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Radioastronomie

Imaging capabilities of future radio telescopes

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1. Introduction

In the past two decades, radio astronomical instruments have typically featured a superior resolution and adequate imaging performance, compared to the instruments working in other spectral bands. The situation may be quite different in the next few decades. The next generation interferometric facilities, ALMA, LOFAR and SKA, will become operating at the time when several new large optical (LBT, CELT, EURO50, OWL) X–ray (CONSTELLATION-X, XEUS, MAXIM) and Gamma–ray (GLAST) telescopes are expected to be working. The future optical telescopes like CELT and OWL would both surpass the dynamic range offered in the radio by the connected interferometers and match the resolution of ground-based centimeter wavelength VLBI. This is a compelling argument for designing future radio instruments to match the imaging capabilities of the top-level facilities in the optical and other spectral bands.

At the sensitivity levels envisaged for the LOFAR and the SKA, the imaging performance has to be evaluated using not only the "common" combination of the limiting resolution and dynamic range but also the *spatial dynamic range* (SDR) defined as a ratio between the sizes of the maximum and minimum detectable structures. For interferometric instruments, the SDR is affected by a number of factors, most critical of which are the channel bandwidth, integration time and coverage of the Fourier domain (Lobanov 2003). The limitations posed by these factors should be carefully considered. A brief discussion of imaging capabilities of LOFAR and SKA is presented here, with special attention paid to comparison of spatial dynamic ranges of LOFAR, SKA and other major astrophysical facilities planned to be built in the next decades.

A detailed comparison of imaging capabilities of LOFAR, SKA, and various other telescopes can be found at
`http://www.mpifr-bonn.mpg.de/staff/alobanov/ska/telescopes.html`

2. Resolution

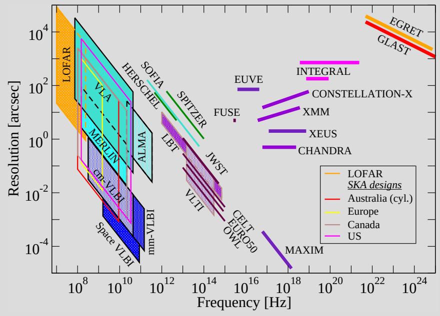

Figure 1. Resolution of LOFAR and SKA compared with the resolution of other main existing and future astronomical instruments. The resolution of LOFAR will provide an excellent match to resolutions of most of the IR, UV, and high-energy facilities, but it will be inferior to the resolutions of major optical telescopes. All of the basic SKA designs are enlisting longest baselines of $\gtrsim 3000$ kilometers, which results, for most of them, in a ≈ 1 milliarcsecond resolution at the highest observing frequency. This resolution is going to provide and adequate counterpart to the resolution of the largest projected optical telescopes, and may be inferior to the resolution of the proposed X–ray interferometer mission MAXIM. The imaging capability of MAXIM is however rather modest ($D \lesssim 700$), and it should be matched easily by radio observations using millimeter– or space–VLBI. For the optical telescopes, the SKA would be able to present a reasonable match, provided that the longest baseline limit is not pushed below 3000 km. More strictly: the SKA that matches well the resolution of the large future optical telescopes must provide roughly a $\lesssim 1$ mas resolution at its highest observing frequency.

3. Spatial dynamic range

The theoretical limit for the maximum achievable SDR of an interferometer is limited by the field-of-view (FOV) of its primary elements, and it is given by

$$D_{\rm FOV} \approx 0.80 \frac{B_{\rm long}}{\eta_{\rm a}^{0.5} d_{\rm an}},$$

where $\eta_{\rm a}$ is a measure of the aperture efficiency, $B_{\rm long}$ is the length of the longest baseline, and $d_{\rm an}$ is the size of receiving element (the diameter of an individual antenna or the size of a beam forming element, for tile and dipole arrays). Observations in which $D_{\rm FOV}$ can be reached are FOV-limited.

The FOV-limited SDR can be typically achieved only at the shortest baselines, but it is significantly reduced at the highest instrumental resolution, due to a finite bandwidth ($\delta\nu$) and integration time (τ) and an incomplete Fourier domain sampling expressed by the *filling factor*, $\Delta u/u$, of the Fourier plane (where Δu is the average gap in the Fourier domain coverage at a Fourier frequency u).

The compound effect of these factors has to be understood in order to make a quantitative assessment of a realistic SDR of an interferometric instrument. Numerical models should be applied to study specific details related to individual instruments, but an analytical approach can be developed to estimate, to the first order, the realistic SDR of interferometric instruments.

4. Realistic SDR

Consider an interferometer with an ideal, FOV-limited SDR $D_{\rm FOV}$. At a given integration time, τ[sec], the maximum achievable SDR is

$$D_\tau \approx 1.13 \cdot 10^4 \tau^{-1}\,.$$

A finite fractional bandwidth, $\Delta\nu$, yields the limiting SDR

$$D_{\Delta\nu} \approx \frac{1}{\Delta\nu}\left[\left(\frac{I_0}{I}\right)^2 - 1\right]^{0.5},$$

where I_0/I is the maximum acceptable peak brightness reduction. Finally, an incomplete coverage of the Fourier space sets the limiting SDR to

$$D_{uv} \approx 3\,D_{\rm FOV}\left\{\exp\left[\frac{16\ln 2}{\pi^2}\left(1+\frac{\Delta u}{u}\right)^2\right]\right\}^{-1},$$

where $\Delta\,u$ is the size of the largest gap in the coverage of the Fourier domain. For a regular array (i.e., a logarithmic spiral) with N stations, $\Delta u/u$ can be roughly estimated by $(B_{\rm max}/B_{\rm min})^{3/N} - 1$. In case of multifrequency synthesis, $\Delta u/u$ should be substituted by $\Delta u/u + \Delta\nu_{\rm mfs}$, where $\Delta\nu_{\rm mfs}$ is the fractional bandwidth over which the multifrequency synthesis is performed. In a real observation, the actual spatial dynamic range $D_{\rm m}$ is determined by the most conservative of the estimates provided above. This effect is illustrated in Figure 2 comparing the realistic SDR of LOFAR and some of the basic SKA designs. The SDR of major future astrophysical facilities are compared in Figure 3.

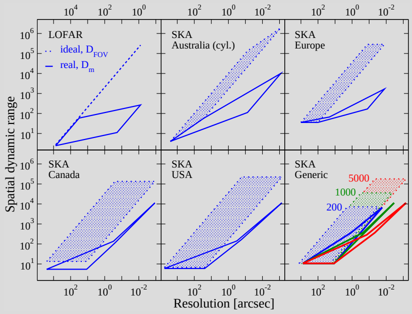

Figure 2. Maximum ($D_{\rm FOV}$, dotted polygons) and realistic ($D_{\rm m}$, solid polygons) spatial dynamic ranges of LOFAR and several basic SKA designs plotted against the resolution (the panel with "generic" designs describes a "typical" SKA with baselines extended up to 200, 1000 and 5000 km, respectively). A reduction of SDR due to finite bandwidth and integration time is visible in all of the panels (a bandwidth, BW = 1 MHz and an integration time, $\tau = 1$ s are assumed). For most of the SKA designs, this results in a factor of ∼100 reduction of the SDR at the highest instrumental resolution (the Canadian design suffers from the SDR reduction at all resolutions, due to a small number of stations and poor $\Delta u/u$ ratio). For LOFAR, the decrease is even larger, but it can be alleviated by applying the multibeaming capabilities of the instrument. Removing the discrepancy between $\rm SDR_{FOV}$ and $\rm SDR_m$ requires reducing BW and/or τ. The reductions necessary to achieve SDR_{uv} are given in Table 1 by BW_{uv} and τ_{uv}. This, correspondingly, leads to an increased point source sensitivity and longer observing times required.

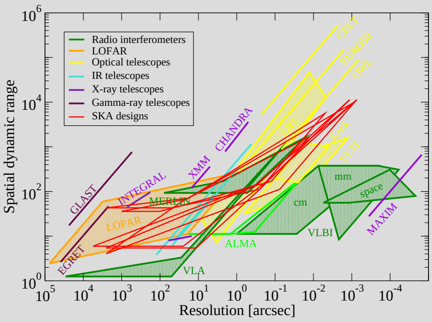

Figure 3. Spatial dynamic range of the SKA designs ($\rm SDR_m$ for observations with a bandwiwdth, BW = 1 MHz and an integration time, $\tau = 1$ s) compared to other major astrophysical facilities. $D_{\rm m}$ of the SKA will be higher than that in most of the existing instruments. Future optical instruments however will be able to achieve much higher SDR levels, comparable with the maximum $D_{\rm FOV}$ of LOFAR and SKA (see Figure 2). This implies that the SDR reductions due to finite bandwidth and integration time should be considered carefully already at the stage of the instrument design, in order to optimize the imaging performance of an interferometer. For most of the high–dynamic range observations with LOFAR and SKA, the bandwidth and integration time may have to be significantly reduced, if one would require to reach SDR similar to that of the largest optical instruments.

References

Lobanov, A.P. 2003, SKA Memo 38.
`http://www.skatelescope.org/PDF/ska_memo38.pdf`

5. Imaging performance

The SDR and imaging performance of LOFAR and four basic designs of the SKA are compared in Table 1. The technical parameters are compiled from the LOFAR system definitions, German LOFAR white paper, and from the design concepts of the SKA. The imaging performance is evaluated by calculating the SDR from the analytical approximations given in Sections 2 and 3. The resolutions of LOFAR and SKA are compared in Figure 1 to the resolutions of various existing and future telescopes. The SDR of LOFAR and different SKA designs are presented in Figure 2 and compared in Figure 3 with the SDR of other instruments. The SDR are affected by the observing bandwidth, averaging time and filling factor of the Fourier domain, $\Delta u/u$. The effect of the latter parameter is illustrated in Figure 4, in which the filling factors and the limiting SNR of "trustable" pixels are compared for different instruments.

Table 1. LOFAR and SKA: Imaging performance

Parameter	**LOFAR**	**Australia (cyl.)**	**Europe Tiles**	**Canada LAR**	**USA SAR**
Frequency and baseline coverage					
$\nu_{\rm low}$[MHz]	10	100	150	100	150
$\nu_{\rm high}$[MHz]	240	9000	15000	22000	34000
$B_{\rm short}$[km]	0.1	0.3	0.5	0.5	0.1
$B_{\rm long}$[km]	400	10000	4000	3000	3000
Configuration and uv–coverage					
$N_{\rm an}$	8300	600	1.0E+6	60	4400
$N_{\rm st}$	83	60	100	60	160
$(\Delta u/u)_{\rm best}$	0.16	0.04	0.10	0.50	0.17
$(\Delta u/u)_{\rm worst}$	0.16	0.11	0.33	0.50	0.24
Spatial dynamic range and image sensitivity					
$D_{\rm FOV}$	2.6E+05	1.9E+06	2.9E+05	1.3E+05	2.2E+05
$\Delta I_{\rm m}[\mu$Jy]	120	5.7	4.5	4.5	3.4
$D_{\rm m}$	2.7E+02	1.1E+04	1.7E+03	1.1E+04	1.1E+04
D_{uv}	2.4E+05	1.9E+06	1.8E+05	5.4E+04	1.7E+05
BW_{uv}[Hz]	1020	830	940	2050	980
τ_{uv}[ms]	50	83	63	207	66
$\Delta I_{uv}[\mu$Jy]	536.6	197.5	147.5	100.4	108.2

Parameter designation: ν – observing frequency; B – baseline; $N_{\rm an}$ – total number of array elements; $N_{\rm st}$ – total number of array stations; $\Delta u/u$ – Fourier plane filling factor of the array; $D_{\rm FOV}$ – theoretical spatial dynamic range, unconstrained by the array design; $\Delta I_{\rm m}$ – point source sensitivity (with $\Delta t = 1$ hour and BW = 1 MHz); $D_{\rm m}$ – spatial dynamic range of observations reaching $\Delta I_{\rm m}$; D_{uv} – spatial dynamic range limited by the uv–coverage of the array; BW_{uv} – maximum channel bandwidth required by D_{uv}; τ_{uv} – maximum correlator integration time required by D_{uv}; ΔI_{uv} – point source sensitivity reached with D_{uv}.

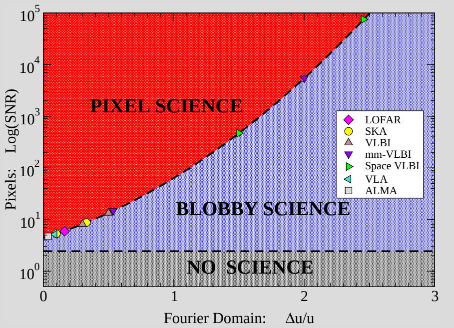

Figure 4. The effect of Fourier domain coverage, $\Delta u/u$ on image fidelity. The lowest SNR of "trustable" pixel in an interferometric image is given by $\ln(\mathrm{SNR_{low}}) \approx 16 \ln 2/\pi^2[(\Delta \mathrm{u/u}) + 1]^2$. This defines the separation between the "pixel" science (where information can be extracted effectively from every pixel) and the "blobby" science (where structures observed in an image must be represented by simplified patterns, or "blobs", such as, for instance, two-dimensional gaussians). Different symbols indicate the best performance values for various interferometers (pairs of symbols indicate ranges for the best/worst imaging performance). LOFAR and ALMA will provide a sufficient pixel fidelity down to pixel SNR of 4–5, while SKA will be able to reach this performance only at low resolutions

Conclusions

Comparison of the SDRs of the next generations of radio telescopes (LOFAR, ALMA, and SKA) to those of other future large astronomical facilities shows that sometimes uncomfortably short integration times ($\tau = 50$ ms, for LOFAR at 10 MHz) and small channel widths ($\Delta\nu \approx 1$ kHz) have to be used to match the instrument output in the radio domain to those that would be achieved in other spectral bands. This issues, however, can be resolved after the initial construction, by implementing more powerful correlators.

The limitations imposed on SDR by the Fourier domain coverage, on the other hand, can only be improved by introducing additional elements into the interferometric arrays, which is traditionally a more difficult problem. At present, the basic configurations envisaged for both LOFAR and ALMA provide sufficient ratios of $\Delta u/u$, while this problem is more severe for the SKA designs, and careful configuration planning is needed there.

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