

Understanding Radio Interferometry

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RADIO NOISE AND RADIO WAVES

RADIO SOURCE EMISSION

RECEIVED NOISE POWER

UNIT OF FLUX DENSITY, S jansky J_y
 $= 10^{-26} \text{ W Hz}^{-1} \text{ m}^{-2}$

($\equiv$ optical magnitude)

SURFACE BRIGHTNESS (of extended source)

FLUX DENSITY / UNIT SURFACE AREA b

UNIT J_y / beam area
 (in radio maps)

PHYSICAL UNIT Brightness temperature T_b

The temperature at which a black body would give the same radio emission per unit area

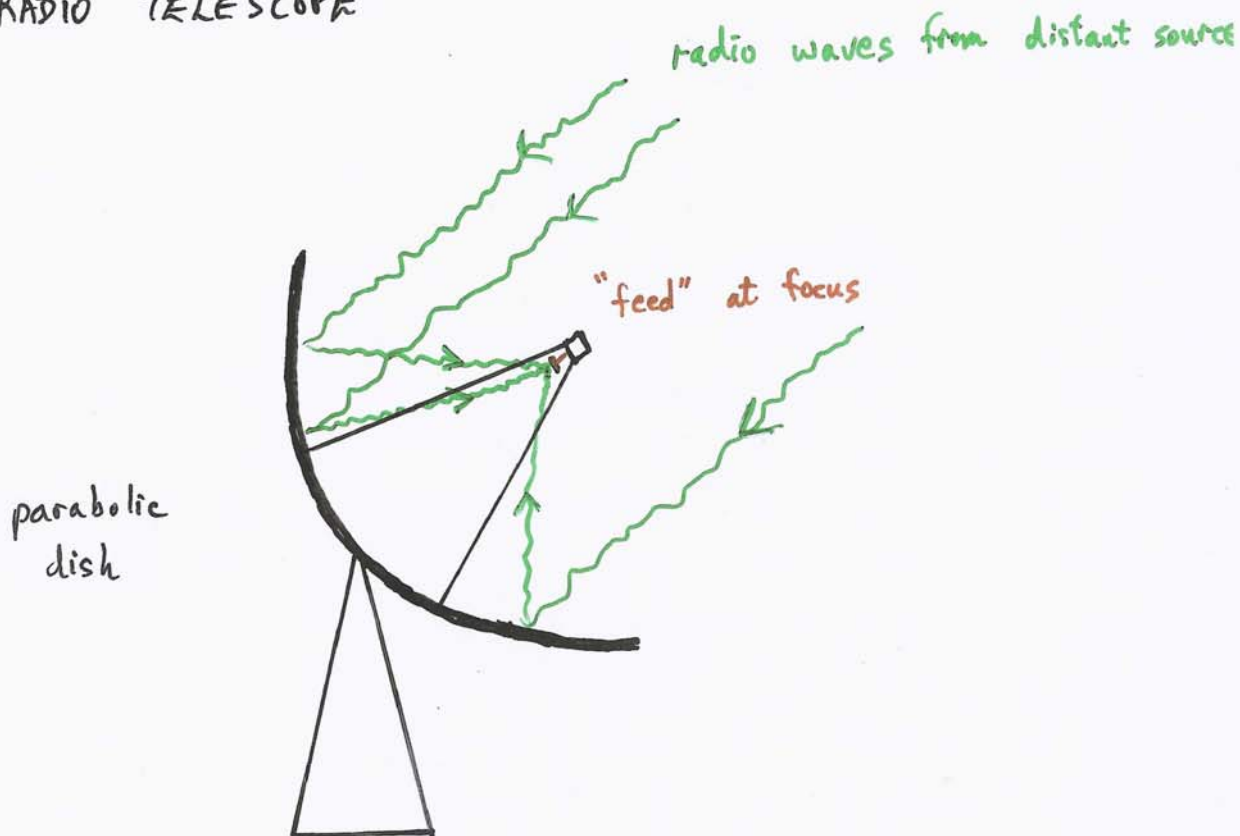
$$T_b = \frac{S \lambda^2}{2 k \Omega}$$

k = Boltzmann's constant
 Ω = observed solid angle

SURFACE BRIGHTNESS IS INTRINSIC (DISTANCE INDEPENDANT)

MEASURING RADIO SOURCE EMISSION

RADIO TELESCOPE



DESCRIPTION IN TERMS OF NOISE POWER



FEED PRODUCES NOISE VOLTAGE REPRESENTING NOISE POWER FROM RADIO SOURCE

COMPARE WITH JOHNSON NOISE FROM RESISTOR OF TEMPERATURE T ($= kT$ per unit bandwidth)

T_{ant} from source

$$T_{ant} = g \cdot S$$

T_{rec} from receiver

T_{sys} = total system temperature = $T_{rec} + T_{ant}$

$$S = g^{-1} (T_{sys}(\text{on-source}) - T_{sys}(\text{off-source}))$$

DESCRIPTION IN TERMS OF VOLTAGES



$$\underline{E} = \underline{E} \cos(2\pi f_R t + \phi_R)$$

Amplifier

$$V_R = V_R \cos(2\pi f_R t + \phi_R)$$

f_R

Mixer

f_L

Local oscillator

$$V_L = V_L \cos(2\pi f_L t + \phi_L)$$

$f_I = f_R - f_L$

Amplifier

$$V_I = V_I \cos(2\pi (f_R - f_L) t + \phi_R + \phi_L)$$

Square-law Detector

$$\langle V_I^2 \rangle \propto T_{sys}$$

RF

IF

THE FEED

(e.g. a dipole)

THE E FIELD IS A 2-D VECTOR
BUT V_R IS A SCALAR

⇒ THE FEED ONLY SAMPLES ONE COMPONENT OF THE E FIELD (A LINEAR OR CIRCULAR POLARIZATION MODE)

THE RF SIGNAL BAND

$$f_R \Rightarrow \underline{f_R \pm \frac{b}{2}}$$

$b = \text{bandwidth}$

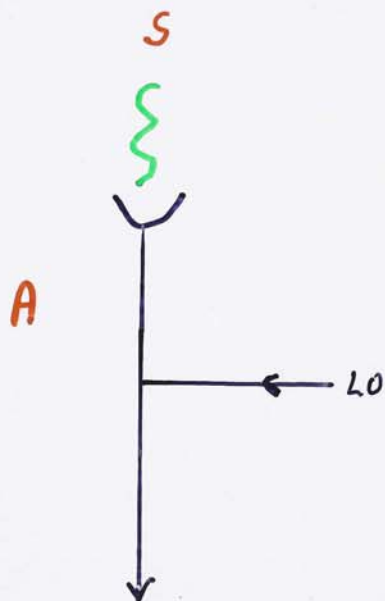
- NOISE VOLTAGE IS THE SUM OF SINUSOIDAL OSCILLATIONS AT ALL FREQUENCIES WITHIN THE BAND b , EACH WITH ARBITRARY RELATIVE PHASE
- FOR A TIME SHORTER THAN b^{-1} THE VOLTAGE BEHAVES LIKE A PURE SINGLE FREQUENCY AT f_R
- AFTER A TIME LONGER THAN b^{-1} THE SIGNAL PHASE HAS CHANGED ARBITRARILY (DUE TO RELATIVE ROTATION ACROSS THE BAND OF THE INDIVIDUAL COMPONENT FREQUENCIES)

$$\langle V_A V_B \rangle = 0$$

THE IF SIGNAL BAND

$$\underline{f_i \pm b}$$

THE NOISE CHARACTERISTICS OF THE IF
SIGNAL PRESERVE THOSE OF THE RF SIGNAL
BUT CONTAIN A PHASE-SHIFT DUE TO THE
PHASE OF THE LOCAL OSCILLATOR.



$$V_I = V_A + V_S$$

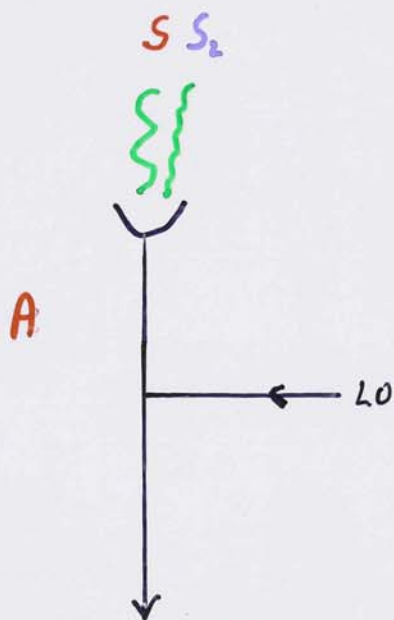
receiver source

$$\langle V_I^2 \rangle = \langle (V_A + V_S)^2 \rangle \quad t > b^{-1}$$

$$= \langle V_A^2 \rangle + \langle V_S^2 \rangle + \langle 2V_A V_S \rangle$$

$$= \langle V_A^2 \rangle + \langle V_S^2 \rangle$$

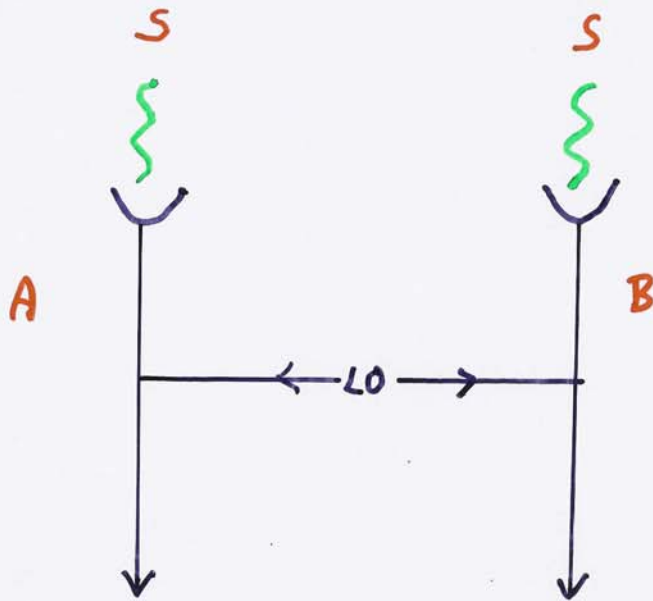
$$\propto T_{\text{rec}} + T_{\text{nr}}$$



$$V_I = \underset{\text{receiver}}{V_A} + \underset{\text{source}}{V_S}$$

$$\begin{aligned} \langle V_I^2 \rangle &= \langle (V_A + V_S)^2 \rangle & t > b^{-1} \\ &= \langle V_A^2 \rangle + \langle V_S^2 \rangle + \langle 2V_A V_S \rangle \\ &= \langle V_A^2 \rangle + \langle V_S^2 \rangle \\ &\propto T_{\text{REC}} + T_{\text{ANT}} \end{aligned}$$

$$\begin{aligned} \langle V_I^2 \rangle &= \langle (V_A + V_S + V_{S_2})^2 \rangle \\ &= \langle V_A^2 \rangle + \underbrace{\langle V_S^2 \rangle + \langle V_{S_2}^2 \rangle}_{T_{\text{ANT}}} \\ &\propto T_{\text{REC}} + T_{\text{ANT}} \end{aligned}$$



$$V_1 = \underset{\text{receiver}}{V_A} + \underset{\text{source}}{V_S}$$

$$V_2 = \underset{\text{receiver}}{V_B} + \underset{\text{source}}{V_S}$$

ADDING INTERFEROMETER (MICHELSON)

$$\langle (V_1 + V_2)^2 \rangle = \langle V_A^2 \rangle + \langle V_B^2 \rangle + 4 \langle V_S^2 \rangle$$

MULTIPLYING INTERFEROMETER

$$\begin{aligned} \langle V_1 \cdot V_2 \rangle &= \langle (V_A V_S + V_A V_B + V_S V_B + V_S^2) \rangle \\ &= \langle V_S^2 \rangle \\ &\propto S \end{aligned}$$

WITH A PHASE ERROR ϕ_i

$$V_1 = V_A + V_S$$

$$V_2 = V_B + V_S$$

$$V_S = V_s \cos(2\pi f_I)$$

$$V_S = V_s \cos(2\pi f_I + \phi_i)$$

$$\underline{\langle V_1 \cdot V_2 \rangle = \langle V_s^2 \rangle \cos \phi_i \propto S \cos \phi_i}$$

MULTIPLIED OUTPUT DECREASED BY $\cos \phi_i$

RECOVER FULL POWER WITH SINE AND COSINE MULTIPLIERS

$$V_S = V_s \cos(2\pi f_I)$$

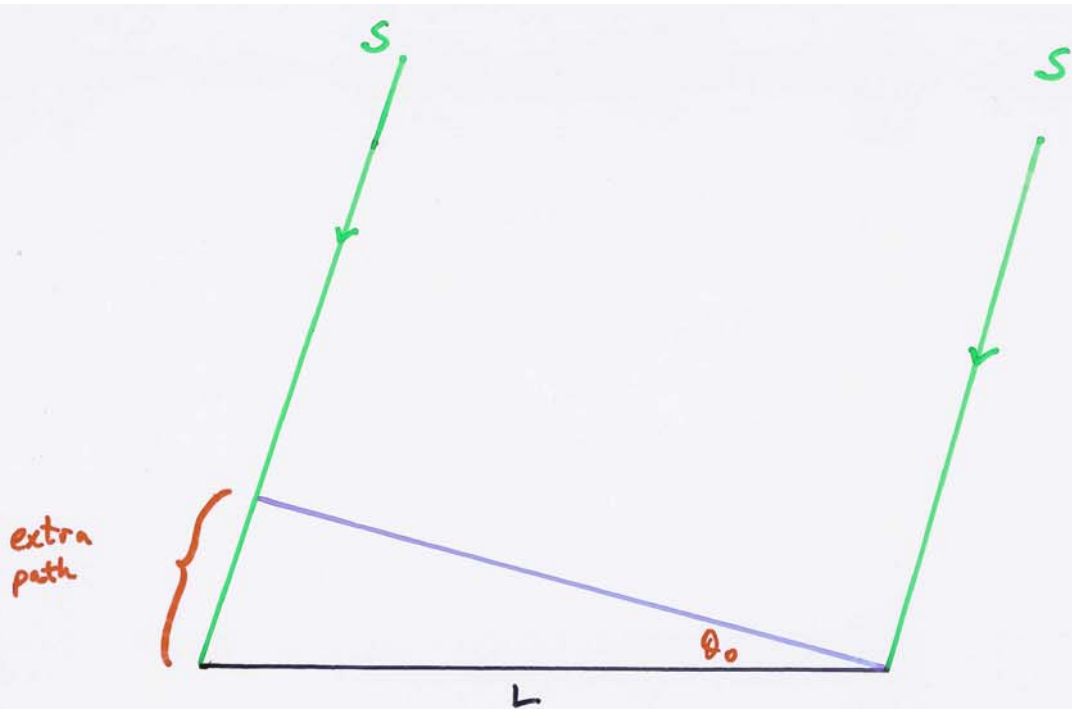
$$V_S = V_s \cos(2\pi f_I + \phi_i - \frac{\pi}{2})$$

$$\langle V_1 \cdot V_2 \rangle = \langle V_s^2 \rangle \sin \phi_i$$

COSINE AND SINE MULTIPLIERS PRODUCE
COMPLEX OUTPUT

$$\langle V_s^2 \rangle e^{i\phi_i} \propto S e^{i\phi_i}$$

IGNORE SUCH PHASE ERRORS FOR NOW



$$\text{EXTRA PATH} = L \sin \theta_0$$

$$\text{EXTRA DELAY, } \tau_c = \frac{L \sin \theta_0}{c}$$

$$\text{ADDITIONAL PHASE, } \phi = 2\pi L \frac{\sin \theta_0}{c} \cdot f_R = 2\pi L \frac{\sin \theta_0}{\lambda_R}$$

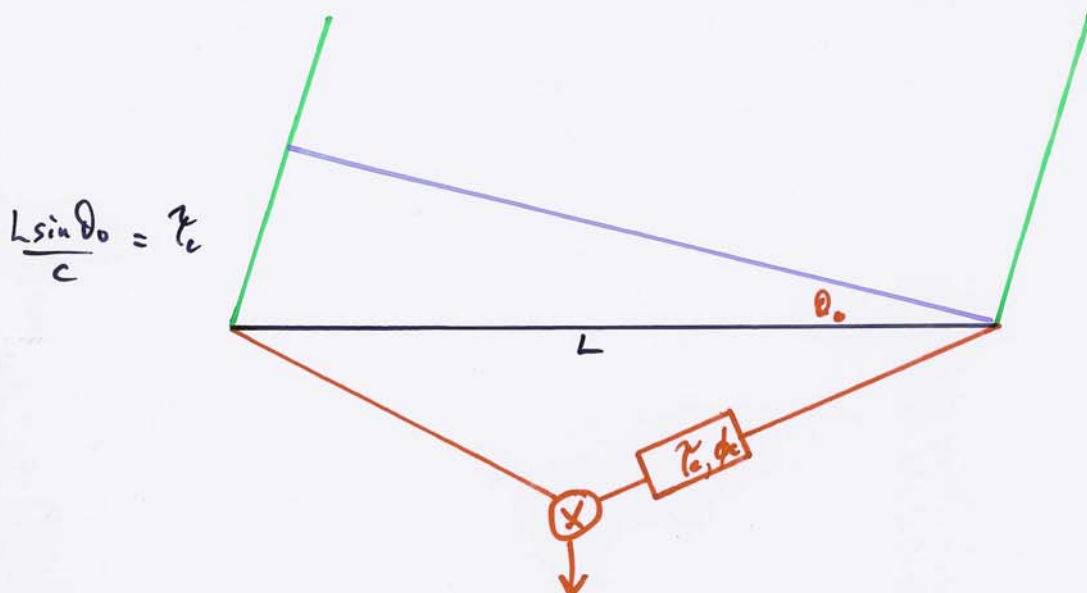
MULTIPLYING INTERFEROMETER OUTPUT

$$\cos x \quad S_c = S \cos \left(\frac{2\pi L \sin \theta_0}{\lambda_R} \right)$$

$$\sin x \quad S_s = S \sin \left(\frac{2\pi L \sin \theta_0}{\lambda_R} \right)$$

$$S = S e^{i \left[\frac{2\pi L \sin \theta_0}{\lambda_R} \right]}$$

RECOVER FULL SIGNAL WITH PATH COMPENSATION



IF PATH DELAY COMPENSATION τ_c

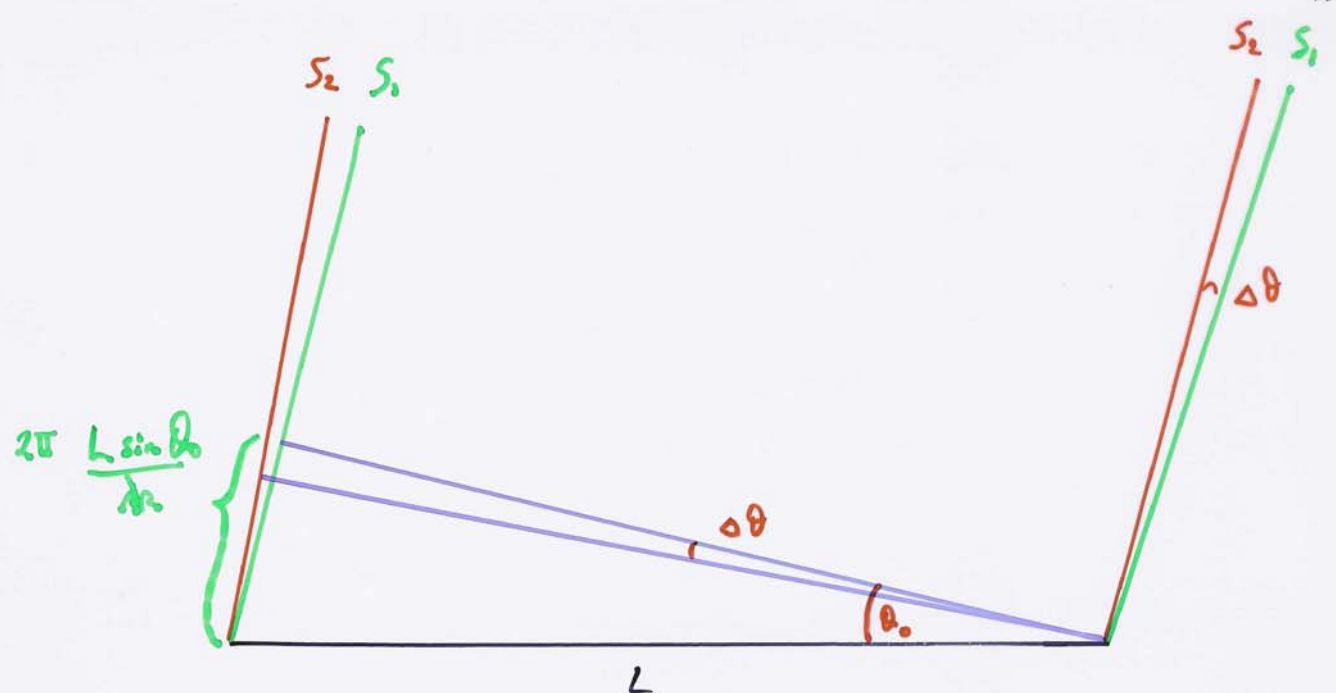
$$\phi_I = 2\pi \frac{L \sin \theta_0}{c} \cdot f_I$$

R.F. PHASE PATH

$$\phi_R = 2\pi \frac{L \sin \theta_0}{c} \cdot f_R$$

EXTRA PHASE ROTATION ϕ_c

$$\begin{aligned} \phi_c &= 2\pi \frac{L \sin \theta_0}{c} [f_R - f_I] \\ &= 2\pi \frac{L \sin \theta_0}{c} \cdot f_c \end{aligned}$$



PATH COMPENSATION FOR S_1

$$S_c = S_1$$

UNCOMPENSATED PATH FOR S_2

$$\Delta \phi = \frac{d}{d\theta} \left[2\pi \frac{L \sin \theta_0}{\lambda_r} \right] \Delta \theta$$

$$\Delta \phi = 2\pi \frac{L \cos \theta_0}{\lambda_r} \cdot \Delta \theta$$

$$\Delta \phi = 2\pi q \cdot \Delta \theta$$

$$q = \frac{L \cos \theta_0}{\lambda_r}$$

(units: wavelengths)

$$S = S_2 e^{i[2\pi q \Delta \theta]}$$

q IS A MEASURE OF THE INTERFEROMETER RESOLUTION

- IT IS THE FACTOR BY WHICH ANGULAR CHANGES ON THE SKY ARE MAGNIFIED TO PRODUCE PHASE CHANGES IN THE INTERFEROMETER OUTPUT
- IT IS THE PROJECTED BASELINE LENGTH IN THE DIRECTION OF THE SOURCE, MEASURED IN WAVELENGTHS.

TOTAL INTERFEROMETER RESPONSE TO S_1 AND S_2

$$S_{\text{tot}} = S_1 + S_2 e^{i2\pi q \Delta\theta}$$

FOR MANY POINT SOURCES

$$S_{\text{tot}} = S_1 + S_2(\Delta\theta_2) e^{i2\pi q \Delta\theta_2} \dots + S_i(\Delta\theta_i) e^{i2\pi q \Delta\theta_i}$$

FOR CONTINUOUS DISTRIBUTION

$$dS = b(\Delta\theta) d\theta$$

$$S_{\text{tot}} = \int b(\Delta\theta) e^{i2\pi q \Delta\theta} \cdot d\theta$$

$$\theta = \theta - \theta_0$$

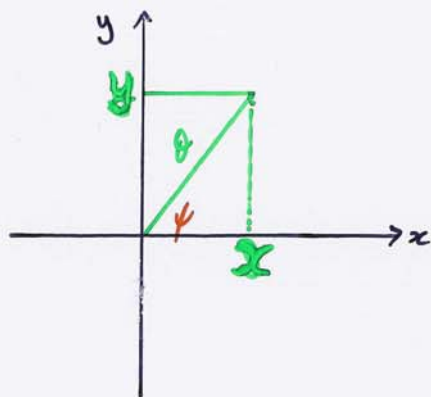
$$S_{\text{tot}} = \int b(\theta) e^{i2\pi q \theta} d\theta$$

INTERFEROMETER FLUX IS ONE COMPONENT OF
THE FOURIER TRANSFORM OF $b(\theta)$
SPATIAL FREQUENCY q

2-D FORMALISM

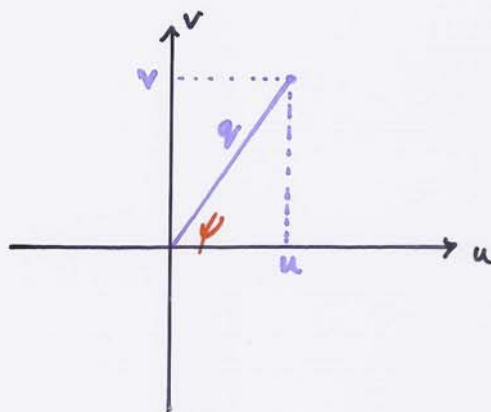
- TELESCOPES A, B AND SOURCE DIRECTION DEFINE AN ARC ON THE SKY

(θ and ψ refer to this direction)



ANGLES ON THE SKY

$$\begin{aligned} x &= \theta \cos \psi \\ y &= \theta \sin \psi \end{aligned}$$



RESOLUTION ON THE SKY

$$\begin{aligned} u &= \theta \cos \psi \\ v &= \theta \sin \psi \end{aligned}$$

$$S_{\text{Tot}} = \int b(x, y) e^{i2\pi(ux + vy)} dx dy$$

EFFECT OF EARTH ROTATION

$$\theta_0 = \theta_0(t)$$

RESOLUTION CHANGES WITH TIME

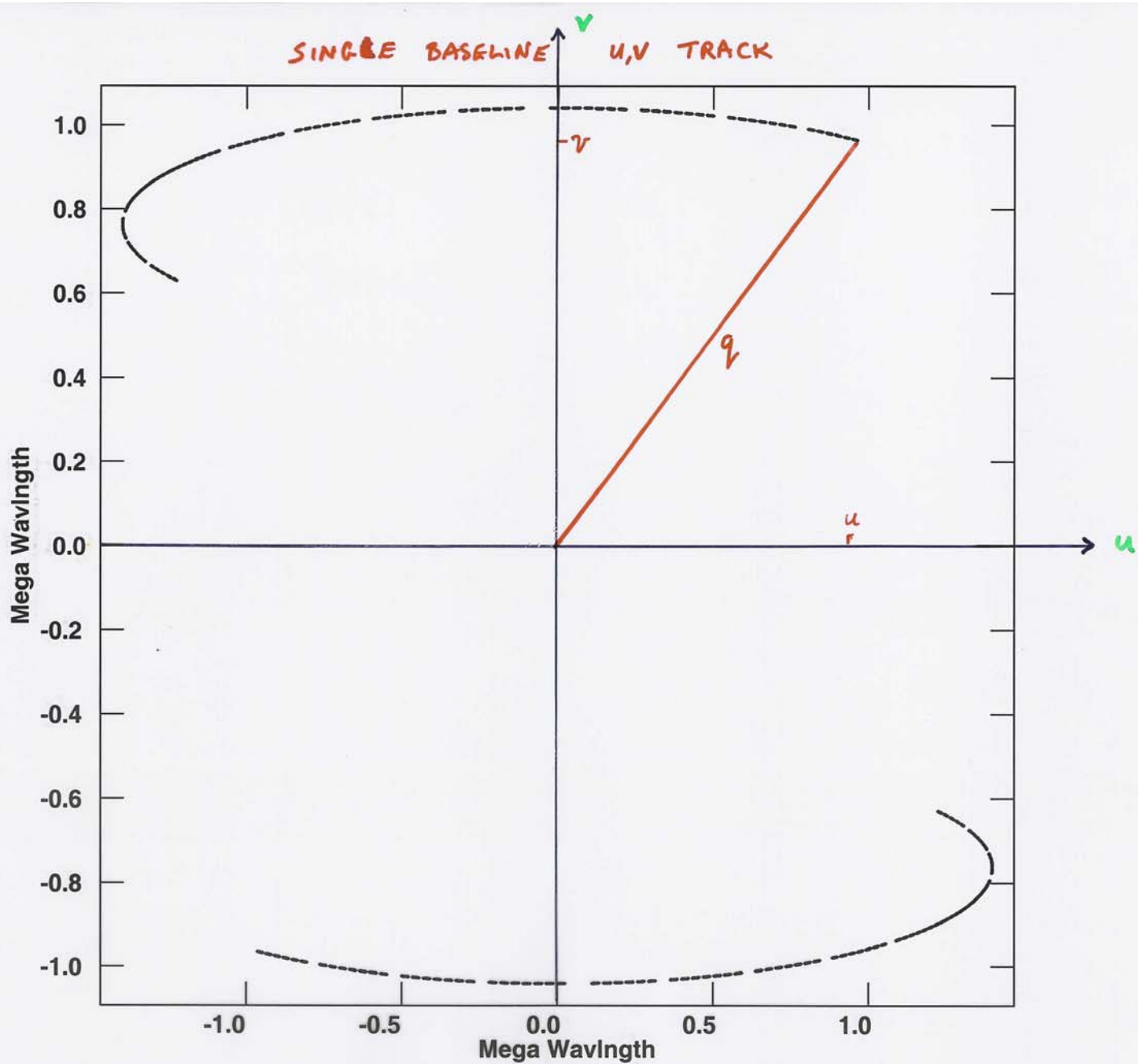
$$\frac{L \cos \theta_0}{\lambda_R} = q = q(t) \quad \begin{matrix} u(t) \\ v(t) \end{matrix}$$

u, v track with time traces an ellipse

REQUIRED PATH COMPENSATION CHANGES WITH TIME

$$\frac{d}{dt} \left[\frac{L \sin \theta_0}{c} \right] = \frac{\partial \tau_c}{\partial t} \quad \text{DELAY TRACKING (AT IF)}$$

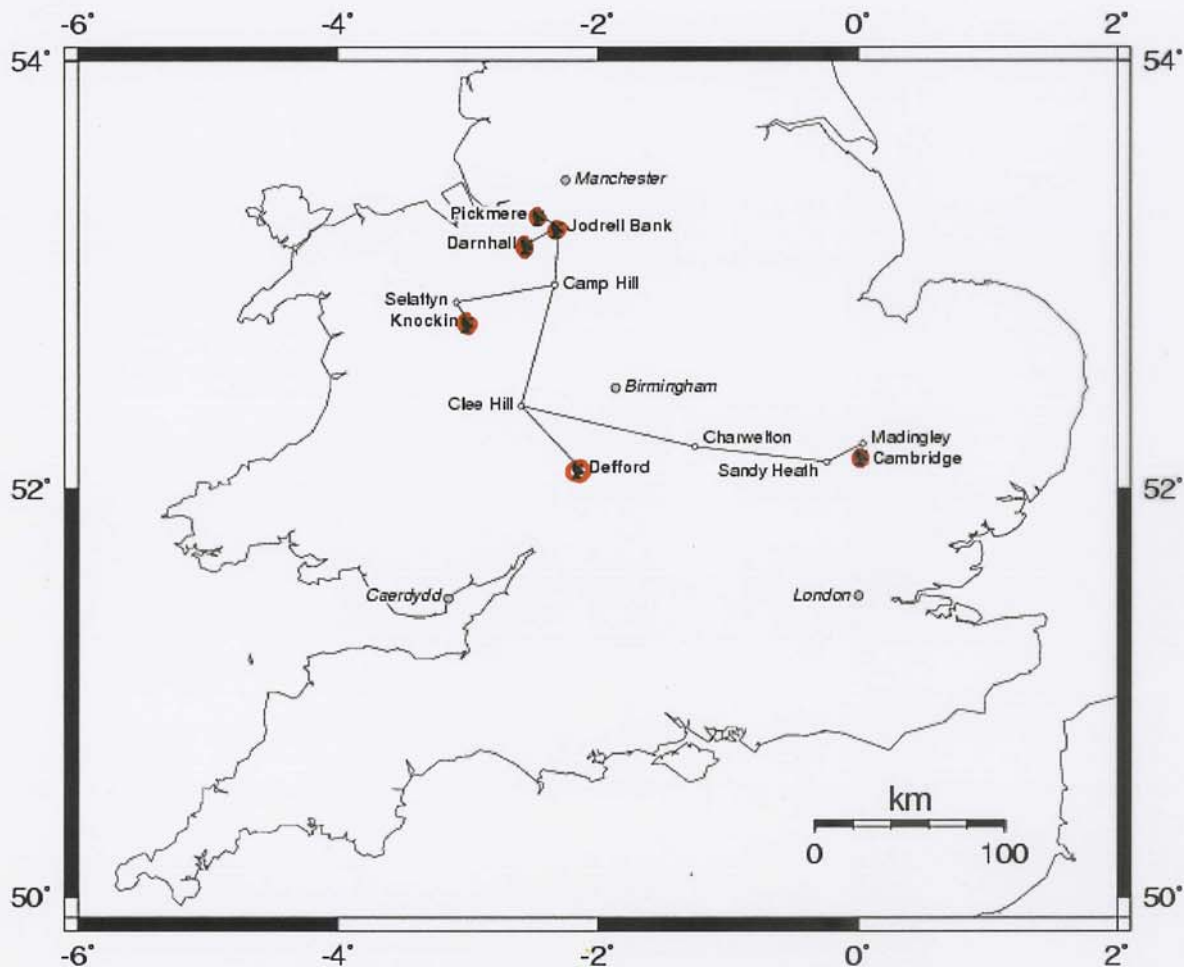
$$\frac{d}{dt} \left[\frac{2\pi L \sin \theta_0}{c} \cdot f_L \right] = \frac{\partial \phi}{\partial t} \quad \text{FRINGE ROTATION}$$



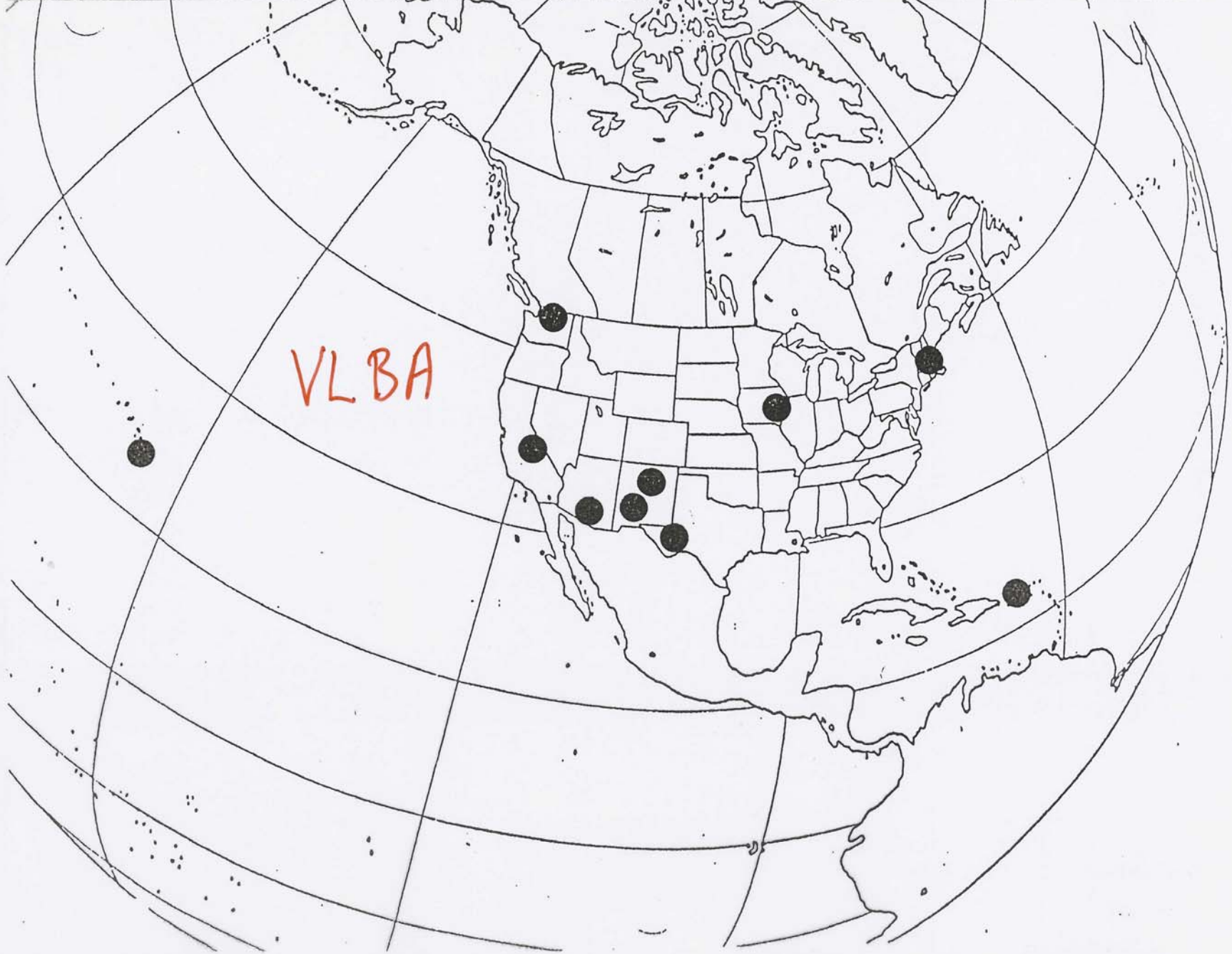
MERLIN

MERLIN

Merlin stands for the Multi-Element Radio-Linked Interferometer Network, Jodrell Banks array of six observing stations that together form a powerful telescope with an effective aperture of over 217 kilometres.



At a wavelength of six centimeters MERLIN has a maximum resolution of 40 milli arc seconds, about twenty times better than can commonly be achieved by the best ground-based telescopes, and comparable to the Hubble Space Telescope. Such resolving power is equivalent to measuring the diameter of a one-pound coin from a distance of 100 kilometres.



MULTI-ANTENNA INTERFEROMETER ARRAYS

ARRAY	TELESCOPES	BASELINES
MERLIN	6	15
VLA	27	351
VLBA	10	45

MERLIN

15 BASELINES

15 SIMULTANEOUS MEASUREMENTS OF
DIFFERENT FOURIER COMPONENT q_i
(u_i v_i)

15 TRACKS IN uv PLANE
(uv COVERAGE)
(SAMPLING FUNCTION)

AFTER A (e.g.) 12-h SOURCE TRACK

U,V COVERAGE

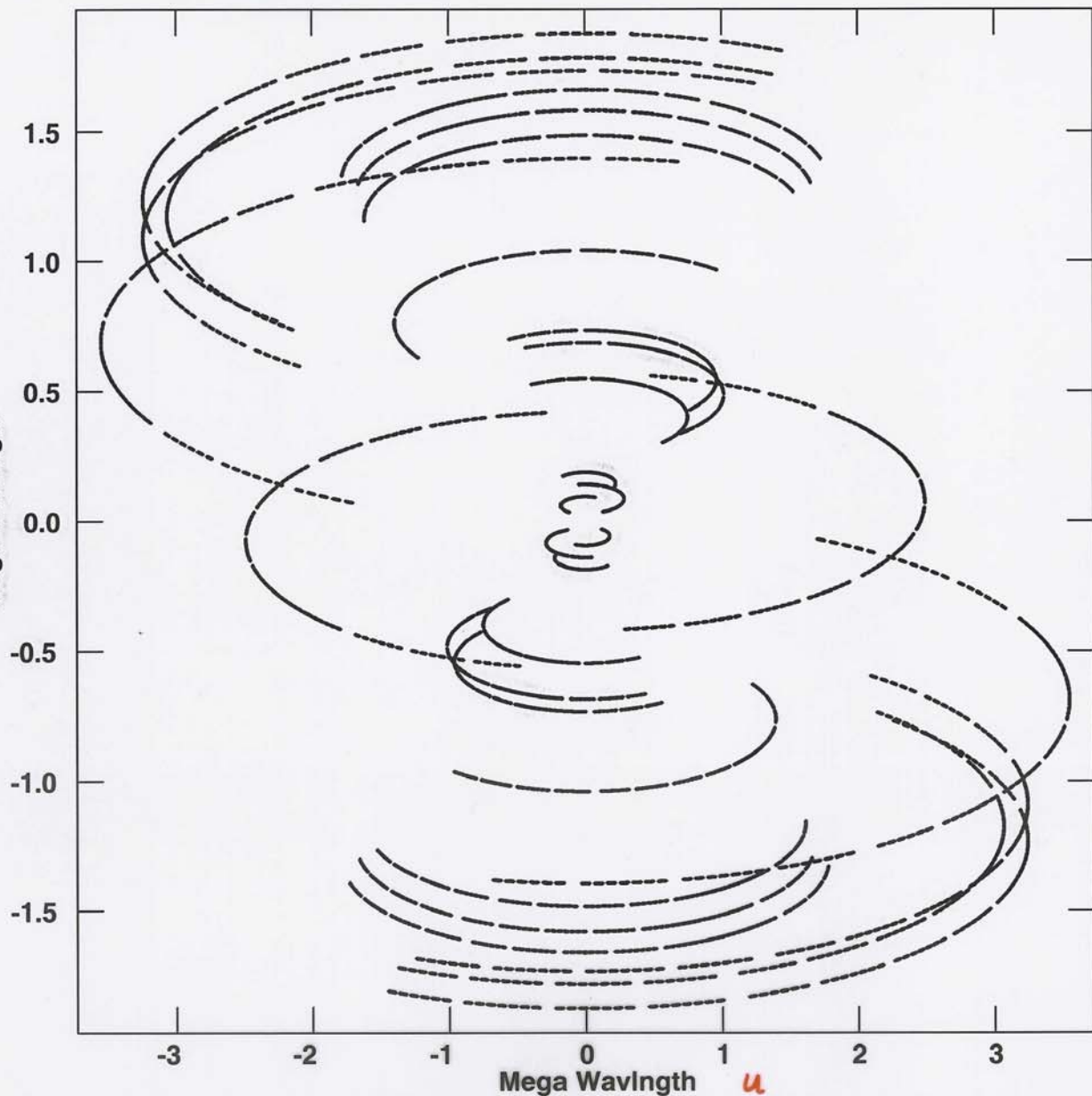
SAMPLING FUNCTION

v

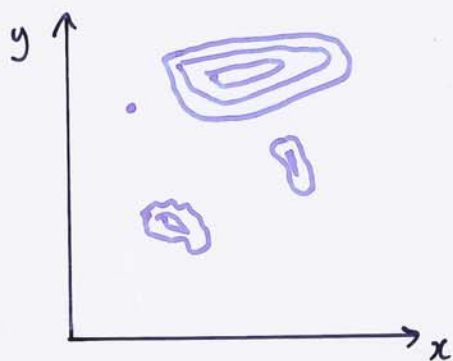
Mega Wavelength

Mega Wavelength

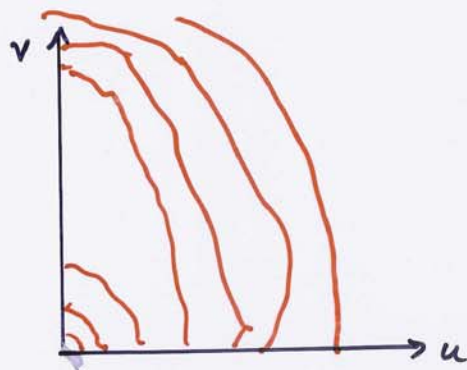
u



FROM FOURIER COMPONENTS TO SKY MAP


 $b(x,y)$

FT


 $S(u,v)$

WITH FULL u,v COVERAGE

 $S(u,v)$

FT

 $b(x,y)$

WITH INCOMPLETE u,v COVERAGE

SAMPLE FUNCTION

 $T(u,v)$

 (= u,v coverage)

 $S(u,v) \cdot T(u,v)$

FT

 $\tilde{T}(x,y) \otimes b(x,y)$

data

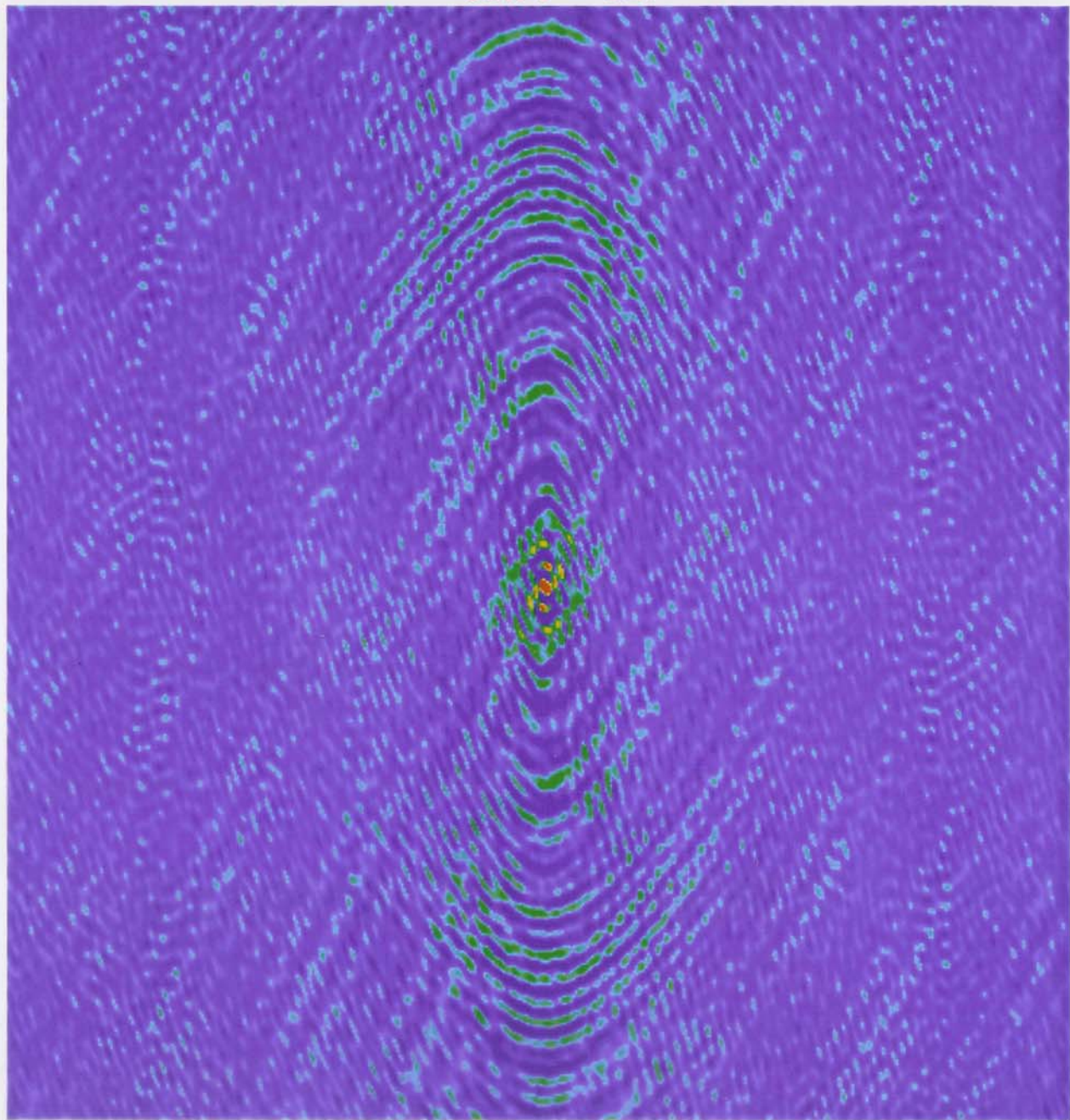
FT

 sky convolved with $\tilde{T}$
= "DIRTY MAP"

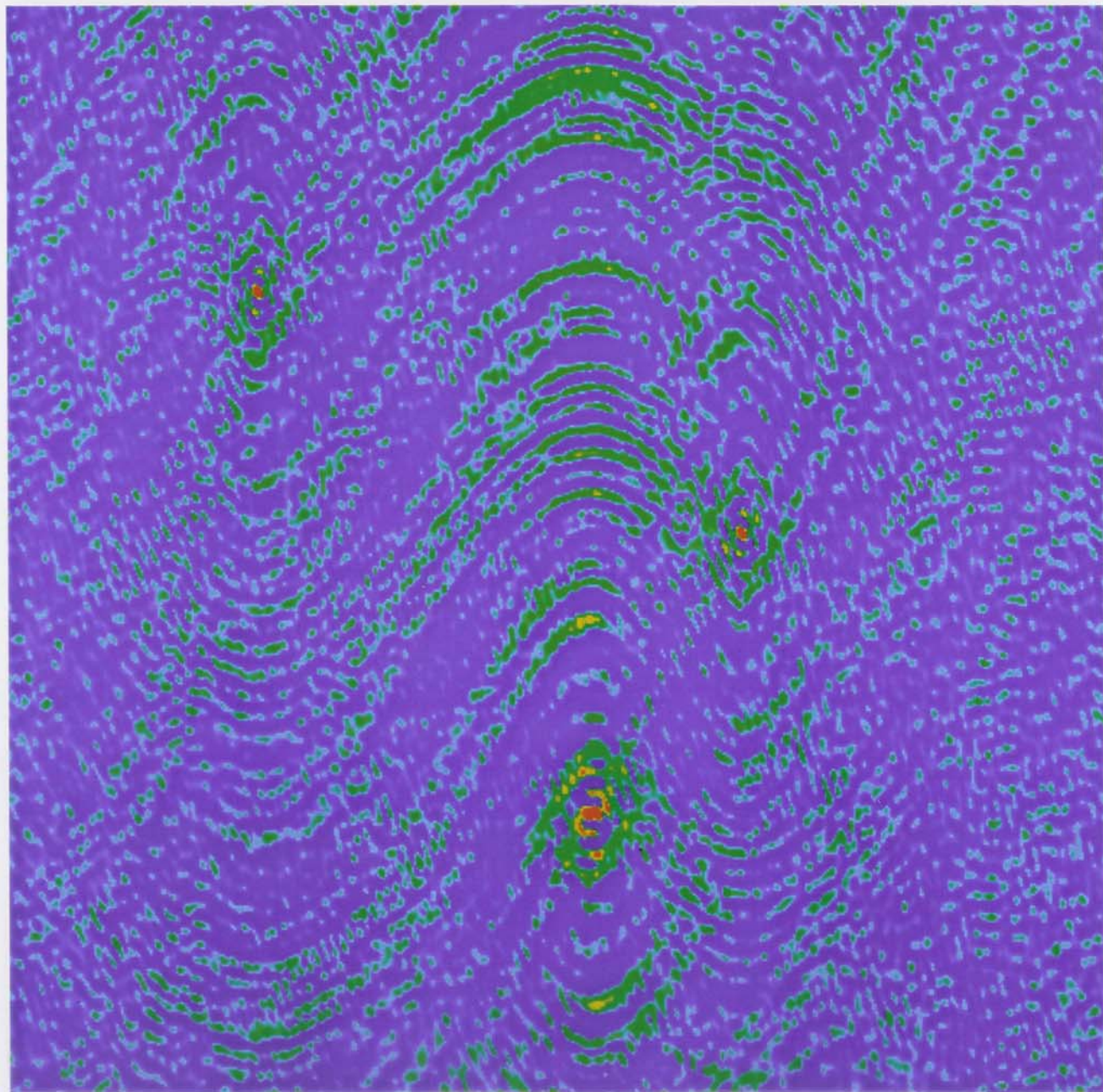
 $\tilde{T}(x,y)$ is KNOWN AS THE "DIRTY BEAM"

= RESPONSE TO A POINT SOURCE

DIRTY BEAM



DIRTY MAP



DECONVOLUTION ALGORITHM "CLEAN"

- COMPUTE DIRTY BEAM $T(u,v)$ FT $\tilde{T}(x,y)$
- COMPUTE DIRTY MAP $S(u,v) \cdot T(u,v)$ FT $\tilde{T}(x,y) * b(x,y)$

(•) TRY TO RECOGNIZE DIRTY BEAM RESPONSE IN DIRTY MAP

LOCATE POSITION OF BRIGHTEST POINT IN DIRTY MAP

SUBTRACT SCALED VERSION OF DIRTY BEAM
CENTRED AT THAT POINT

- SCALED BY DIRTY MAP VALUE
- SCALED BY $L \times (\text{DIRTY MAP VALUE})$
 $L = \text{"LOOP GAIN"} \sim 0.1$

STORE THE VALUE AND LOCATION OF THE
POINT-SOURCE RESPONSE REMOVED
= CLEAN COMPONENT $CC(x,y)$

• REPEAT LAST 3 STEPS UNTIL THE BRIGHTEST
POINT IN THE DIRTY MAP IS BELOW THE
MAP NOISE LEVEL

- WE ARE LEFT WITH A DENUDED DIRTY
MAP (= "RESIDUAL MAP") AND
A LIST OF CLEAN COMPONENTS.
RESIDUAL MAP IS HOPEFULLY JUST NOISE.

- THE $CC(x,y)$ REPRESENT A MULTI-POINT-SOURCE MODEL OF $b(x,y)$ WHICH, WHEN CONVOLVED WITH THE DIRTY BEAM, WOULD REPRODUCE THE DIRTY MAP.
- IN GENERAL A POOR REPRESENTATION OF A CONTINUOUS BRIGHTNESS DISTRIBUTION $b(x,y)$
- INSTEAD WE CONVOLVE THE $CC(xy)$ WITH AN IDEALIZED BEAM FUNCTION - THE CLEAN BEAM
- CLEAN BEAM SIZE CHOSEN TO CORRESPOND TO THE DIRTY BEAM RESOLUTION (CENTRAL PEAK REGION OF DIRTY BEAM)
- ADD TO RESIDUAL MAP TO INDICATE NOISE

$$CM(x,y) = CB(x,y) \otimes CC(x,y) + RM(x,y)$$

clean map

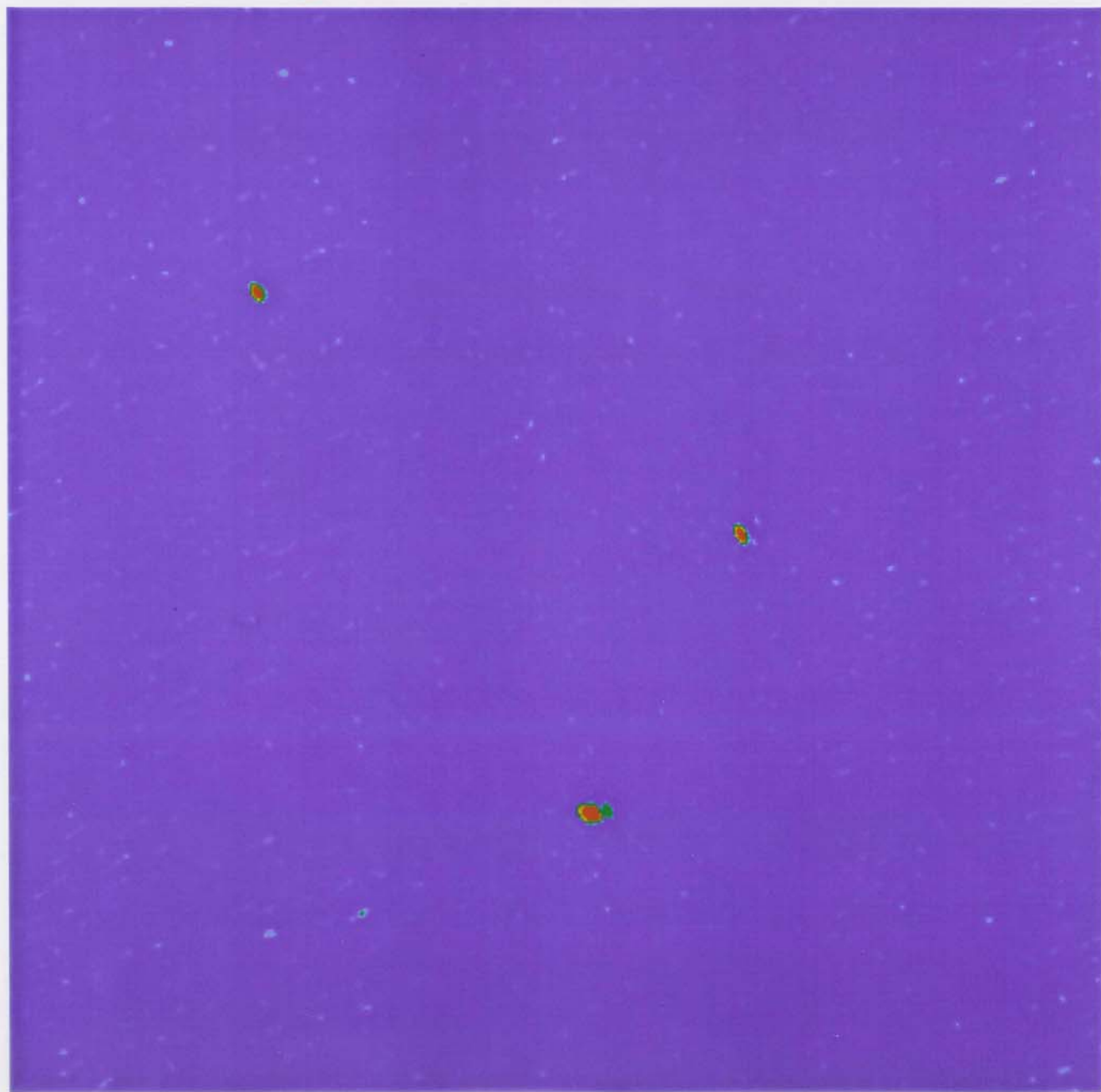
clean beam

clean
components

residual map

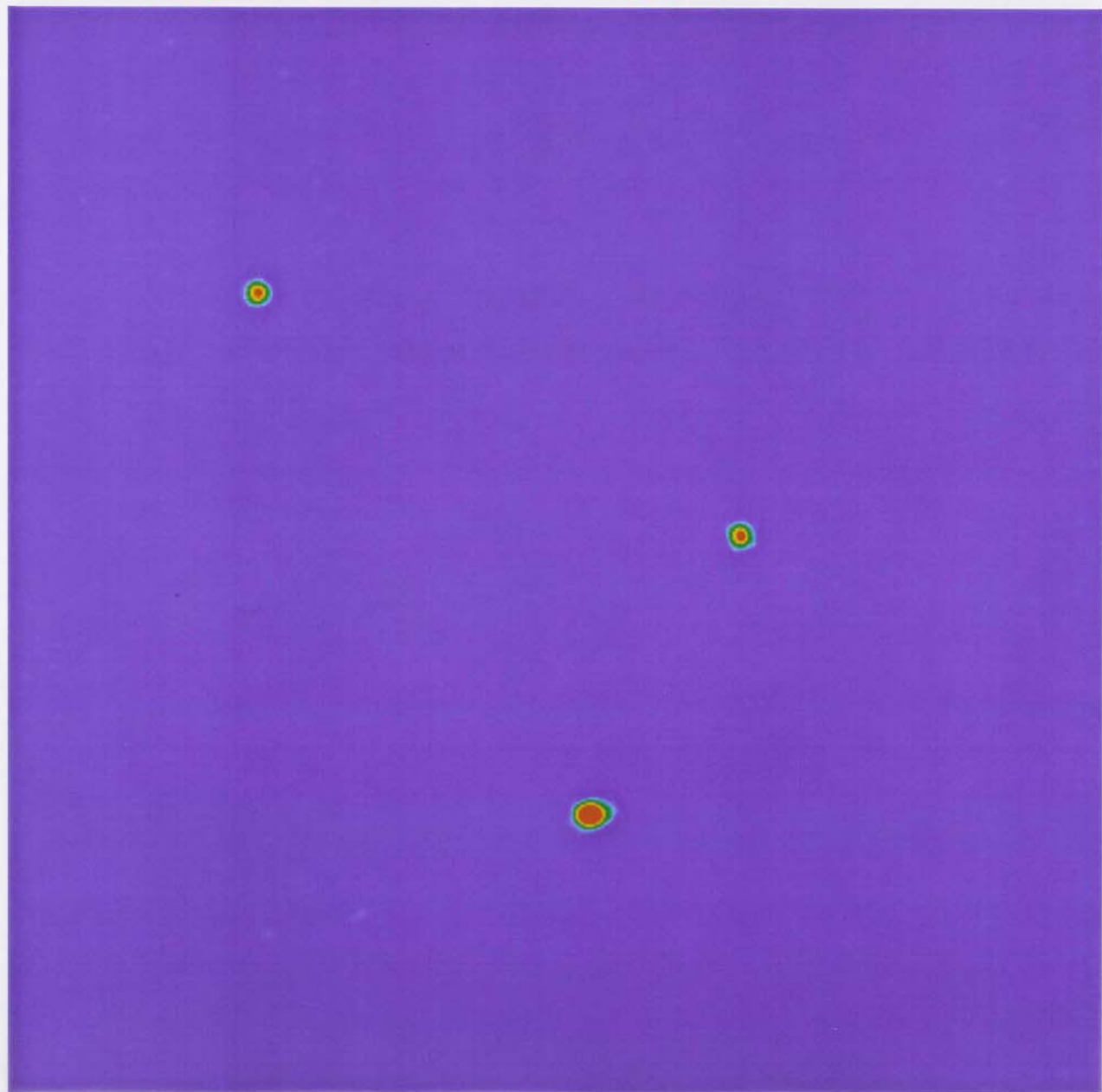
CLEAN MAP

CLEAN BEAM $0''.071 \times 0''.044$ PA 31°



CLEAN MAP

CLEAN BEAM $0.1'' \times 0.1''$



INSTRUMENTAL PHASE ERRORS

$$S_{ij} = \int b(\theta) \cdot e^{i2\pi q_{ij}\theta} \cdot d\theta$$

$$S_{ij} = a_{ij}, \phi_{ij}$$

$$S_{ij} = a_{ij}, \phi'_{ij} \quad \phi'_{ij} = \phi_{ij} + \phi_{ij}^e$$

CALIBRATION ON POINT SOURCE OF KNOWN POSITION

$$S_{ij}^{CAL} = a_{ij}^{CAL}, \phi_{ij}^{CAL} = 0$$

INTERFEROMETER PHASE MEASURED ON CALIBRATOR IS

THE ERROR PHASE ϕ_{ij}^e

PHASE SELF-CALIBRATION

A bit of a fiddle ?

.... or a powerful algorithm ?

- BASED ON A SIMPLE IDEA
- WE OBSERVE A SOURCE WITH N ANTENNAS
 $N \geq 3$
- AT TIME t WE HAVE $\frac{N(N-1)}{2}$ BASELINES OF DATA
- THE MEASURED VISIBILITY PHASES, ϕ'_{ij} ARE CORRUPTED VERSIONS OF THE TRUE PHASES, ϕ_{ij}
- ASSUME THAT THE CORRUPTING PHASES ARISE AT TELESCOPES, ϕ_i , AND CONTRIBUTE TO ALL BASELINES TO ANY TELESCOPE IN THE SAME WAY
- $\phi'_{ij} = \phi_{ij} + \phi_i - \phi_j$

measured
true
antenna corruptions

- NOW THE TRICK
- SUPPOSE WE KNOW THE TRUE VISIBILITY PHASES, ϕ_{ij} (because we know the source structure and can thus compute them for each baseline for time t)
- THEN WE CAN SOLVE THE $\frac{N(N-1)}{2}$ SIMULTANEOUS EQUATIONS TO DETERMINE THE CORRUPTING PHASES, ϕ_i , BECAUSE THERE ARE MORE EQUATIONS THAN UNKNOWN

ϕ'_{12}	=	ϕ_{12}	+	ϕ_1	-	ϕ_2	equations $\frac{N(N-1)}{2}$
ϕ'_{13}	=	ϕ_{13}	+	ϕ_1	-	ϕ_3	
ϕ'_{14}	=	ϕ_{14}	+	ϕ_1	-	ϕ_4	
ϕ'_{23}	=	ϕ_{23}	+	ϕ_2	-	ϕ_3	
$\vdots$		$\vdots$		$\vdots$		$\vdots$	

measured calculated unknowns (N)

- $\frac{N(N-1)}{2} \geq N$ SO SYSTEM IS

OVER-DETERMINED : USE LEAST SQUARES

MINIMIZATION

- BUT WE DO NOT KNOW THE

SOURCE STRUCTURE YET SO

THE ALGORITHM

- MAKE A GUESS FOR THE SOURCE STRUCTURE

(Point source,)

- CALCULATE SOURCE VISIBILITY PHASE FOR EACH BASELINE (FOR EACH TIME t)

- SOLVE THE EQUATIONS FOR THE ϕ_i
FOR EACH TIME t

- CORRECT THE MEASURED VISIBILITY FUNCTION

$$\phi_{ij}^c = \phi_{ij}' - \phi_i + \phi_j$$

- MAKE A MAP OF THE SOURCE USING THE CORRECTED VISIBILITIES
IMAGR CLEAN

- USE THIS NEW SOURCE STRUCTURE AS A BETTER INPUT GUESS

- HOPE THAT THIS CONVERGES TO

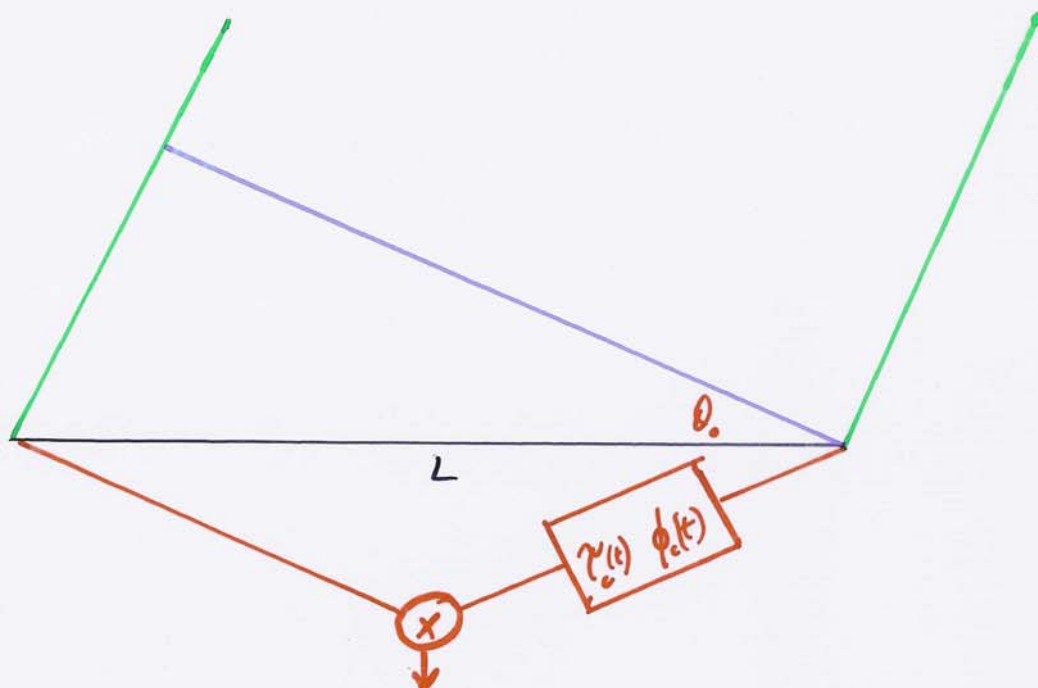
$$\phi_{ij}^c = \phi_{ij}$$

corrected true

- THE ALGORITHM IS KNOWN AS
"HYBRID MAPPING"
- IT IS BASED ON PHASE SELF-CALIBRATION
- IT ONLY WORKS IF THERE IS GOOD
 SNR FOR THE VISIBILITIES
- IT DESTROYS THAT PART OF THE
 PHASE INFORMATION WHICH GIVES THE
 POSITION OF THE SOURCE
- THE POSITION OF A SOURCE IN A
 HYBRID MAP SIMPLY REFLECTS THE
 POSITION OF THE INITIAL MODEL
 GUESS (e.g. POINT AT THE MAP ORIGIN)
- ONLY $N-1$ VALUES OF ϕ_i CAN BE
 DETERMINED; WE SET THAT OF A
 REFERENCE ANTENNA TO ZERO ($\phi_{\text{ref}} = 0$)
 SINCE FOR ANY SOLUTION SET ϕ_i ,
 THE VALUES $\phi_i + k$ ARE ALSO A SOLUTION
 SET

IMPERFECT PATH COMPENSATION

$$\tau = \frac{L \sin \theta_0}{c}$$



IF DELAY COMPENSATION

$$\tau_c(t) = \frac{L \sin \theta_0(t)}{c}$$

IF PHASE ROTATION

$$\phi_c(t) = \frac{2\pi L \sin \theta_0(t) \cdot f_L}{c}$$

τ_c, ϕ_c

CALCULATION
OF L, θ_0

DEPENDS ON KNOWLEDGE

Source position (α, δ)

sidercal time

baseline vector (antenna coordinates)

[VLBI: clock errors]

- FOR LONG BASELINES THE EFFECT OF ANGULAR ERRORS IN THE MODEL PRODUCE LARGER ERRORS IN $\gamma_c(t)$, $\phi_c(t)$.

EXAMPLE

$$\frac{50}{1''}$$

$$L = 5 \text{ km} \quad \lambda = 6 \text{ cm}$$

$$\frac{\delta L_{\text{in } \lambda}}{\lambda}$$

$$\rightarrow 2.4 \text{ cm} \quad (144^\circ)$$

$$1''$$

$$L = 5000 \text{ km} \quad \lambda = 6 \text{ cm}$$

$$\rightarrow 24 \text{ m}$$

$$(\delta \gamma_c = 80 \text{ ns}) = \text{residual delay}$$

c.f. coherence time for $b = 20 \text{ MHz} = 50 \text{ ns}$

$\Rightarrow$ LOSS OF COHERENCE

$\Rightarrow$ NO OUTPUT FROM MULTIPLIER

- OVERCOME WITH "MULTI-LAG" INTERFEROMETER

$$\langle V_1 \cdot V_2 \rangle$$

$$\langle V_1(t) \cdot V_2(t + \frac{1}{2b}) \rangle$$

$$\langle V_1(t) \cdot V_2(t + \frac{2}{2b}) \rangle$$

$\vdots$

$$\langle V_1(t) \cdot V_2(t + \frac{i}{2b}) \rangle$$

$$i = -16 \rightarrow 16$$

- NEED TO CHOOSE WHICH VALUE OF i HAS THE SIGNAL
- SIGNAL MUST BE STRONG ENOUGH TO RECOGNIZE (e.g. $S/N\text{-ratio} > 5$)
- WE MAY NEED TO INTEGRATE THE SIGNAL TO ACHIEVE SUFFICIENT $S/N\text{-ratio}$.
- BUT ERRORS IN MODEL PRODUCE ERRORS IN FRINGE ROTATION PHASE, ϕ_c , WHICH CHANGE WITH TIME AND PREVENT (VECTOR) INTEGRATION

EXAMPLE

SB

1"

$L = 5000 \text{ km}$

$\lambda = 6 \mu\text{m}$

$\delta \phi_c$

33 mHz

(turn is 30°)

= residual fringe rate

- NEED TO TRY DIFFERENT RESIDUAL FRINGE RATES TO FIND WHICH IS THE CORRECT ONE!

FRINGE - FITTING

(FRINGE)

- SEARCH TO FIND THE SIGNAL IN
INTEGRATION TIME t (SOLINT)
- SIMULTANEOUS SEARCH OF RESIDUAL DELAY
SAMPLES ($i = -16 \rightarrow 16$) AND A RANGE
OF POSSIBLE RESIDUAL FRINGE-RATES
- SIGNAL MUST BE ABOVE A DETECTION THRESHOLD
- SEARCH IDENTIFIES CORRECT RESIDUAL DELAY
AND FRINGE-RATE