POLARIMETRY FOR APERTURE SYNTHESIS OBSERVATION

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1. Describing Polarization.

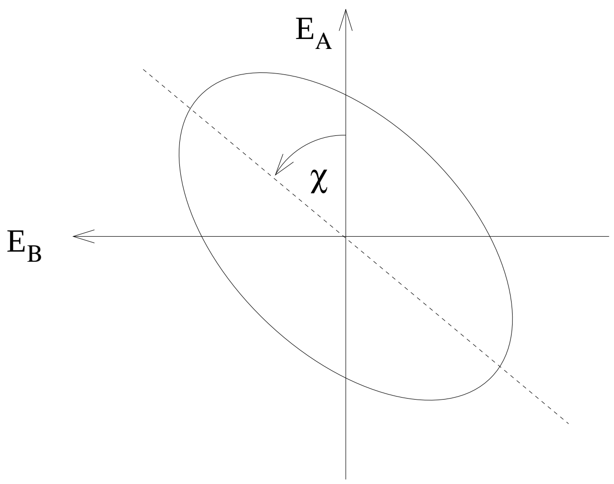
The Polarization of an EM wave is the distribution of the Electric field in the plane perpendicular to its direction of propagation (the 'sky' plane).

Unpolarized or **Randomly polarized** radiation has Electric field which is randomly oriented in the sky plane.

Fully polarized radiation is, in general, elliptically polarized. The fields can be described in terms of components along two orthogonal axes, usually taken to be North (A) and East (B).

$$
\begin{aligned}
E_A &= E_1 \cos(\omega t - \epsilon_1) \\
E_B &= E_2 \cos(\omega t - \epsilon_2)
\end{aligned}
$$

A Monochromatic wave must be fully elliptically polarized, but when independent sources

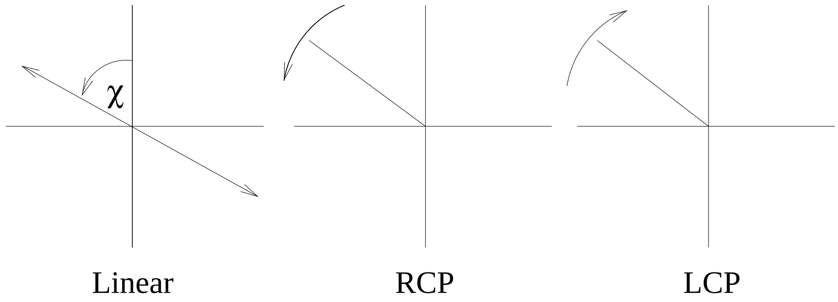


contribute with differing polarization states the radiation is no longer fully polarized.

2. Linear and Circular Polarization.

Let $\delta = \epsilon_2 - \epsilon_1$.
If $\delta = 0$ or $n\pi$ then E_A and E_B describe a straight line, resulting in pure **Linear Polarization**, with polarization angle χ measured North to East.

If $\delta = \pm\pi/2$ and $E_1 = E_2$, then $E_A^2 + E_B^2$ =constant, giving pure **Circular Polarization**. If $\delta = +\pi/2$, E circulates anticlockwise $\rightarrow$ Right circular polarization (RCP). If $\delta = -\pi/2$, E circulates clockwise $\rightarrow$ Left circular polarization (LCP). The convention for RCP and



sky plane − radiation approaching

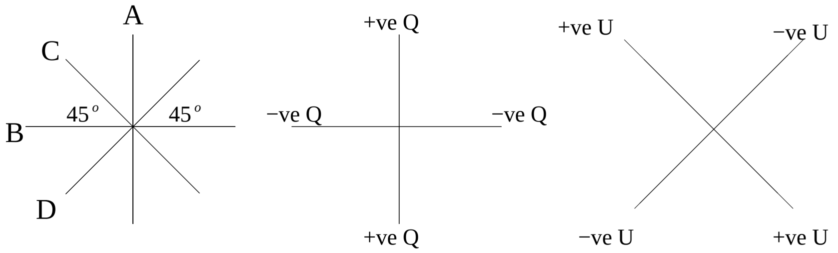
LCP described here is that adopted by engineers and normally used in radio astronomy. Many physics text books adopt the opposite convention.

3. Stokes parameters

These form a useful means to describe the observed radiation field. They are:

$$
\begin{aligned}
I =< E_A^2 > + < E_B^2 > &= < E_R^2 > + < E_L^2 > \\
Q =< E_A^2 > - < E_B^2 > & \\
U =< E_C^2 > - < E_D^2 > &= 2 < E_A E_B \cos\delta > \\
V =< E_R^2 > - < E_L^2 > &= 2 < E_A E_B \sin\delta >
\end{aligned}
$$

where $<>$ denotes the average value. (See Conway & Kronberg (1969), Pacholczyk (1970) for derivations).



States of pure Q and U

For unpolarized radiation:

$$Q = U = V = 0$$

For fully polarized wave:

$$Q^2 + U^2 + V^2 = (< E_A^2 > + < E_B^2 >)^2 = I^2$$

the squared intensity of the wave.
For a partially polarized wave, therefore

$$Q^2 + U^2 + V^2 < I^2$$

We can define the linearly polarized flux density as

$$p = \sqrt{(Q^2 + U^2)}$$

Then:

$$Q = p \cos 2\chi$$
$$U = p \sin 2\chi$$

The degree of linear polarization is

$$m = p/I$$

The complex polarization is

$$P = pe^{2i\chi} = mIe^{2i\chi} = Q + iU$$

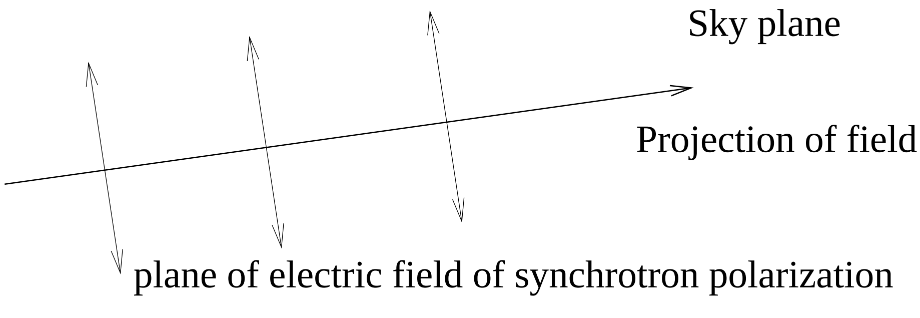
4. Sources of polarization.

Polarized radio emission arises chiefly from:

- Synchrotron Emission from radio galaxies and supernova remnants strong linear polarization $\simeq 70\%$ for uniform field. Weak circular polarization.
- Scattering - strong linear polarization possible. Weak polarization of the cosmic background radiation due to scattering just before p and e combined to form H.
- In astrophysical masers - strong linear and circular polarization possible - percentage polarization up to 100%

5. Polarization from Synchrotron radiation.

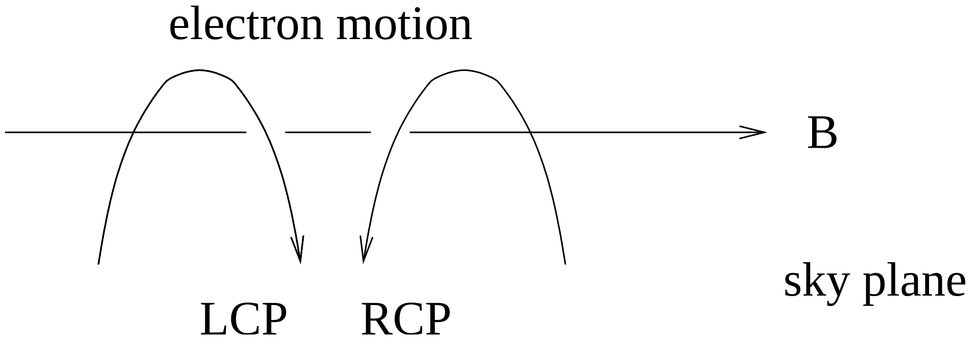
For optically thin radiation, the plane of linear polarization is parallel to the projection of the mean acceleration direction, ie. perpendicular to the sky projection of the magnetic field.



Interpretation is complicated by:

- **Opacity** - for optically thick sources, polarization is weak and plane of polarization 'flips' by 90°. This change has been observed, eg. by Gabuzda & Gomez (2001). Most resolved emission is likely to be optically thin.
- **Field disorder** Regions with different field directions within beam or along line of sight leads to depolarization. Levels are often below 10%.
- **Faraday rotation** - see later.

Since synchrotron radiation is due to electrons with $\gamma \gg 1$, the observer sees radiation from electrons with a narrow range of pitch angles, giving nearly equal numbers either side of the line of sight and thus nearly equal intensities of RCP and LCP.



6. Faraday Rotation

Propagation on a path parallel to a component of magnetic field $B_{||}$ in the presence of (non-relativistic) plasma of electron number density n (and electron mass m) causes a phase change between the two hands of circular polarization. This leads to a rotation of the plane of linear polarization by

$$\Delta\chi = RM\lambda^2$$

where

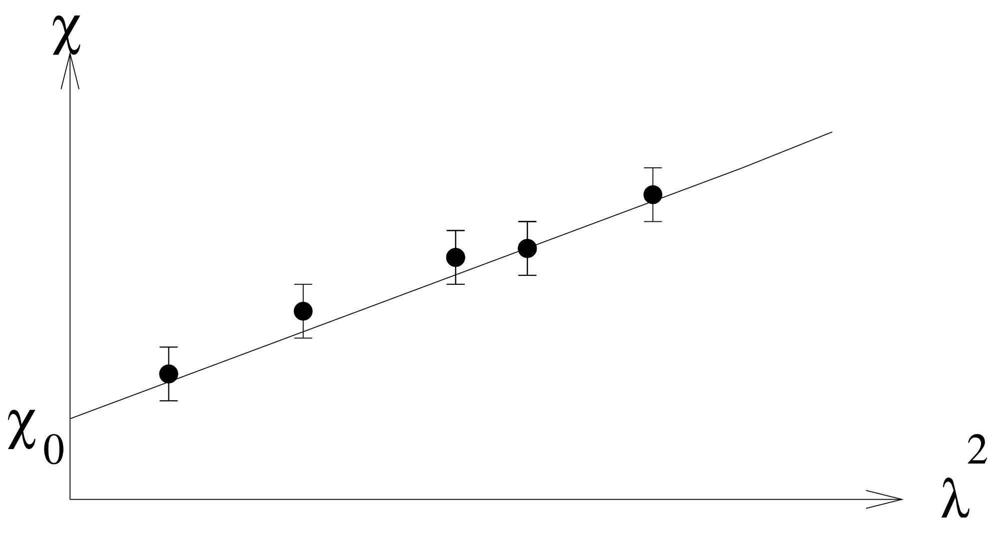
$$RM = \frac{e^3}{8\pi^2\epsilon_0 m^2 c^3}\int nB_{||}dl$$

is called the rotation measure (all units are SI) (Rybicki & Lightman, 1979).

Since $RM \propto e^3$, electrons and positrons produce equal and opposite effects. Hence an e^+, e^- plasma produces zero RM.

Faraday rotation is a nuisance, since it stops us seeing the true direction of the projected field.

For **external** Faraday rotation, this can be corrected by observing χ over a range of frequencies. To avoid 180° ambiguities, at least 3 frequencies are required.



Faraday rotation can also be useful, as an indicator of gas that might otherwise be undetected.

For **internal Faraday rotation** χ depends on the depth from which the radiation is emitted. A range of χs results in a reduction in the degree of polarization, known as Faraday depolarization.

Depolarization can also result from a range of RMs within the beam (side-to-side depolarization).

See Burn (1966) and Tribble (1991,1992) for discussion of Faraday rotation and depolarization.

7. Faraday Conversion

Propagation perpendicular to a component of magnetic field in the presence of relativistic plasma results in a change in phase δ between the two orthogonal components of linear polarization. Since

$$U = 2 < E_A E_B \cos\delta >$$
$$V = 2 < E_A E_B \sin\delta >$$

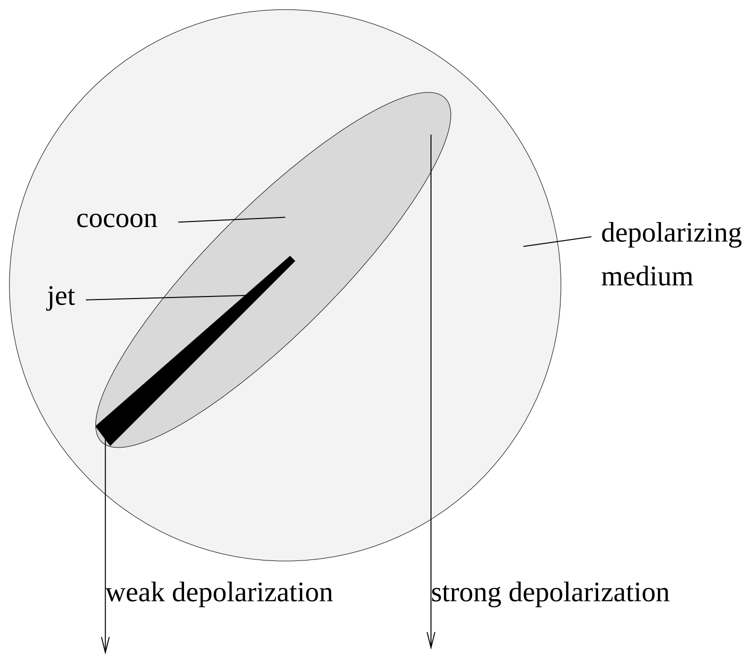
the result is conversion between U and V, ie. generating circular polarization from linear.

Wardle et al. (1998) find observed circular polarization too large to be intrinsic (ie. $m_c > 1/\gamma$) and probably due to Faraday conversion.

In 3C279 this requires a very low cut off in the electron population $\gamma_{min} < 20$ which, in turn, requires an electron positron jet (or there is too much Faraday rotation and too much energy flowing up the jet).
Faraday Conversion acts on U, but synchrotron radiation generates Q, hence some Faraday rotation (or a tangled field) is required.

8. Depolarization of Radio Lobes.

A nice illustration of science based on polarization, this provides evidence that the one-sidedness of jets in radio galaxies really is due to relativistic boosting (Garrington et al., 1988). The most strongly depolarized lobe in a radio galaxy is shown to be the one with



no observed jet, and is therefore the receding lobe. The jet is therefore on the approaching side.

9. Polarization and Aperture Synthesis

Many radio telescope arrays (VLBA, MERLIN, VLA, EVN) use **circularly polarized** feeds, and this discussion is for this case. Note that the Australia telescope uses crossed linear feeds.

For circularly polarized feeds, four correlations are produced for each baseline:

$$
\begin{gathered}
< LL^*(u,v) > \\
< RR^*(u,v) > \\
< RL^*(u,v) > \\
< LR^*(u,v) >
\end{gathered}
$$

where L and R stand for the signals from the right and left circular feeds.

From the definition of the Stokes parameters, it is clear that the parallel hands correlations between telescopes i and j are

$$
\begin{aligned}
< L_i L_j^*(u,v) > &= \tilde{I_{ij}} - \tilde{V_{ij}} \\
< R_i R_j^*(u,v) > &= \tilde{I_{ij}} + \tilde{V_{ij}}
\end{aligned}
$$

where $\tilde{I_{ij}}$ and $\tilde{V_{ij}}$ are the Fourier transforms of the sky distributions of Stokes parameters I and V.

By writing the amplitude of the components of field E_A,E_B,E_C and E_D in terms the right and left circular components E_R and E_L one can show that cross-hands correlations between telescopes i and j are

$$
\begin{aligned}
< R_i L_j^*(u,v) >= &\ \tilde{Q_{ij}} + i\tilde{U_{ij}} &= \tilde{P_{ij}}(u,v) \\
< L_i R_j^*(u,v) >= &\ \tilde{Q_{ij}} - i\tilde{U_{ij}} &= \tilde{P_{ij}}^*(u,v)
\end{aligned}
$$

where $\tilde{Q}$ and $\tilde{U}$ are the Fourier transforms of the sky distribution of Stokes parameters Q and U. $\tilde{P}$ is the Fourier transform of the complex polarization. (See Roberts, Wardle & Brown, 1994, for further details.)

Hence:

$$
\begin{aligned}
< L_i L_j^*(u,v) > &= \int\int (I(x,y) - V(x,y)) e^{2\pi i(ux+vy)} dxdy \\
< R_i R_j^*(u,v) > &= \int\int (I(x,y) + V(x,y)) e^{2\pi i(ux+vy)} dxdy \\
< R_i L_j^*(u,v) > &= \int\int p(x,y) e^{2i\chi(x,y)} e^{2\pi i(ux+vy)} dxdy \\
< L_i R_j^*(u,v) > &= \int\int p(x,y) e^{-2i\chi(x,y)} e^{2\pi i(ux+vy)} dxdy
\end{aligned}
$$

For imaging synchrotron sources it is usually acceptable to set $V = 0$ in the first equation, unless you're trying to measure it!

It is clear that

$$< L_i L_j^*(-u,-v) >=< L_i L_j^*(u,v) >^*$$

(and similarly for RR) Hence the UV plane can always be filled symmetrically for the imaging RR or LL.

The same symmetry does not apply to the cross hands correlations. However from the Fourier transforms above it is clear that

$$
\begin{aligned}
< R_i L_j^*(-u,-v) > &= < L_i R_j^*(u,v) >^* \\
< L_i R_j^*(-u,-v) > &= < R_i L_j^*(u,v) >^*
\end{aligned}
$$

For this reason, for a symmetrical UV coverage, both RL and LR correlations must be measured for every UV point. If one or more telescopes in the array records only one hand of polarization (eg. the orbiting VLBI station, HALCA, or due to a receiver failure), then a complex P image must be made (rather than U and Q separately) and a complex clean performed.

Note that circular feeds are good for measurement of weak linear polarization, since the linear polarization appears as the cross correlation of the two feeds rather than the difference between two signals which may have poor relative calibration.

However, for the same reason, circular feeds are poor for measurement of weak circular polarization, since the circular polarization appears as the difference between $< RR^* >$ and $< LL^* >$, which may have poor relative calibration. Special techniques have been developed for imaging circular polarization using circular feeds (Wardle et al (1998), Homan et al. (1999)).

10. Calibration of Polarization data

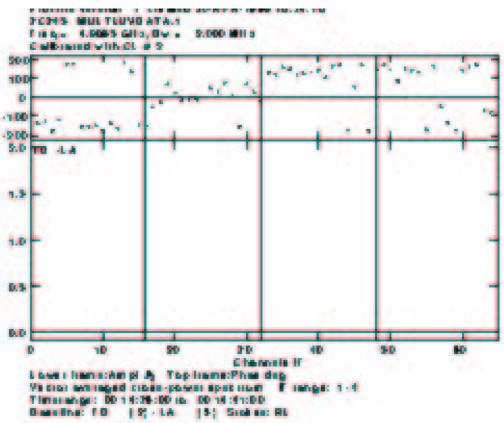
This discussion applies to polarization imaging of discrete continuum sources. For 'Recipes' see AIPS cookbook, chapter 9, or the Brandeis website:
pc.astro.brandeis.edu/BRAG/pol_cal

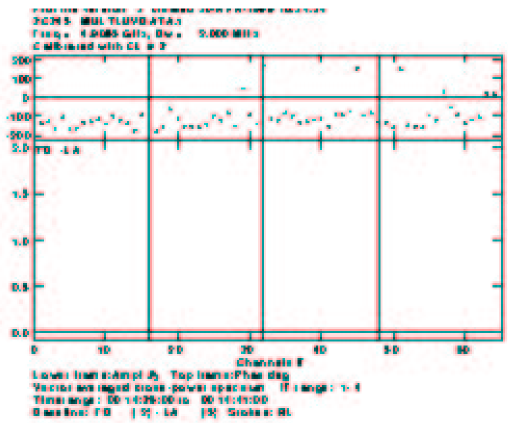
Several calibration tasks must be performed following those required for I imaging:

- Correction for R-L delay
- Removal of effects of instrumental polarization
- Calibration of the polarization angle χ

10.1 The effect of R-L delays.

Fringe fitting together with use of telescope delay measurements (eg. using VLBA pulse-cal data) should remove all delays and rates for each hand of polarization, but there may be residual delay between the R and L data channels. The effect of these will be to produce a phase gradient across the RL and LR bands which prevents integration across the band. This is illustrated in POSSM plot figure below.





At most arrays (eg. MERLIN, VLA, recent VLBA data) the on-line monitoring of the telescope takes account of this effect and no such gradient should be present (now included in VLBA pulse cal).

If such effects are present they can be determined using a scan on a strongly polarized source that includes the reference antenna and removed using AIPS task VLBACPOL (formerly CROSSPOL).

We have assumed the signals are perfectly split into RCP and LCP. Sadly this is not true. Nominal RCP channels will be contaminated by a small amount of LCP and vice versa.

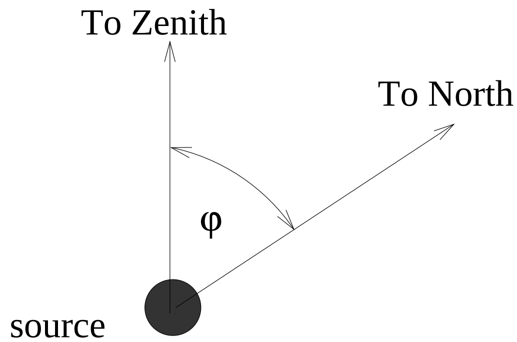
There are two common representations, the elliptical feed model and the D-term model. These two can be shown to be equivalent (Roberts, Wardle & Brown, 1994). The D-term model is most commonly used.

The level of contamination expressed by the complex **D-terms**.

$$V_R = G_R(E_R e^{-i\phi} + D_R E_L e^{i\phi})$$
$$V_L = G_L(E_L e^{i\phi} + D_L E_R e^{-i\phi})$$

The G terms are the complex gains, determined by self-calibration or using a compact calibrator nearby.

The ϕ angles are the **parallactic angles** and describe the rotation of the feeds of an alt-az telescope relative to the source as it tracks a source across the sky.
For equatorial mounts (rare nowadays) the parallactic angles can be taken to be zero. Information about the mounts is contained in the antenna tables.



Parallactic angle φ subtended at source between local vertical and North.

In calibration using AIPS, parallactic angle corrections are applied at an early stage, rotating phases by $-\phi$ for L and $+\phi$ for R feeds. The corrupted correlations then become

$$\begin{aligned} L_i L_j^* &= G_{Li} G_{Lj}^* I \\ R_i R_j^* &= G_{Ri} G_{Rj}^* I \\ R_i L_j^* &= G_{Ri} G_{Lj}^* [P + I(D_{Lj}^* e^{2i\phi_j} + D_{Ri} e^{2i\phi_i})] \\ L_i R_j^* &= G_{Li} G_{Rj}^* [P + I(D_{Rj}^* e^{-2i\phi_j} + D_{Li} e^{-2i\phi_i})] \end{aligned}$$

These expansions ignore terms of order DP and $D^2 I$ and are valid for small D terms (a few percent or less).

If uncorrected, the influence of the D-terms is to scatter total intensity flux into the P image. (See Roberts et al., 1994, for a full discussion of effects).

The cross-hands correlations each contain 3 terms that depend differently on the parallactic angles. This is the key to determination of the D-terms. A good range of parallactic angles is therefore desirable.

10.3 Estimation of D-terms.

Use of an unpolarized source. At reasonably low frequencies (eg. < 15GHz) there are some compact radio sources with weak polarization $\ll 1\%$. eg. 3C84 and OQ208. For these the source terms disappear and the cross-hand expressions can be solved for the D-terms. At higher frequencies, sufficiently weakly polarized sources do not exist.

Use of a compact polarized calibrator.

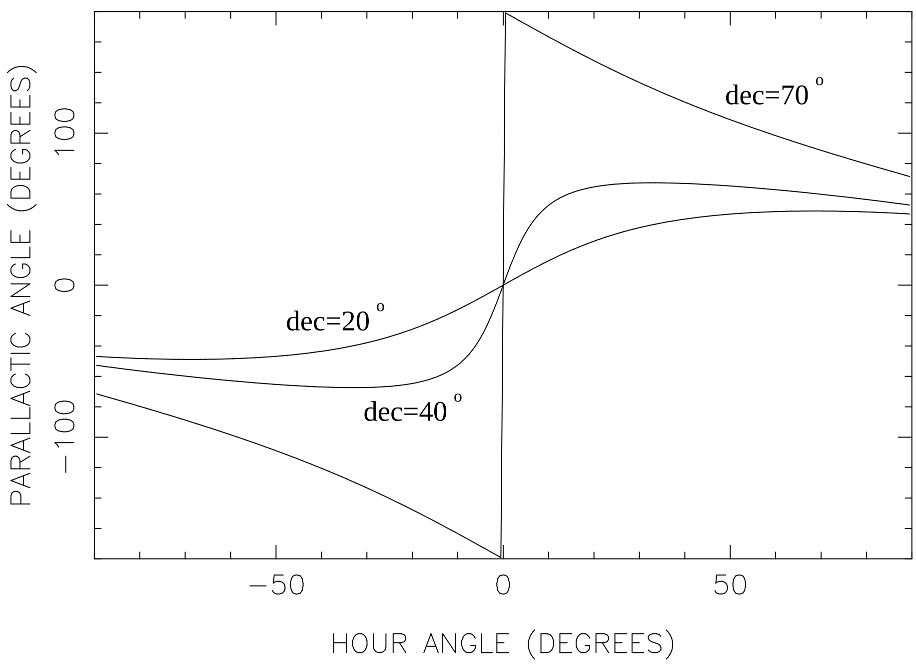
- A compact, strongly polarized calibrator must be imaged in I,
- divided into a small number of regions
- a clean component table constructed for each region
- LPCAL uses the clean components and visibility data to solve for the percentage polarization of each region and the D-terms. It uses only the linear approximation to the full D-term model.
  PCAL can provide solution for a full elliptical feed model but can only use a point-source model. For VLBI, LPCAL is normally used.

When using non-VLBI arrays (VLA, MERLIN, WSRT) 'point source' calibrators may be available and a point source model can be specified in the solution.

10.4 Choice of polarization calibrator.

The polarization calibrator should be

- strongly polarized
- compact to the array being used
- provide good parallactic angle coverage during the course of your observation.



The parallactic angle coverage depends on the telescope latitude and calibrator declination. The figure above shows parallactic angle against hour angle for a telescope at latitude 45° and calibrators at declinations 20°, 40° and 70°. At low declinations the range of parallactic angles is small. At very high declinations, the range is large, but the variation is slow, so that for a short observation a limited range of parallactic angles is obtained. Intermediate declinations are probably the best, but beware of sources that pass too close to the zenith: for these, the parallactic angles may change too rapidly.

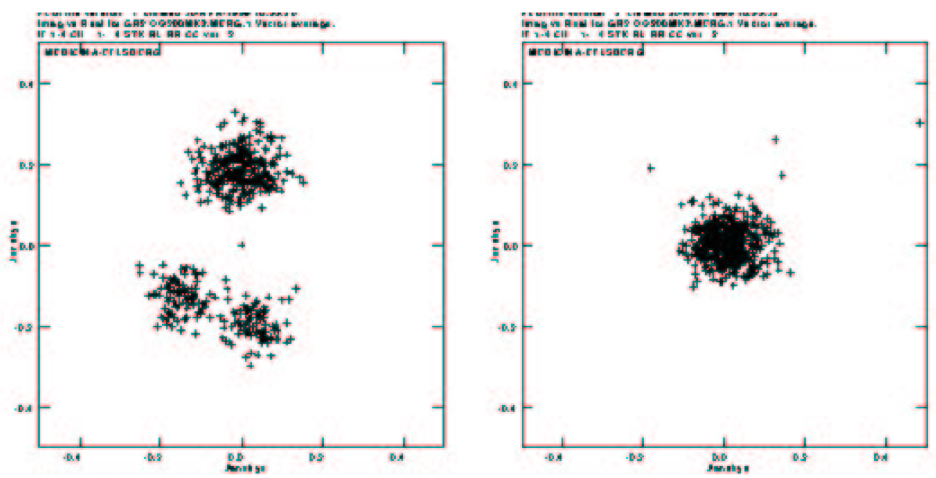
At VLBA telescopes, D-terms are of order 1% or less, even at 43GHz.
Note that some EVN antennas have, in the past, been found to have D-terms of up to 25%. In this case, the linear approximations are only marginally valid. Care should be taken in use of any data resulting from such large D-term corrections.

The D-terms are applied when the data is 'SPLIT' to form a single source dataset. Corrections can be applied to parallel hands and as well as cross hands fringes provided that all four correlations (LL, RR, RL, LR) are present. This is important for attempts to measure the weak circular polarization, $V = RR - LL$.

10.5 Has the calibration worked?

AIPS is not well provided with diagnostics for checking the calibration. One option is to plot the imaginary v. real parts of the cross-hand visibilities. For calibrated data on a compact source these should be clustered around a point. For reasonably large D-terms, the difference in the distributions before and after calibration should be apparent.

The plot below shows real and imaginary parts of the cross hand fringes for the unpolarized source OQ208 before and after calibration. The baseline is Effelsberg-Medicina with D-terms of order 5% and 15% respectively.



For VLBA data, the effects may be harder to detect with cross-talk signal comparable to the noise on single data points.

The effects on the map should be checked. (See Roberts, Wardle & Brown, 1994, for a means to estimate the map errors resulting from instrumental polarization.)

10.6 Polarization angle calibration.

At this point one remaining phase correction must be made. This is the phase difference θ between the right and left hand gains at the reference antenna, which results in a rotation of the polarization angles in the image by $\Delta\chi = \theta/2$.

There are several methods to evaluate this correction.

Observation of a source of known position angle. At the VLA and MERLIN this is straightforward: there are several compact, strongly polarized sources that are known to have highly stable values of χ. 3C48 (0137+331) and 3C236 (1331+305) are examples and a short scan on one of these should suffice. Their polarization angles at different frequencies are tabulated on the NRAO website. Provided the sources are unresolved, LISTR matrix averages of RL and $(LR)^*$ phases can be obtained and the correction is found by subtracting these from the expected value of 2χ.

For VLBI, this is more difficult because polarized sources compact to VLBI arrays are generally (i) resolved and (ii) highly variable. If the observation includes an array such as MERLIN or VLA, a compact, strongly polarized source for which all polarized flux detected by the smaller array is also detected by the VLBI array can be used. The source can be imaged using both arrays and the absolute polarization angle set using the result of the smaller array. Being compact and highly polarized, BL Lac objects are ideal for this purpose. A good choice is to be found in Gabuzda & Cawthorne (1996).

If a compact array is not available then one of a number of standard calibrators can be observed. Their polarization is monitored approximately monthly by NRAO in bands at $5, 8, 15$ and 43GHz, and values can be obtained from the website:
www.aoc.nrao.edu/∼smyers/calibration.
However note that many active galaxies do vary on timescales shorter than this. Therefore, several of these calibrators should be included and checked for consistency.

Another source of polarization monitoring is the University of Michigan database: **www.astro.lsa.umich.edu/radiotel/umrao.html**. This contains observations at 5, 8 and 15GHz (though the bands are slightly offset in frequency than those used by NRAO). This may be a useful backup if NRAO fail to make the observation you were depending on.

Use of the known RL phase differences. At some telescopes (including several VLBA telescopes) this is known to be fairly stable. So far as I know, NRAO do not publish the R-L phases of VLBA antennas, but there is a list of values at 15 GHz from the MOJAVE AGN project maintained by Matt Lister at:
www.physics.purdue.edu/astro/MOJAVE/data.html.

At other frequencies the values are known to regular observers such as Denise Gabuzda, Alan Marscher, Thomas Krichbaum, etc. who will tell you the latest values. This can be a useful backup but should NOT be depended on since large variations in RL phase difference can and do sometimes occur.

In general: design a strategy for PA calibration and incorporate this into your .key file. Aim to have at least two determinations to check for consistency.

11. Imaging polarization.

Once calibration is complete sources should be imaged again in I. Q and U maps can be made straightforwardly using IMAGR if RL and LR fringes are present at each UV point. Otherwise a complex Fourier transform and clean must be performed.

Polarization angles are normally represented by the orientation of polarization sticks. Polarized flux density can be represented by the stick length if superimposed on an I contour map, or otherwise by a p contours.

12. Calibration for Circular Polarization.

Circular polarization provides useful constraints on the properties of synchrotron sources, but detection of this weak signal is made more difficult by use of R and L feeds at many arrays. The a priori calibration of R and L channels may only be accurate to $\sim 5\%$ at best, and the small circular polarization signal

$$V = RR - LL$$

of only $< 1\%$ of I is therefore lost.

Special techniques have been developed for detection of circular polarization (Homan & Wardle, 1999).
Application of D term corrections to parallel hand data is essential for strongly polarized sources. Following this, several techniques for gain calibration have been developed, and the results from these should be checked for consistency. The methods include:

- **Self-calibration assuming zero V.** This process may reduce or relocate V, but cannot create it.
- **Gain transfer.** Gains are determined for a source believed to have zero V and transferred to the target source.
- **Phase only mapping.** This involves setting all the amplitudes to unity. The resulting structures are determined by the closure phases alone. Prior knowledge of the location of the V component is required to accurately determine its flux density.

13. Appendix: A formalism for calibration.

Here I present a very brief summary of the formalism for describing the polarization of interferometer output which is used at many observatories. My advice is to ignore this section until you're fairly confident about the material presented earlier!

The polarization state of a wave can be represented by a vector using a formalism known as Jones algebra (eg. see Grant & Fowles).

The base vectors can be regarded as either the orthogonal E_A or E_B vectors or the Right and Left Circular components.

Adopting the second option as appropriate to most aperture synthesis telescopes nowadays, an RCP wave is given by

$$\mathbf{E} = \begin{pmatrix} R \\ L \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

an LCP wave is given by

$$\mathbf{E} = \begin{pmatrix} R \\ L \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and a pure linearly polarized wave with polarization angle χ is given by

$$\mathbf{E} = \begin{pmatrix} R \\ L \end{pmatrix} = \frac{1}{2}\begin{pmatrix} e^{i\chi} \\ e^{-i\chi} \end{pmatrix}$$

so that $E_A = Re(R + L) = \cos\chi$ and $E_B = Im(R - L) = \sin\chi$ as expected.

The effects of propagation through space and through the telescope can be described by matrices, known as Jones matrices, which multiply these vectors. For example, the effects of Faraday rotation can be described by the matrix

$$\mathbf{F} = \begin{pmatrix} e^{i\Delta\chi} & 0 \\ 0 & e^{-i\Delta\chi} \end{pmatrix}$$

A similar matrix $\mathbf{P}$ can include the effect of parallactic angle variation.

The effect of polarization leakage can be described by the matrix

$$\mathbf{D} = \begin{pmatrix} 1 & D_R \\ D_L & 1 \end{pmatrix}$$

In fact all propagation and transmission effects can be included in this way so that the (R,L) signal input to the correlator can be written as the product of the Jones matrix $\mathbf{J}_i = \mathbf{BGDAPTF}$ and the Jones vector for the source (R,L) signal, where, in addition to those matrices mentioned above, $\mathbf{B}$ describes the bandpass response, $\mathbf{G}$ describes the gains, $\mathbf{A}$ describes the antenna beam response, bfB describes the bandpass and $\mathbf{T}$, describes effects of the troposphere (refraction, opacity). Hence the signal input to the correlator is $\mathbf{E}_{i,corr} = \mathbf{J}_i\mathbf{E}_{source}$.

One such Jones matrix, $\mathbf{J}_i$ exists for each antenna i. The correlated visibility between two antennas i and j can be represented by a type of matrix product called the Kronecker product (sometimes referred to as a type of outer product - see Jain, 1989):

$$\mathbf{V}_{ij} = \left\langle \begin{pmatrix} R_i \\ L_i \end{pmatrix} \right\rangle \otimes \left\langle \begin{pmatrix} R_j \\ L_j \end{pmatrix} \right\rangle = \begin{pmatrix} < R_i R_j > \\ < R_i L_j > \\ < L_i R_j > \\ < L_i L_j > \end{pmatrix}$$

If the source polarization vectors from the two antennas are E_i and E_j the value of $\mathbf{V}$ output by the correlators will be (using the mixed-product property of the Kronecker product)

$$\mathbf{V}_{ij,obs} =< \mathbf{J}_i\mathbf{E}_i \otimes \mathbf{J}_j\mathbf{E}_j >=< (\mathbf{J}_i \otimes \mathbf{J}_j)(\mathbf{E}_i \otimes \mathbf{E}_j) >=< (\mathbf{J}_i \otimes \mathbf{J}_j) > \mathbf{V}_{source}$$

The above assumes we are dealing with a point source, but if the source has finite angular size, integration over direction cosines l and m gives the correlated visibilities RR, RL, LR and LL as

$$\mathbf{V}_{ij,obs} = \int\int (\mathbf{J}_i \otimes \mathbf{J}_j)\tilde{\mathbf{V}}_{ij,source} e^{2\pi i(ul+vm)} dldm$$

Finally, if the source polarization is, instead, expressed in terms of Stokes parameters, the source visibility can be expressed as

$$\tilde{\mathbf{V}} = \mathbf{SI} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & i & 0 \\ 0 & 1 & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+V \\ Q+iU \\ Q-iU \\ I-V \end{pmatrix}$$

Then the measured visibilities can be expressed as

$$\mathbf{V}_{ij,obs} = \int\int (\mathbf{J}_i \otimes \mathbf{J}_j)\mathbf{S}\,\mathbf{I}_{source} e^{2\pi i(ul+vm)} dldm$$

which is known as the *measurement equation.*

All the quantities required to calibrate the measurement are contained in the Kronecker product of the Jones matrices for the two antennas. Much of the discussion of calibration contained on observatory websites is couched in terms of the measurement equation. It provides a sound formalism to turn to when you want to be sure you've got things right, but it's not (to me, anyway) very useful when trying to *understand* polarization, interferometry and calibration!!

Note that the rule for forming the Kronecker product of 2 by 2 matrices is

$$\begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \otimes \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} = \begin{pmatrix} J_{11}K_{11} & J_{11}K_{12} & J_{12}K_{11} & J_{12}K_{12} \\ J_{11}K_{21} & J_{11}K_{22} & J_{12}K_{21} & J_{12}K_{22} \\ J_{21}K_{11} & J_{21}K_{12} & J_{22}K_{11} & J_{22}K_{12} \\ J_{21}K_{21} & J_{21}K_{22} & J_{22}K_{21} & J_{22}K_{22} \end{pmatrix}$$

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