Self-Calibration and Hybrid Mapping

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Lecture Outline

- Initial calibration and its deficiencies
- Visibility errors and dynamic range in interferometric images
- Closure quantities
- Hybrid imaging
- Basics of self-calibration
- Practical examples

Initial calibration

- Initial calibration: determining the empirical corrections for time-variable instrumental or environmental factors that cannot be measured, or monitored, directly.
- The calibration equation:

$$\tilde{V}_{ij}(t) = g_i(t)\, g_j^{\star}(t)\, V_{ij}(t) + \epsilon_{ij}(t)$$

$\tilde{V}_{ij}$ – visibility measured between antennas i and j.

$V_{ij}(t) = A_{ij}(t)\ \exp[i\,\psi_{ij}(t)]$ – true visibility between antennas i and j.

$g_i(t) = a_i(t)\ \exp[i\,\phi_i(t)]$ – complex gain of antenna i.

$\epsilon_{ij}(t)$ – additive noise.

Measured quantities

- Measured complex visibilities:

$$\tilde{V}_{ij}(t) = a_i(t)\, a_j(t)\, \exp\{i\,[\phi_i(t) - \phi_j(t)]\}\, V_{ij}(t) + \epsilon_{ij}(t)$$

Measured visibility amplitudes:

$$|\tilde{V}_{ij}(t)| = a_i(t)\, a_j(t)\, A_{ij}(t) + \sigma_{\rm a}$$

Measured visibility phases:

$$\tilde{\psi}_{ij}(t) = \psi_{ij}(t) + \phi_i(t) - \phi_j(t) + \sigma_\phi$$

$$\sigma_{\rm a},\ \sigma_\phi \quad - \quad \text{noise terms.}$$

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A priori calibration: problems

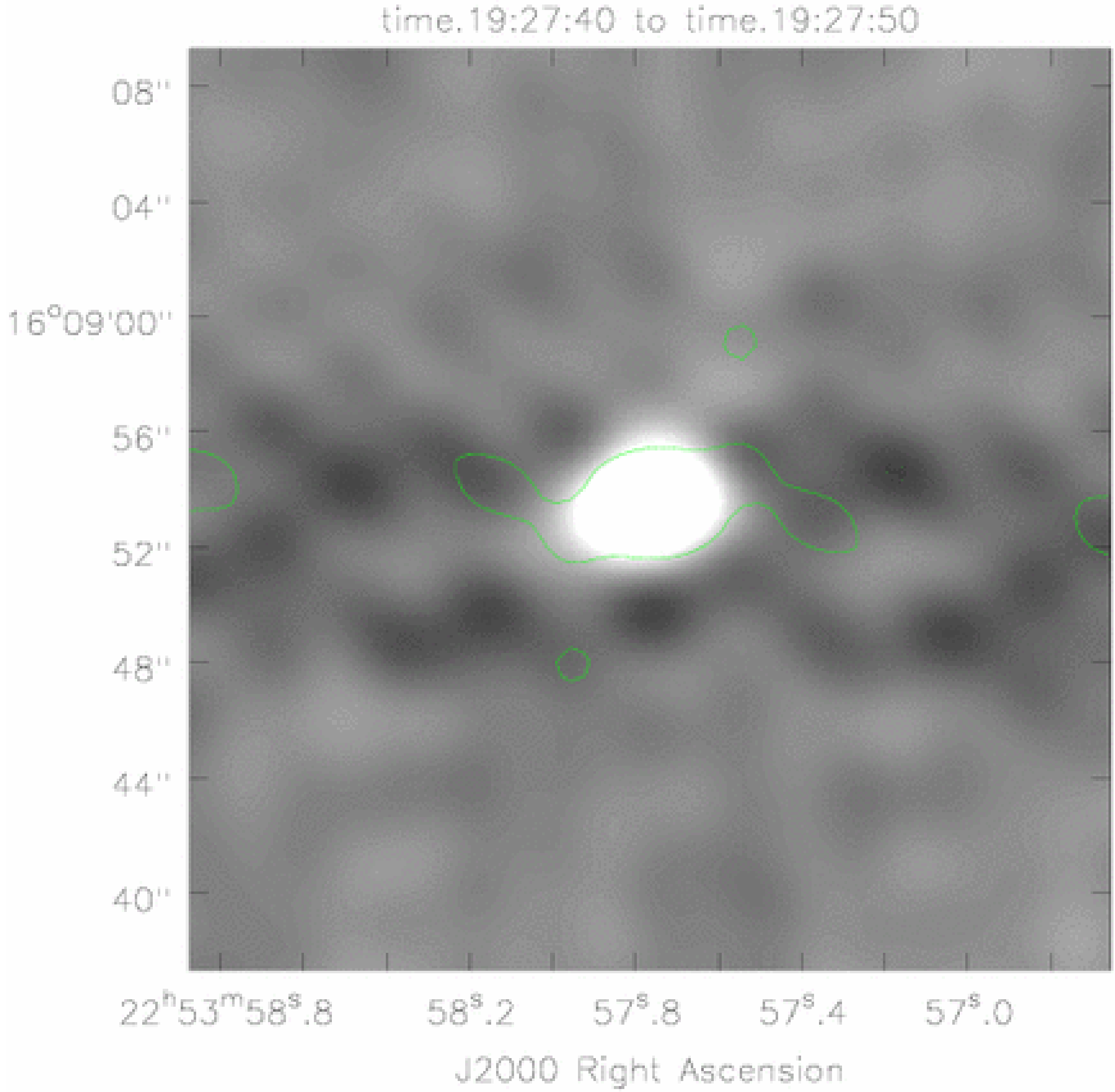
The initial calibration is usually determined with substantial inaccuracies. Main reasons for these inaccuracies are:

- **Instrumental**: antenna and source positions, station clocks rates and offsets are uncertain.

- **Environmental**: tropospheric water vapor fluctuations and ionospheric electron content fluctuations introduce variable phase offsets at different antennas.

- **Logistical**: the gains can hardly be measured in an immediate conjunction with observation of astronomical targets.

A point source at 22 GHz



Should we worry?

Typical errors reach $\phi \approx 0.1$rad in phase, $\varepsilon \approx 1\%$ in amplitude…
Is this too bad?

- A random, antenna-based phase error ϕ present at all antennas would limit the dynamic range to

$$D = \frac{1}{\phi}\sqrt{\frac{N(N-1)}{2}} \approx \frac{N}{\sqrt{2}\phi}$$

The same applies to amplitude errors, since one can substitute $\phi \rightarrow \varepsilon$.

- In an observation consisting of M scans with independent errors:

$$D \sim \frac{\sqrt{M}\,N}{\phi}$$

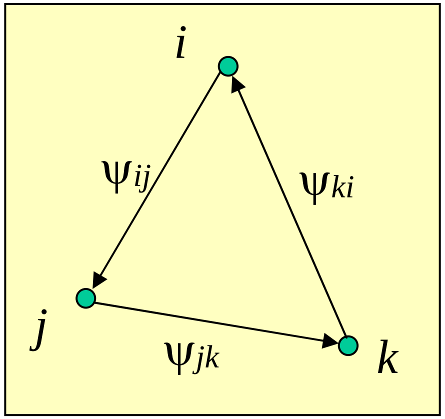
Bottomline: relying on *a priori* calibration may not be sufficient!

Calibration: Remedies

- **Redundant calibration**: solving for the gains using pairs of antennas that measure the same (u,v) sample. Determines variable antenna gains up to a linear phase slope.
  Advantage: model independent.
  Problem: reduces SNR of the estimated true visibilities.
- **Astronomical calibration**: use calibrators to remove instrumental effects.
- **Phase referencing**: frequent observations of a nearby calibrator source. Can only correct phase errors. Significant fraction of observing time is spent on calibrators. Used effectively for imaging weak sources.

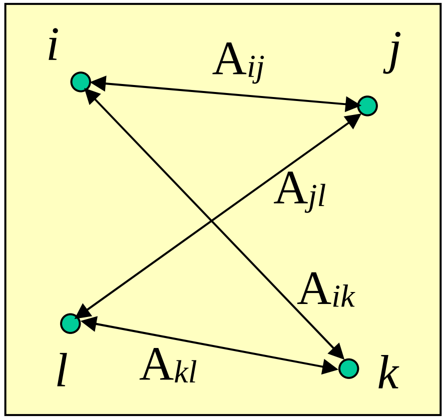
Calibration: Remedies II

- **Hybrid imaging**: relies on the use of *closure quantities* that can be applied to remove instrumental contributions to visibility phases and amplitudes



Closure phases:

$$\begin{aligned}\tilde{\Psi}_{ijk} &= \tilde{\psi}_{ij} + \tilde{\psi}_{jk} + \tilde{\psi}_{ki} \\ &= \psi_{ij}(\phi_i - \phi_j) + \psi_{jk} + (\phi_j - \phi_k) + \psi_{ki} + (\phi_k - \phi_i) \\ &= \psi_{ij} + \psi_{jk} + \psi_{ki} \\ &= \Psi_{ijk}\end{aligned}$$



Closure amplitudes:

$$\tilde{\Gamma}_{ijkl} = \frac{|\tilde{V}_{ij}|\,|\tilde{V}_{kl}|}{|\tilde{V}_{ik}|\,|\tilde{V}_{jl}|} = \frac{A_{ij}\,A_{kl}}{A_{ik}\,A_{jl}} = \frac{|V_{ij}|\,|V_{kl}|}{|V_{ik}|\,|V_{jl}|} = \Gamma_{ijkl}$$

Hybrid Mapping

1. Choose an initial model: I^{Model}
2. Predict visibilities for the model: $V^{\mathrm{Model}} = \mathrm{FT}(I^{\mathrm{Model}})$
3. Keep observed visibility amplitudes
4. Select new visibility phases by modifying model visibility phases to be consistent with the observed closure phases.
5. Produce new dirty image from new visibilities.
6. Produce a new model I^{Model} by CLEAN-ing the new dirty image.
7. If the process has not converged, go back to Step (2).

Hybrid Mapping: Difficulties

- Proper treatment of noise is difficult (noise is additive only in vector visibility, not in the amplitude or phase).
- Not easy to choose the optimal set of closure quantities.
- Calibration effects in radio imaging are really related to antennas, not baselines.

... We shold probably look for an antenna-based method for improving *a priori* calibration

Calibration: Remedies III

- **Self-calibration**: antenna gains are considered free parameters that can be iteratively adjusted to provide optimal correspondence between a model of the target source and the set of visibilities observed. Most commonly applied method.

  Advantages: universality. Uses implicitly the constraints provided by the closure quantities.

  Difficulty: depends on *a priori* knowledge of source structure.

Self-Calibration

Self-calibration uses a model of the target source to solve for improved values for the complex gains of the individual antennas

Advantages

- Gains are derived for correct time, not by interpolation.
- Gains are derived for correct direction on celestial sphere.
- Solution is fairly robust if there are many baselines.

Disadvantages

- Requires a sufficiently bright source.
- Results depend on the assumed model. If the model is incorrect, it will be "built into" the derived gains, leading to incorrect visibilities and images.

Point Source

- For a point source, the calibration equation becomes:

$$\tilde{V}_{ij}(t) = g_i(t)\, g_j^{\star}(t)\, S + \epsilon_{ij}(t)$$

S – flux density of the point source.

- This equation can be solved for antenna gains via least squares algorithm.

- Significant redundancy: N-1 baselines contribute to gain estimate for any antenna.

Extended Source

- Can use a model of the calibrator structure, obtaining:

$$\tilde{V}_{ij}(t) = g_i(t)\, g_j^\star(t)\, V_{ij}^{\mathrm{model}}(t) + \epsilon_{ij}(t)$$

$$V_{ij}^{\mathrm{model}}(t) \quad - \quad \text{model visibilities.}$$

- Can then correct for estimated antenna gains:

$$V_{ij}^{\mathrm{cal}}(t) = \left(g_i(t)\, g_j^\star(t)\right)^{-1}\, \tilde{V}_{ij}(t)$$

These corrections can be smoothed or interpolated if desired.

Extended Source II

- The extended calibrator case transforms effectively into the point-source calibrator case by dividing by the model visibilities:

$$X_{ij}(t) = \frac{V_{ij}(t)}{V_{ij}^{\rm model}} = g_i(t)\, g_j^\star(t) + \epsilon'(t)$$

$\epsilon'(t)$ – modified noise term.

Self-Calibration Cycle

1. Produce an initial source model, using any a priory information of the structure.
2. Use the model to convert observed source into a “pseudo-point source”.
3. Solve for the complex gains.
4. Find corrected visibility

$$V_{ij,\mathrm{corr}}(t) = \frac{\tilde{V}_{ij}(t)}{g_i(t)\, g_j^{\star}(t)}$$

5. Form a new model from the corrected data.
6. Go to Step 2, unless current model is satisfactory.

Self-Calibration Strategies

- **Initial model**

  - Point source, CLEAN components from initial image, *uv*-model fit.

- **Phase/amplitude self-calibration**

  - Start with phase only self-calibration: amplitude errors are relatively unimportant at dynamic ranges of up to ~1000!

- **Tapering and weighting**

  - Simple source: all baselines can be used. Complex source: self-calibrate first most compact structures (use long baselines, uniform weighting, uv-tapering).

- **Solution intervals**

  - Phases: as short as allowed by SNR. Amplitudes: should not go to short solution intervals, as this has a great danger of freezing the model into the data! (especially for arrays with relatively small number of elements).

When to use Self-Cal

- Self-calibration is needed when calibration errors are evident through one or both of the following conditions:
  - The background noise is considerably higher than expected
  - There are convolutional artifacts around objects, especially point sources
- Calibration errors should not be confused with effects of poor Fourier plane sampling such as:
  - Low spatial frequency errors due to lack of short spacings
  - Multiplicative fringes (due to deconvolution errors)
  - Deconvolution errors around moderately resolved sources

Pushing the Limits

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- Self-calibration can be applied if SNR on most baselines is greater than one.
- For a point source, the error in the gain solution is:

Phase only:

$$\sigma_{g,\psi} = \frac{1}{\sqrt{N-2}} \frac{\sigma_{\rm V}}{S}$$

Amplitude and phase:

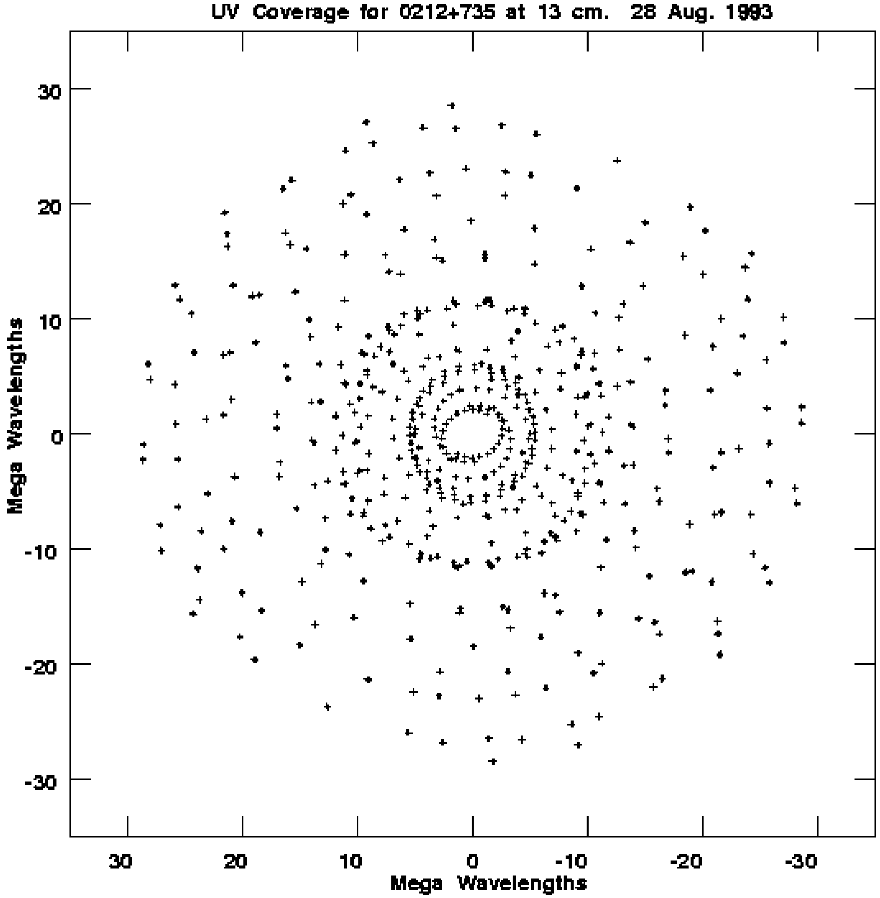
$$\sigma_{g,a} = \frac{1}{\sqrt{N-3}} \frac{\sigma_{\rm V}}{S}$$

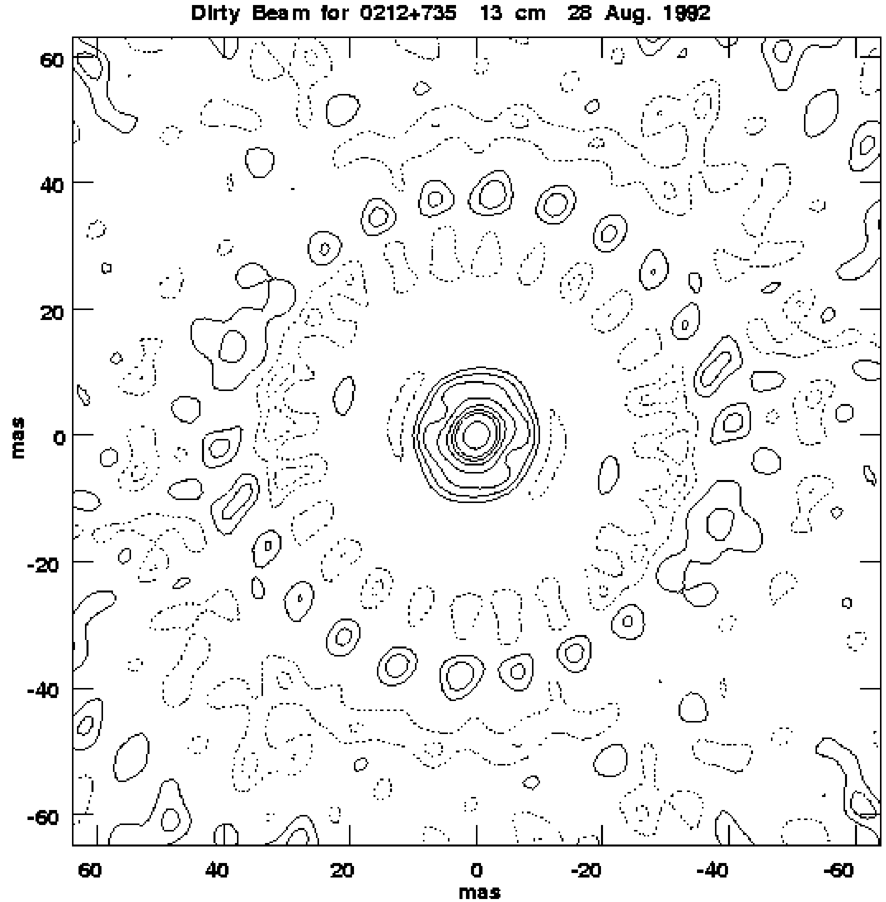
$\sigma_{\rm V}$ – noise per visibility sample.

When the gain errors are significantly smaller than one, then the noise in the final image approaches its expected level.

Example

Self-calibration and hybrid mapping of VLBI data on 0212+735 (Walker 1995)





Contour levels (percent) = -20, -15, -10, -5, 5, 10, 15, 20, 30, 40, 50, 75

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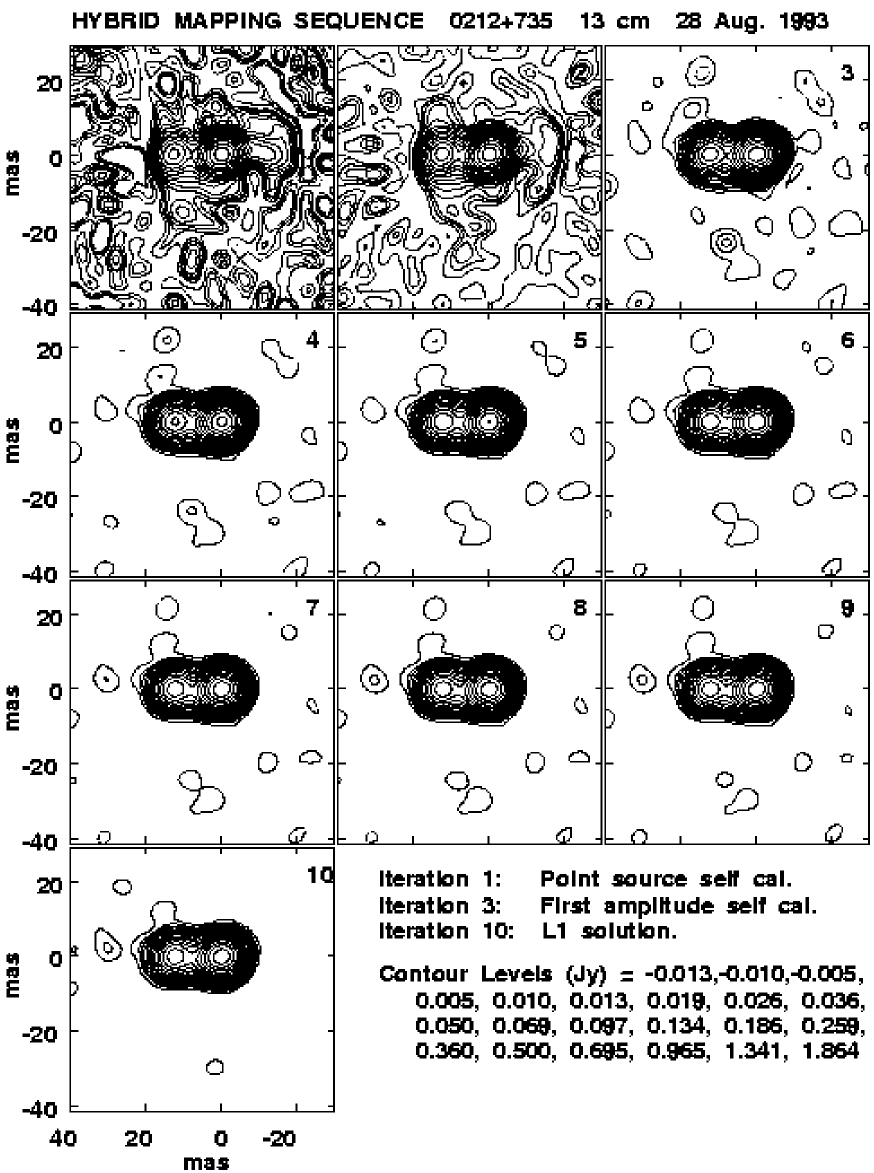
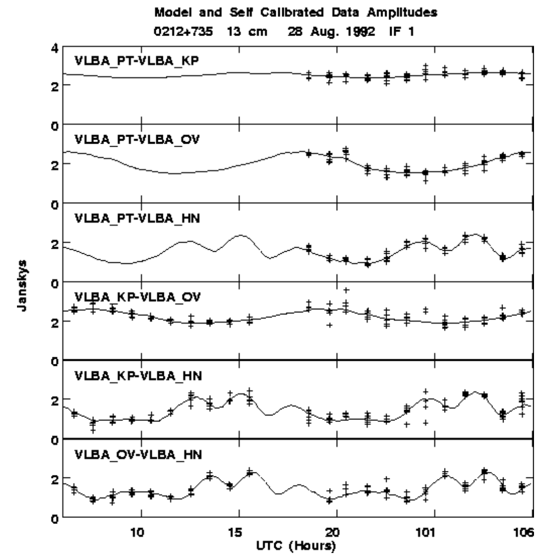
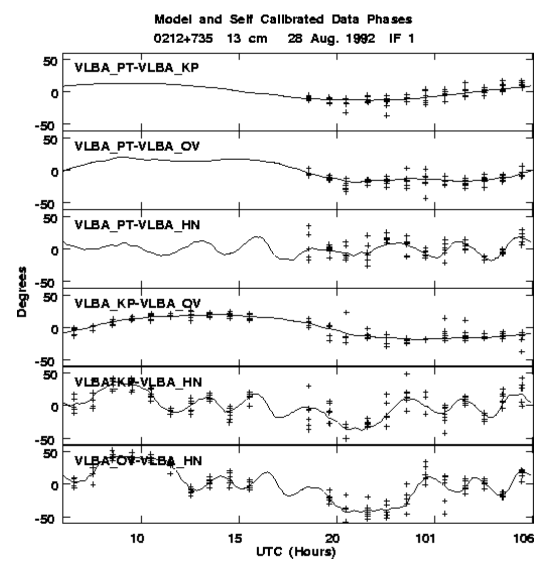
Example II







Summary

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- Direct imaging and CLEAN-ing interferometry data are almost always insufficient for producing high quality images
- Interferometric data suffer from imperfect initial calibration and various time variable factors that must be corrected for in an empirical procedure.
- Self-calibration and hybrid imaging can be used for correcting the initial calibration. Especially important for VLBI observations.

Can something go wrong?

We'll find out on Thursday…