



Astrometric Interferometry

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- VLBI observables for astrometry
- VLBI modeling
- Reference frames and surveys (ICRF, VCS)
- Organisation for data acquisition and processing (IVS)
- Phase-referenced astrometry

Astrometric potential of VLBI

- Geometric time delay

$$c \tau_g = d \cos \theta$$

- Variation of time delay with source direction:

$$\Delta \tau_g = - (d/c) \sin \theta \Delta \theta$$

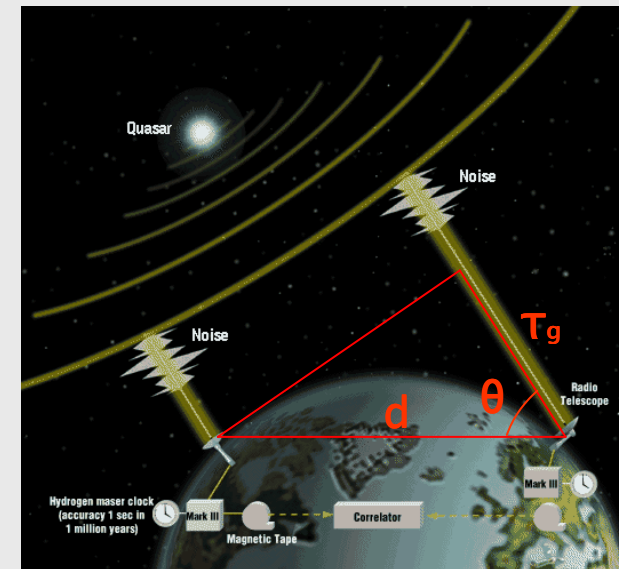
$$\Rightarrow \Delta \theta = - (c/d \sin \theta) \Delta \tau_g$$

- A change of 10% of a wavelength is easily detectable at correlation

$$\Delta \tau_g = 0.1 \lambda / c$$

$$\Rightarrow \Delta \theta = - (0.1 \lambda / d \sin \theta)$$

→ directions theoretically measurable by VLBI to ~ 0.2 mas



Assuming:

$$d = 5000 \text{ km}$$

$$\lambda = 3.6 \text{ cm}$$

$$\Delta \theta = 0.14 \text{ mas} / \sin \theta$$

- Phase delay

$$\tau_{\phi} = \phi / \omega$$

ϕ = fringe phase

$$\omega = 2\pi\nu$$

ν = frequency (e.g. 8 GHz)

- ϕ only known modulo $360^\circ \rightarrow \tau_{\phi}$ ambiguous
- Interpretation of the phase requires **resolving the inherent ambiguity** (i.e. determining the number of phase cycles)
- Prediction of the number of cycles requires a **very accurate geometric model** (< 20 ps or 5 mm at 8 GHz)
- But, generally not the case... $\Rightarrow \tau_{\phi}$ cannot be used directly

Error in phase delay:

$$\sigma_{\tau_{\phi}} = (1 / \omega) \sigma_{\phi} = (1 / \omega) \cdot (1 / \text{SNR})$$

The phase delay VLBI observable is
very accurate but ambiguous

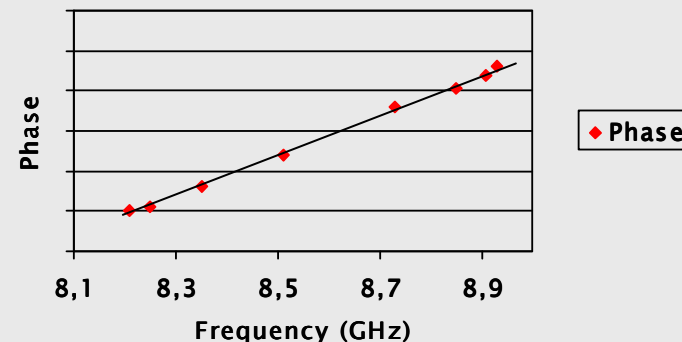
$$\begin{aligned} \nu &= 8.4 \text{ GHz, SNR} = 50 \\ \Rightarrow \sigma_{\tau_{\phi}} &= 0.4 \text{ ps (0.1 mm)} \end{aligned}$$

Astrometric VLBI observables (2)

- Group delay

$$\tau_{\text{group}} = \partial\phi / \partial\omega$$

- τ_{group} not ambiguous (unlike τ_{ϕ})
- In practice, τ_{group} is determined by fitting a straight line to the sequence of phases measured at several “discrete” frequencies



- Error in group delay:

$$\sigma_{\tau_{\text{group}}} = \sigma_{\phi} / (2\pi \Delta\nu_{\text{rms}}) = 1 / (2\pi \Delta\nu_{\text{rms}} \text{ SNR})$$

The group delay is less accurate than the phase delay but not ambiguous

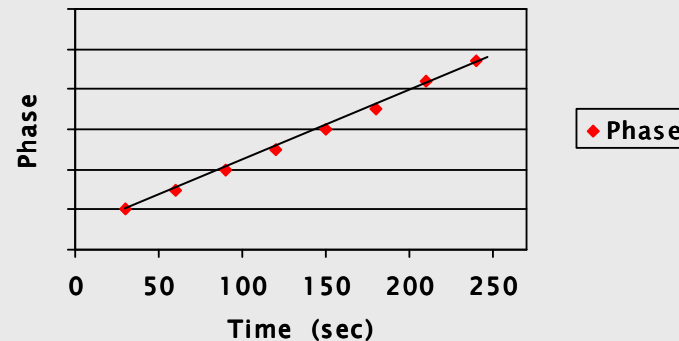
$$\Delta\nu_{\text{rms}} = 0.3 \text{ GHz}, \text{ SNR} = 50 \\ \Rightarrow \sigma_{\tau_{\text{group}}} = 10 \text{ ps (3 mm)}$$

Astrometric VLBI observables (3)

- Phase delay rate (fringe frequency)

$$\dot{\tau}_{\phi} = \partial \tau_{\phi} / \partial t = (1 / \omega) \partial \phi / \partial t$$

- $\dot{\tau}_{\phi}$ not ambiguous
- $\dot{\tau}_{\phi}$ is determined by fitting a straight line to the sequence of phases measured over time (a few minutes)



- Error in phase delay rate:

$$\sigma_{\dot{\tau}_{\phi}} = (1 / \omega) \sigma_{\phi} / \Delta t_{\text{rms}} = 1 / (2\pi\nu \Delta t_{\text{rms}} \text{SNR})$$

$$\nu = 8.4 \text{ GHz}, \Delta t_{\text{rms}} = 60 \text{ s}, \text{SNR} = 50 \Rightarrow \sigma_{\dot{\tau}_{\phi}} = 0.06 \text{ ps/s}$$

Angular sensitivity of VLBI observables



Assuming: $d = 5000 \text{ km}$, $\lambda = 3.6 \text{ cm}$, $\text{SNR} = 50$,
 $\Delta v_{\text{rms}} = 0.3 \text{ GHz}$, $\Delta t_{\text{rms}} = 60 \text{ s}$

- Phase delay

$$\sin \theta \Delta\theta(\tau_{\phi}) = (1/2\pi) \cdot (\lambda/d) \cdot (1/\text{SNR}) \Rightarrow 0.005 \text{ mas}$$

- Group delay

$$\sin \theta \Delta\theta(\tau_{\text{group}}) = (1/2\pi) \cdot (\lambda/d) \cdot (v/\Delta v_{\text{rms}}) \cdot (1/\text{SNR}) \Rightarrow 0.13 \text{ mas}$$

- Phase delay rate

$$\cos \theta \Delta\theta(\dot{\tau}_{\phi}) = (1/2\pi) \cdot (\lambda/d) \cdot (1/\omega_e \Delta t_{\text{rms}}) \cdot (1/\text{SNR}) \Rightarrow 1.1 \text{ mas}$$

(ω_e = angular velocity of the Earth's rotation)

The primary astrometric VLBI observable is the group delay
(also called « bandwidth synthesis » delay)

- Duration of experiments: 24 hours
- Dual-band observations to calibrate ionosphere (8.4/2.3 GHz)
- Bandwidth:
 - 8 frequencies at X band spread over 720 MHz (8.2–8.9 GHz)
 - 6 frequencies at S band spread over 125 MHz (2.22–2.35 GHz)
- 6–10 telescopes
- Observe ~ 50 different sources with a few scans on each source => a few thousand delay observations
- For each measured quantity (τ_{group} or $\dot{\tau}_{\phi}$), calculate corresponding theoretical model

=> Form **O-C (observed – calculated)** and estimate parameters of interest (e.g. source coordinates,...)
- Software: CALC/SOLVE (NASA), MODEST (JPL), OCCAM (Europe)

- Observables:

$$\tau_{\text{group}} = \partial\phi / \partial\omega, \quad \dot{\tau}_{\phi} = (1/\omega) \partial\phi / \partial t$$

- VLBI model (τ_{group} abbreviated as τ)

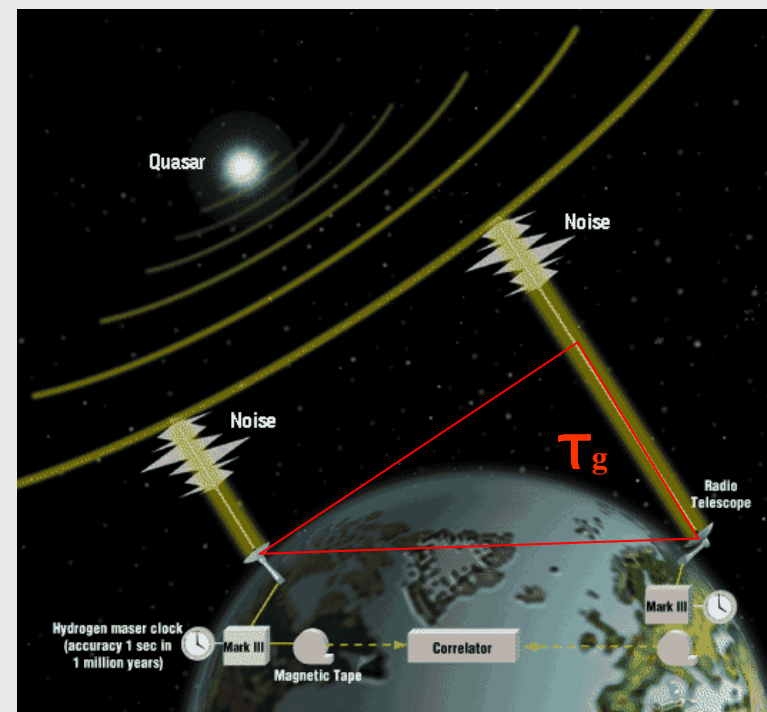
$$\tau = \tau_g + \tau_{\text{inst}} + \tau_{\text{trop}} + \tau_{\text{ion}} + \dots$$

τ_g = geometric delay

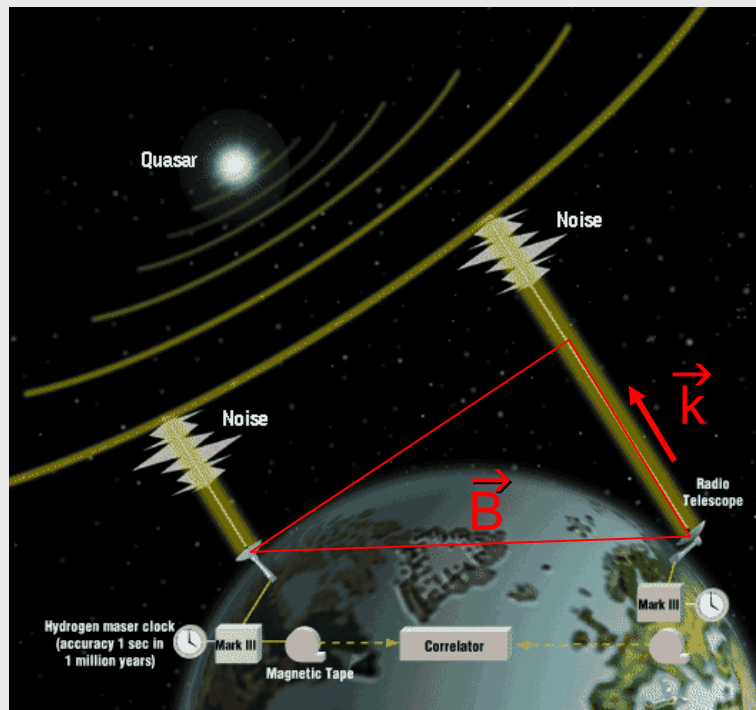
τ_{inst} = instrumental delay

τ_{trop} = tropospheric delay

τ_{ion} = ionospheric delay



VLBI modeling: geometric delay τ_g



$$\tau_g \text{ max} = 0,02 \text{ s}$$

Q = rotational transformation of coordinates from the terrestrial frame to the celestial frame

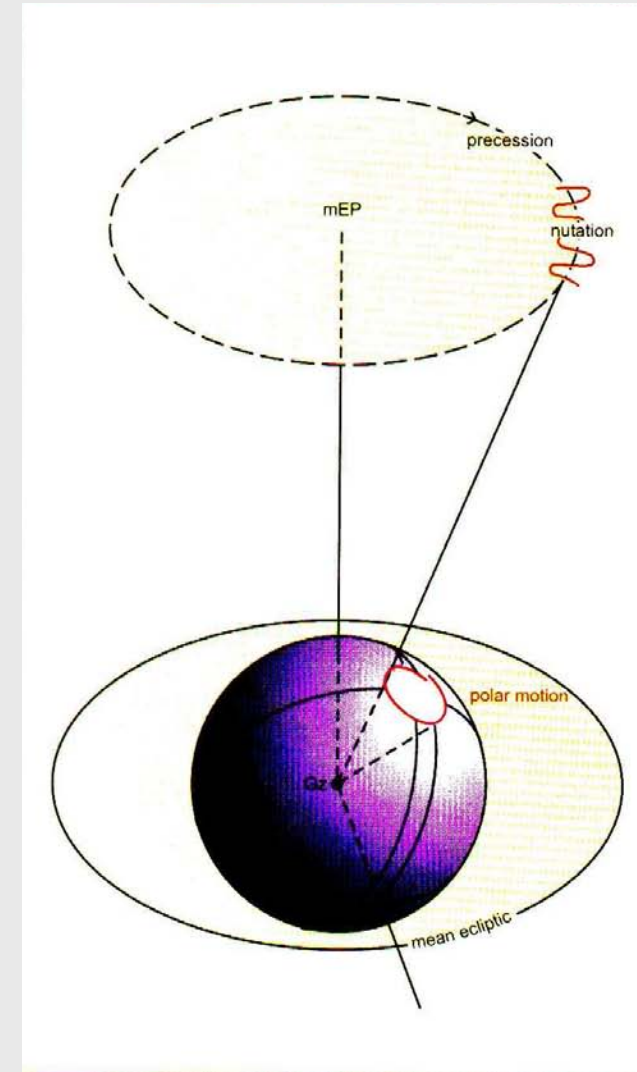
$Q = \text{PNUXY}$ $X, Y = \text{polar motion, } U = \text{Earth's rotation}$
 $N = \text{nutration, } P = \text{precession}$

- $$\tau_g = - (1 / c) \vec{k} \cdot \vec{B} [\dots]$$
- $\vec{B}_{\text{cel}} = Q (\vec{B}_{\text{ter}} + \Delta \vec{B}_{\text{ter}})$
 - Relativistic effects at the level of 10^{-8} s
 - $\Delta \vec{B}_{\text{ter}}$ depends on:
 - Tectonic motions
 - Solid Earth tides
 - Ocean loading
 - Atmospheric loading
 - ...

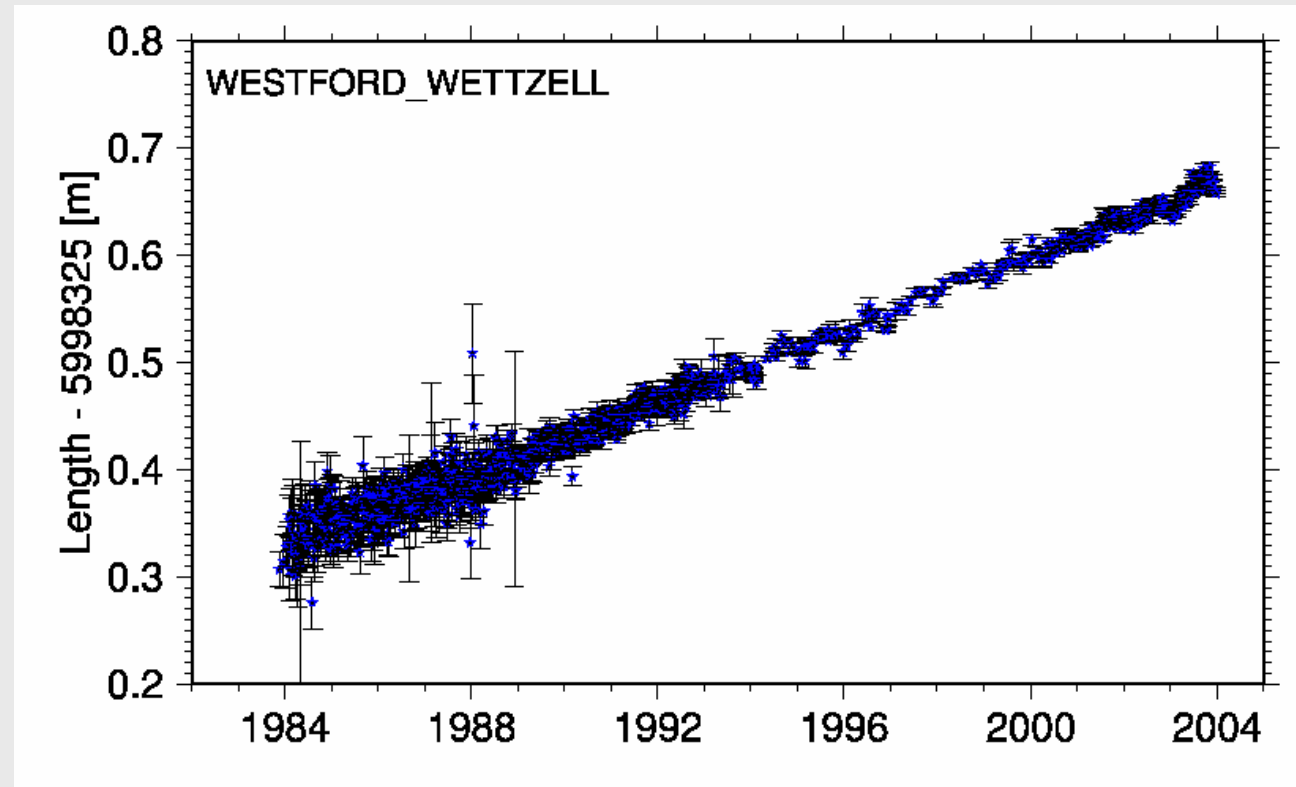
$$Q = PNUXY$$

- X, Y: polar motion
- U: rotation of the Earth around its axis (UT1-UTC)
- N: nutation
→ consists of several hundreds of periodic terms
- P: precession

Q transforms a vector from the terrestrial frame to the celestial frame



Evolution of the distance between Westford (USA) and Wettzell (Germany) between 1984 and 2004



- The instrumental delay comprises two terms:
 - The delay caused by the shift between the clocks at the two stations

$$\tau_{\text{clock}} = \tau_{\text{clock2}} - \tau_{\text{clock1}}$$

- The delay caused by propagation time in cables and electronics at each station

$$\tau_{\text{prop}} = \tau_{\text{prop2}} - \tau_{\text{prop1}}$$

- In practice, the instrumental delay $\tau_{\text{inst}} = \tau_{\text{clock}} + \tau_{\text{prop}}$ is modeled by a linear or quadratic function with the coefficients estimated from the VLBI observations.

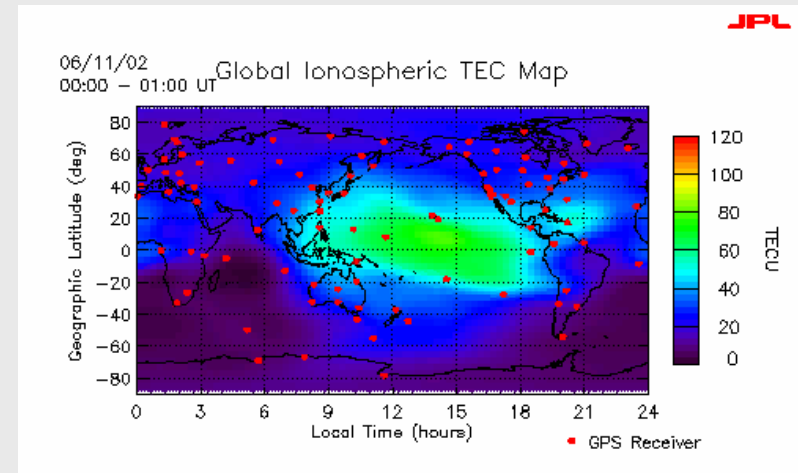
- τ_{trop} may be separated into two components
 - The **dry component**: ~ 7 ns at zenith (210 cm)
 - > may be predicted to ~ 1 cm from meteorological data
 - The **wet component**: < 1 ns at zenith (30 cm)
 - > difficult to predict, quickly variable
- Modeling
 - For each station i : $\tau_{\text{trop } i} = \tau_{\text{trpz } i} R(H_i)$
 - $\tau_{\text{trpz } i}$ = zenith tropospheric delay
 - R = mapping function (H_i = elevation)
 - Differential effect
$$\tau_{\text{trop}} = \tau_{\text{trop } 2} - \tau_{\text{trop } 1}$$
- In practice
 - $\tau_{\text{trpz } i}$ estimated from the VLBI data

- Depends on the total electronic content (TEC) above telescopes
- Diurnal cycle
- Proportional to $1/v^2$
- Differential effect

$$\tau_{\text{ion}} = \tau_{\text{ion } 2} - \tau_{\text{ion } 1}$$

In practice:

τ_{ion} removed by linear combination of the delays measured at 2.3 GHz (S band) and 8.4 GHz (X band)

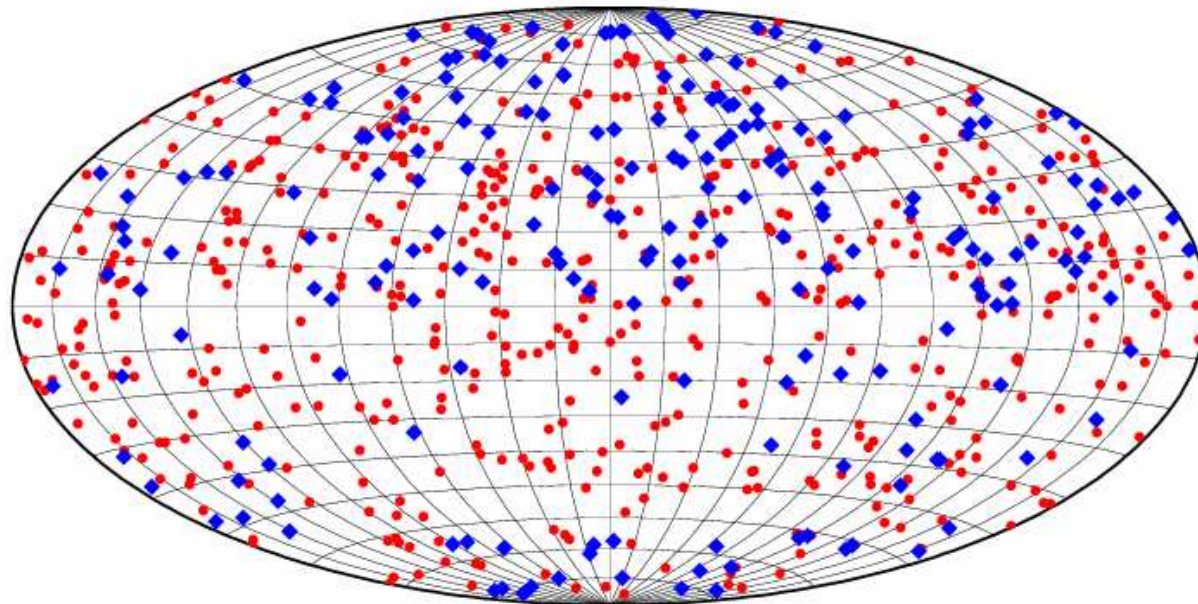


The International Celestial Reference Frame



- ICRF (International Celestial Reference Frame) built in 1995 by a Working Group of the IAU (Ma et al. 1998)
- Based on all VLBI astrometric/geodetic data acquired between 1979 and 1995
- 1.6 Million pairs of group delay and phase delay rate observations
- ICRF comprises 608 extragalactic sources
 - 212 defining sources
 - 294 candidate sources
 - 102 other sources
- Orientation of frame consistent with optical FK5 system
- ICRF-Ext.1 (1999) and ICRF-Ext.2 add another 59 + 50 sources => 717 sources today

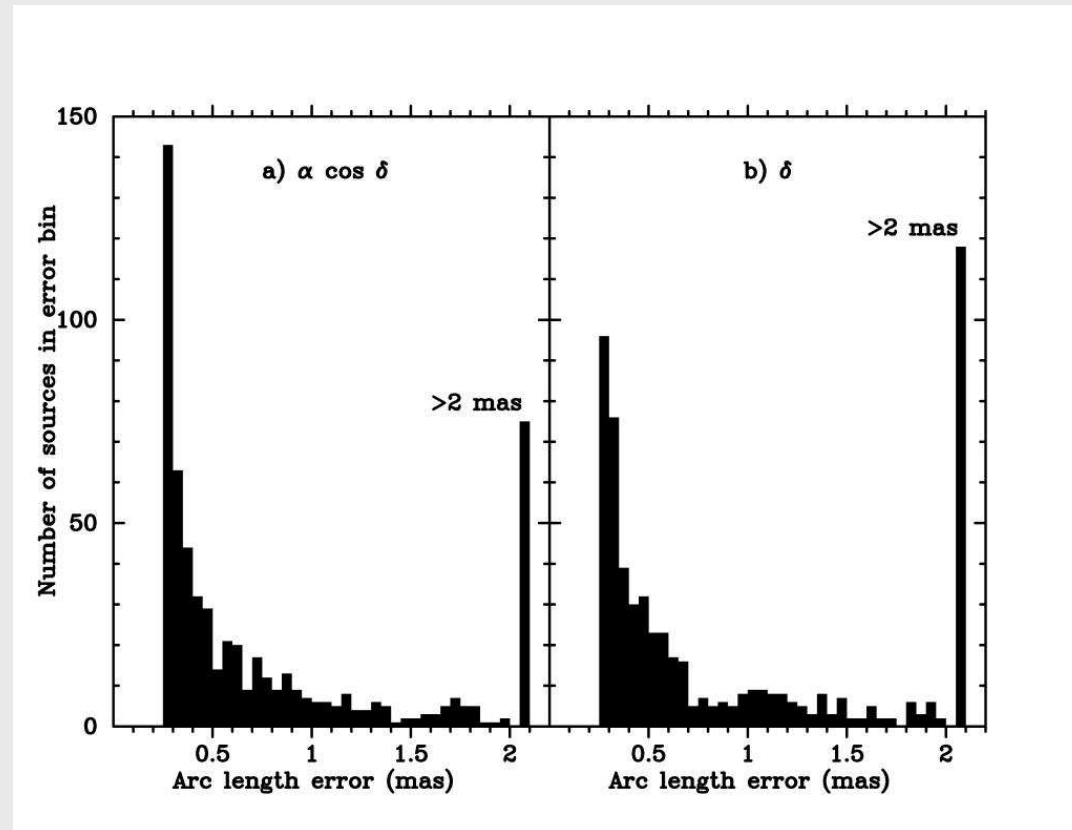
ICRF Ext.2 Sources



◆ 212 defining • 505 non-defining

Ma (2006)

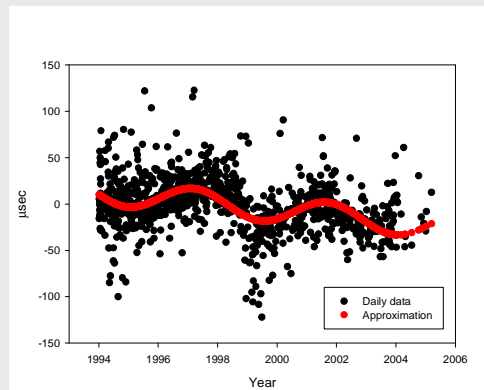
- Individual source position accuracy: ≥ 0.25 mas



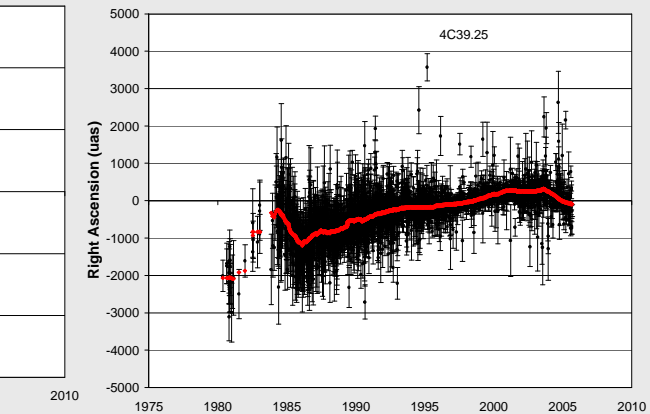
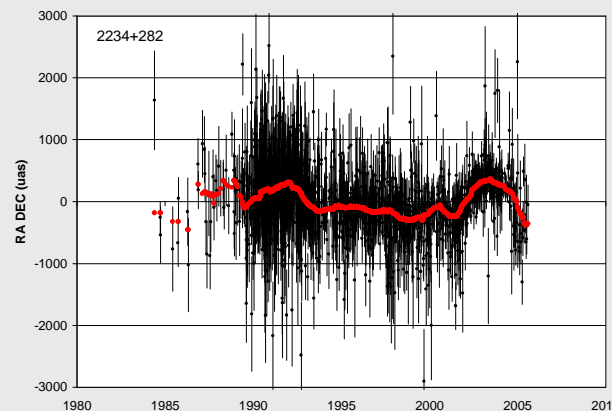
- Orientation of reference frame known to 0.02 mas

Future: ICRF-2

- Currently being built by a Working Group of the IAU and IVS
- Should be presented at IAU 2009 General Assembly
- Will add all VLBI data acquired since 1995
- Improved modeling (troposphere)
- Improved source categorisation
 - => accounts for **source position variability** and **source structure** to identify defining sources
- Source position variability



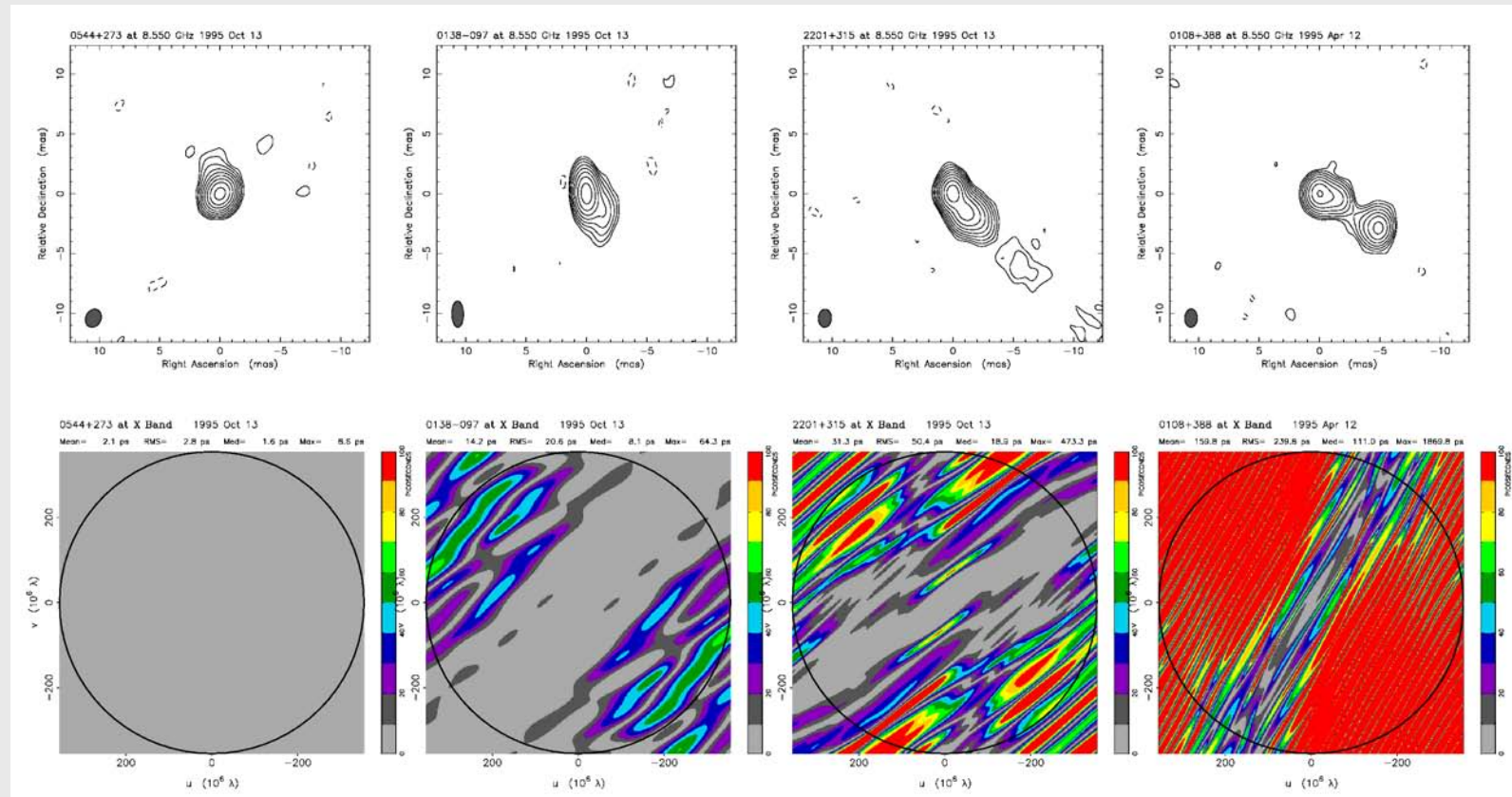
Titov (2006)



MacMillan (2006)

Future: ICRF-2

- Impact of source structure on the VLBI group delay



Four « structure index » categories defined to identify the most compact sources

Charlot (2002)

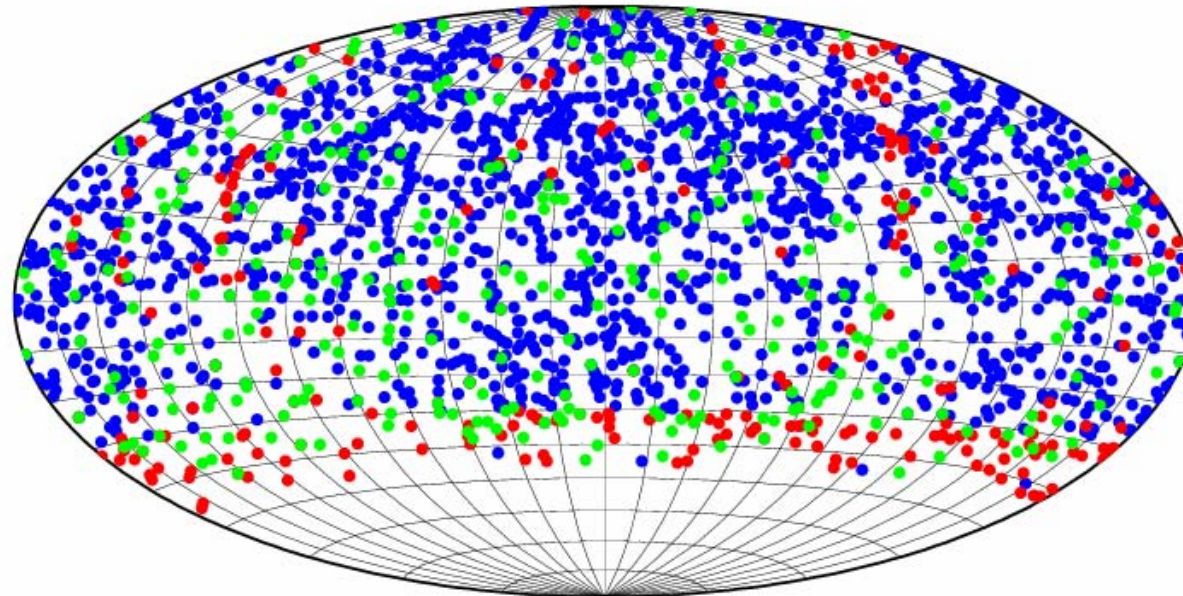
The VLBA Calibrator Survey (VCS)



- Based on a series of VLBA experiments conducted since 1994
- Most of the sources observed only in one experiment
- Declination $> -45^\circ$
- Milliarcsecond position accuracy
- Successive steps:
 - VCS1: 1579 sources (Beasley et al. 2002)
 - VCS2: 243 sources (Fomalont et al. 2003)
 - VCS3: 308 sources (Petrov et al. 2005)
 - VCS4: 258 sources (Petrov et al. 2006)
 - VCS5: 569 sources (Kovalev et al. 2006)
- Includes VLBI images as well
- Provides a dense grid of calibrators for phase-referencing

VCS sky distribution

VCS Sources



• 1579 VCS1

• 243 VCS2

• 308 VCS3

Ma (2006)

+ 258 VCS4 + 569 VCS5 (not shown)

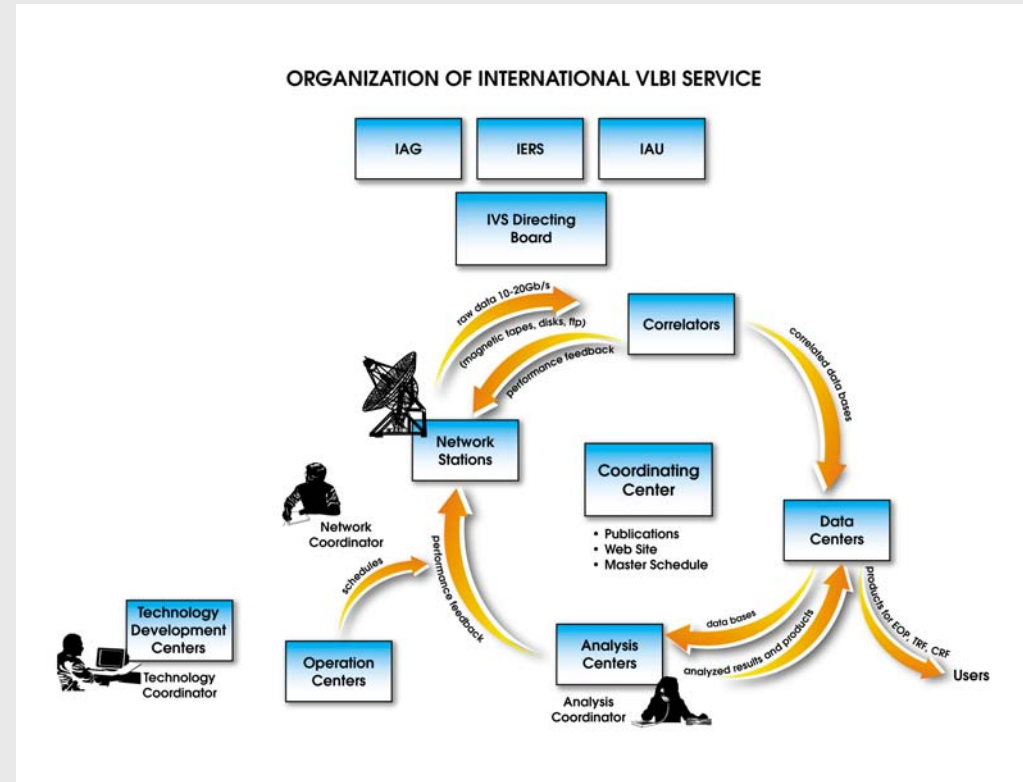
International VLBI Service for Geodesy and Astrometry



> 30 institutes around the world

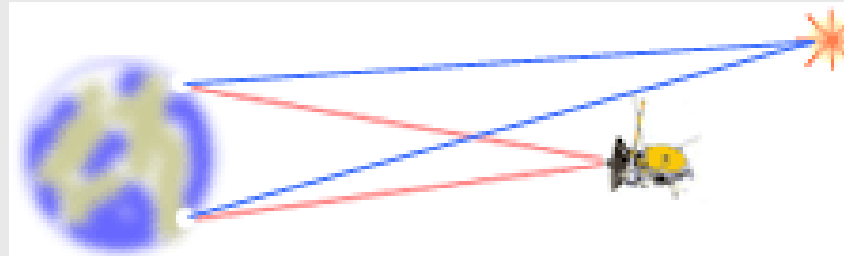
IVS monitors...

- the celestial frame
- the Earth's orientation
- the terrestrial frame



IVS observing

- Several 24-hour sessions conducted each week
- Additional 1 – to 2-hour sessions every day for Earth rotation
- Data immediately available from IVS data centres



- Observe alternately a target and an angularly-close ($\sim 1^\circ$) calibrator
 - target: e.g. weak extragalactic source, stellar object, spacecraft,...
 - calibrator: e.g. ICRF or VCS source
 - cycle calibrator/target of a few minutes
- Then, differentiate the measured phases for the calibrator and target
 - do not require phase connection
 - modeling errors scale with calibrator/target angular separation
- Permits accurate relative (narrow-angle) astrometry
 - see talk/demo by A. Brunthaler tomorrow...

The end



Thanks!