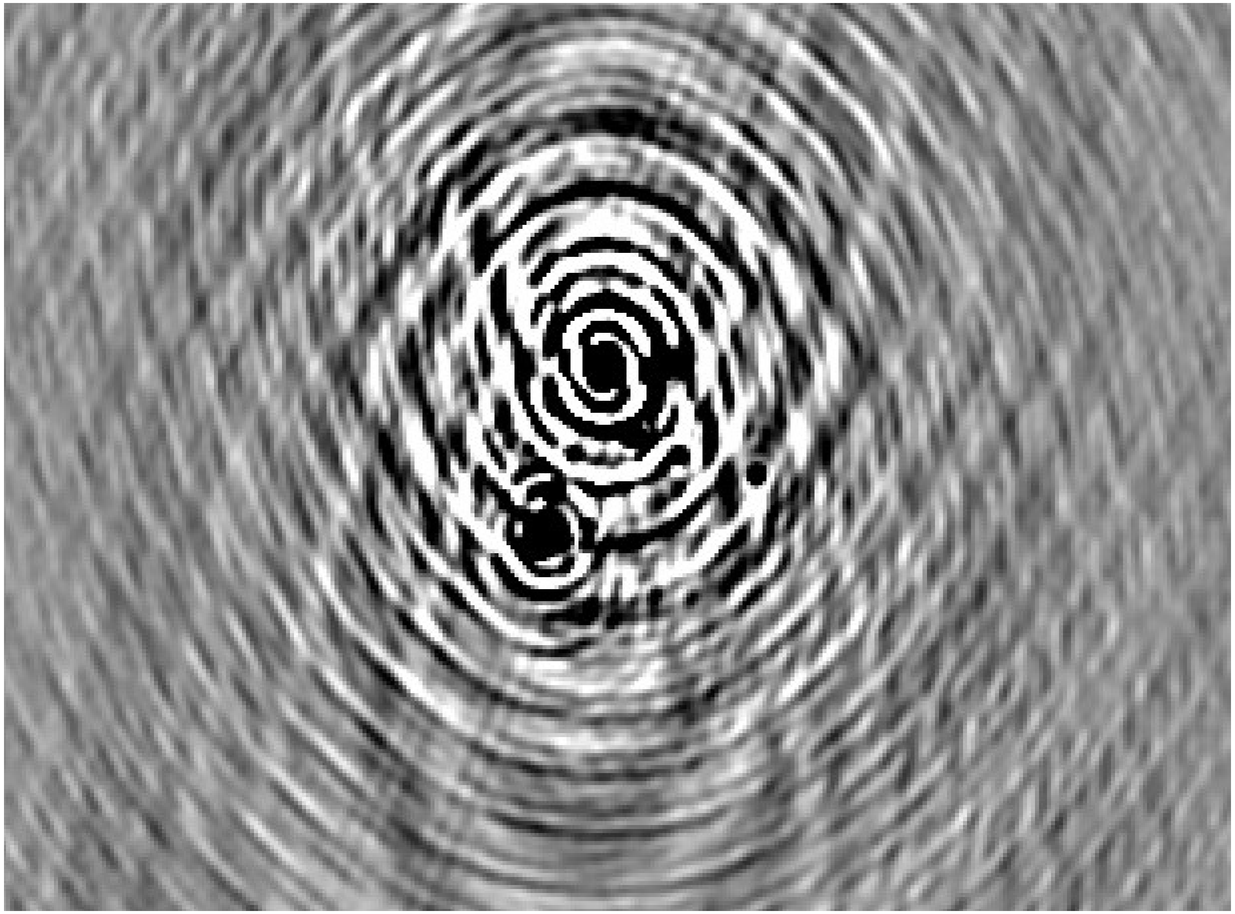


…pretty good, but do we lose faint sources?

S-W clean to various orders

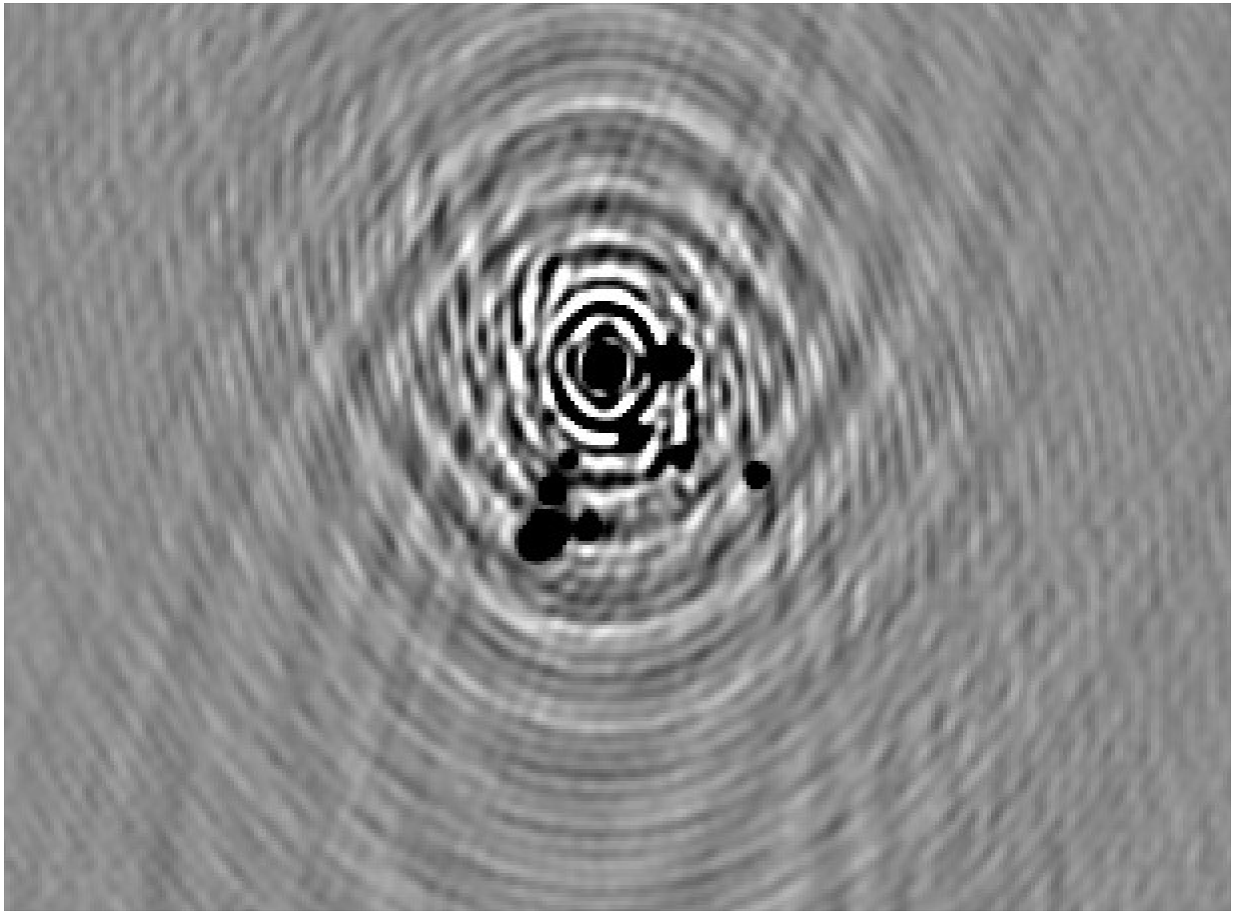
(All 1000 cycles with gain = 0.1)

0th order (equivalent to Hoegbom clean)



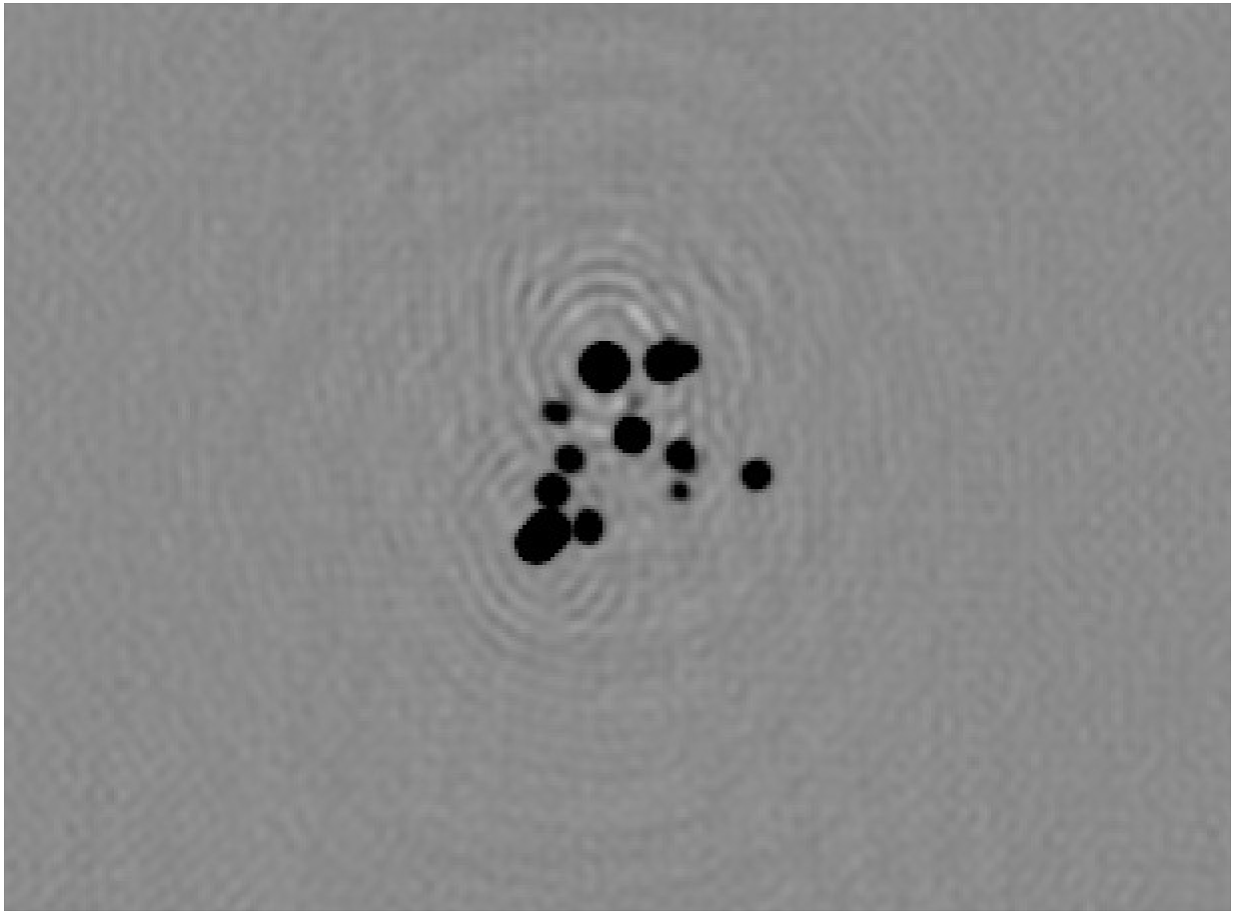
S-W clean to various orders

1st order



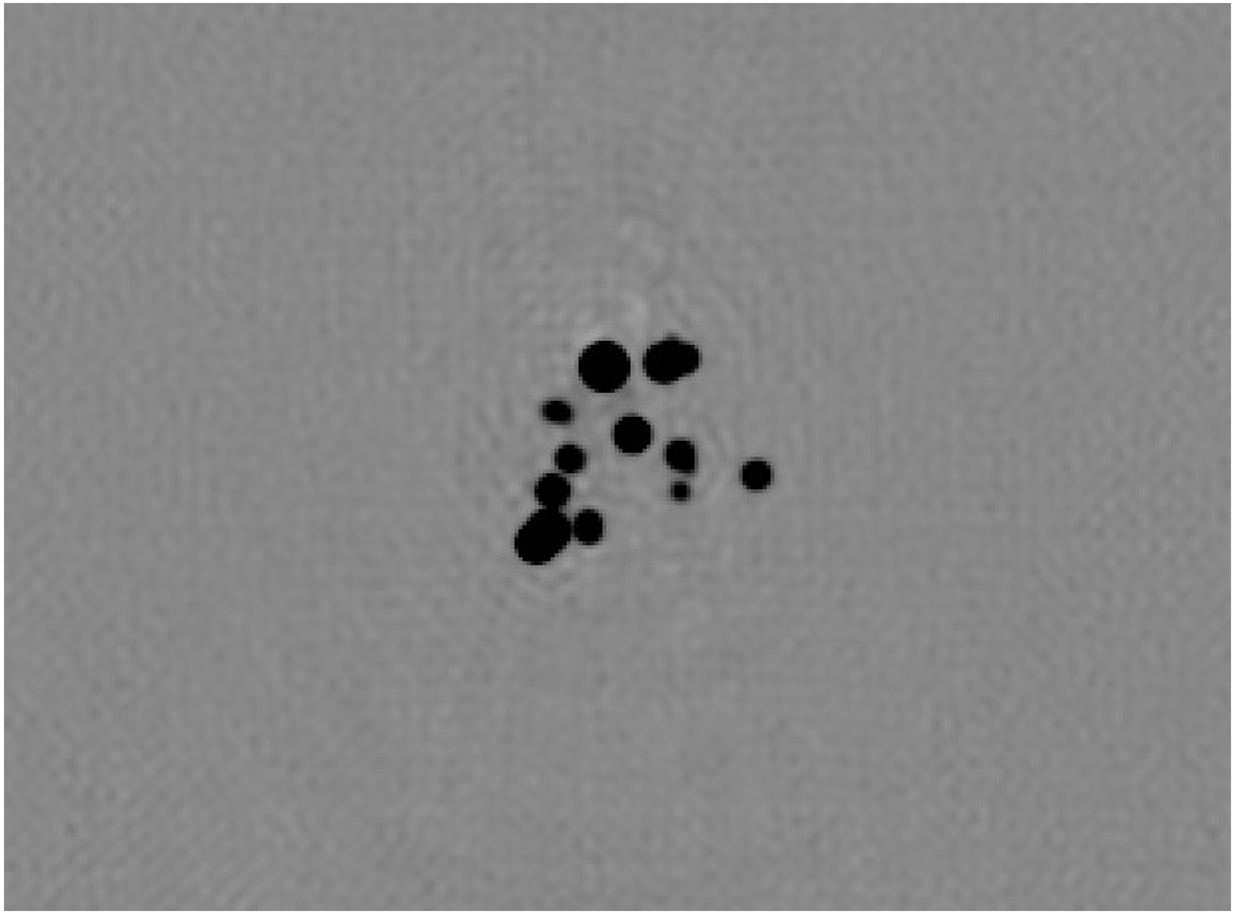
S-W clean to various orders

2nd order



S-W clean to various orders

3rd order



Not much left but numerical noise.

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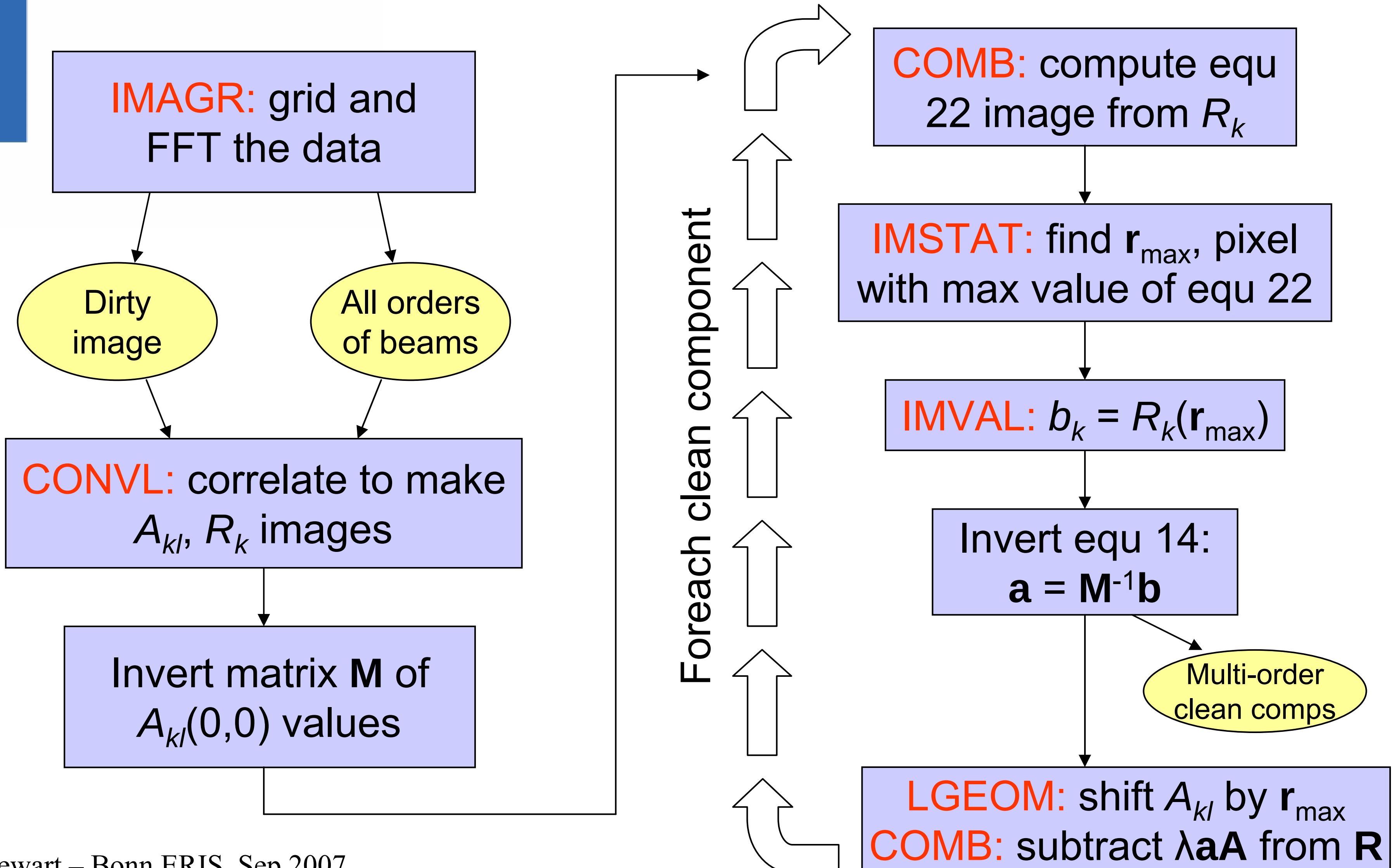
S-W Implementation in Parseltongue



Wide-Band Conclusions:

- Greater sensitivity.
- Better coverage -> cleaner beam. This reduces the need for cleaning;
- but cleaning is more elaborate process.
- Weighting schemes are important.
- Large data sets -> parallel processing needed.
- Primary beam size varies across band.
- Ionospheric Faraday rotation varies across band(?)
- Calibration – easier or harder? Certainly more interesting…!