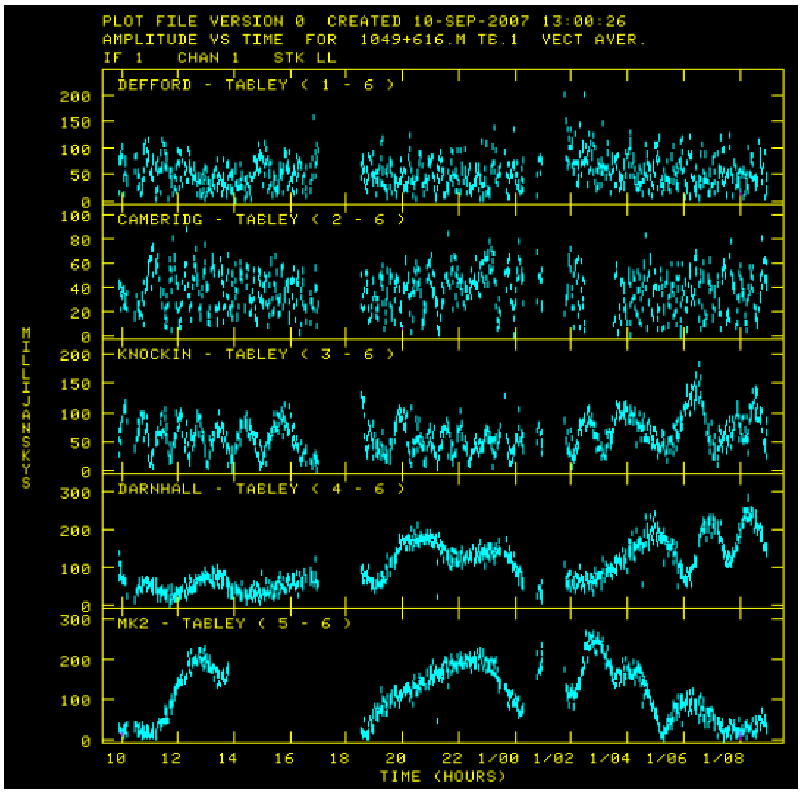
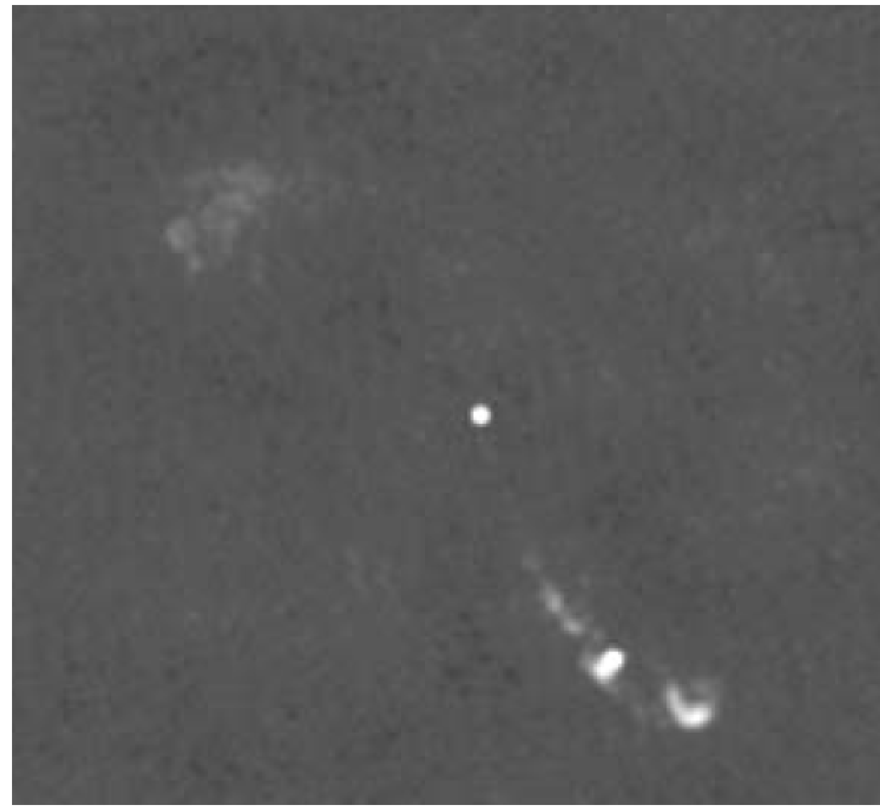
Basic Mapping

Simon Garrington JBO/Manchester





Introduction

- Output from radio arrays (VLA, VLBI, MERLIN etc) is just a table of the correlation (amp. & phase) measured on each baseline every few seconds.
- To make a good image, steps are
  - Initial calibration (few %)
  - Data editing, averaging
  - Making image (Fourier transform) ←
  - Deconvolution ←
  - Refining calibration (self cal)
  - Final image
- Often, all steps essential to make even a recognisable image

Automation

- Automated scripts (pipelines) have been developed (MERLIN, EVN, VLA, VLBI)
  - Used for archives
  - Essential for large surveys
  - In regular use for MERLIN, EVN for calibration and initial imaging
- But…
  - Can be sensitive to data errors
  - Adapted to experiment design
  - Fourier Tranform & deconvolution (mapping) are *flexible*:
    - Can be controlled depending on aims of experiment, type of image, quality of data, nature of radio source etc etc
  - Need to understand process and experiment with your data

Indirect imaging

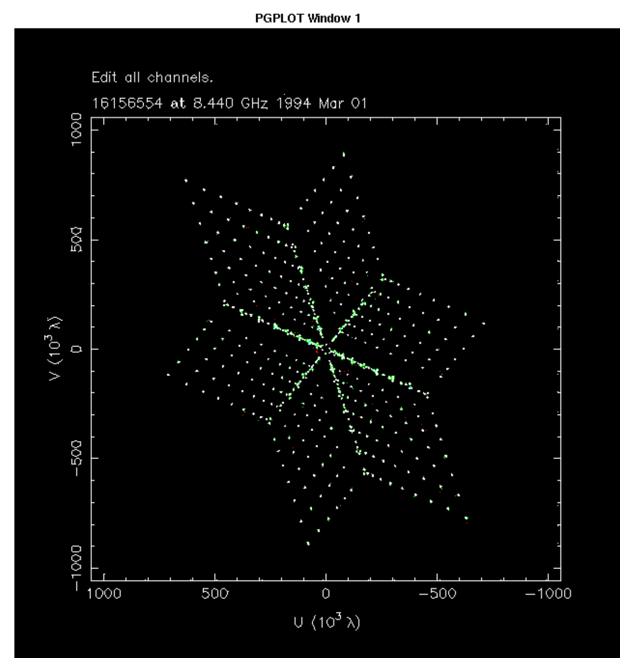
- **Array provides (poorly) sampled Fourier Transform of the radio brightness region of sky**

$$V(u,v) = \iint I(l,m)e^{2\pi i(ul+vm)}dldm$$

- u,v are the co-ordinates in the aperture plane, or visibility plane, perpendicular to the direction to the object, measured in wavelengths.
- At any instant the separation vector between each pair of telescopes can be plotted as a point in the visibility plane
- l.m are sky co-ordinates

- Assumed small region to be mapped
  2D transform
  - Region << individual antenna beam

- If V measured for all u,v to ±∞, inverse FT would yield I(l.m)
- We have set of **samples** of V(u,v). Define sampling function S(u,v) = 1 at measured (u,v), zero elsewhere



Convolution theorem

- Initial image is inverse transform of sampled visibility function

$$I'(x,y) = F^{-1}[S(u,v)V(u,v)]$$

using the convolution theorem

$$I'(x,y) = B(x,y) * I(x,y)$$

where

$$B(x,y) = F^{-1}S(u,v)$$

is the Point Spread Function

response of array to a unit point source at the origin

General description of an imaging system

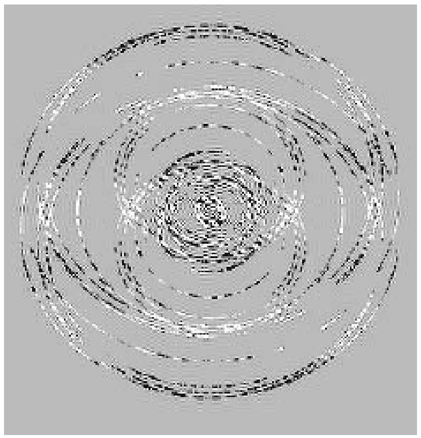
Sometimes quite benign (HST)

But a severe limitation for radio arrays

Will want to minimize its effect

Illustration

Measured visibilities V’



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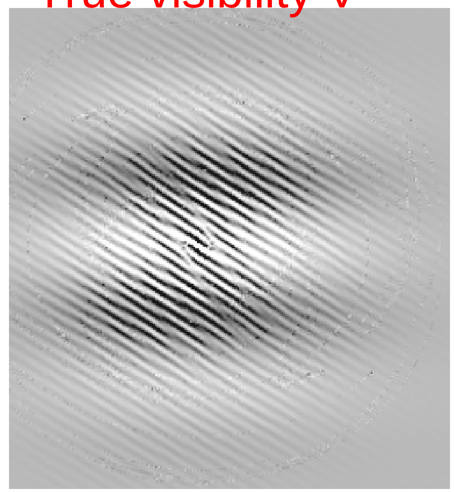
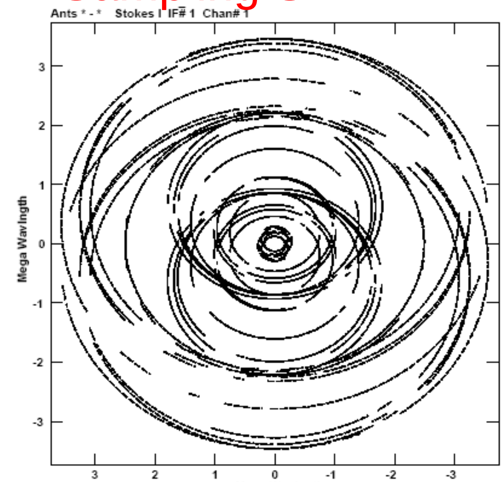
True visibility V

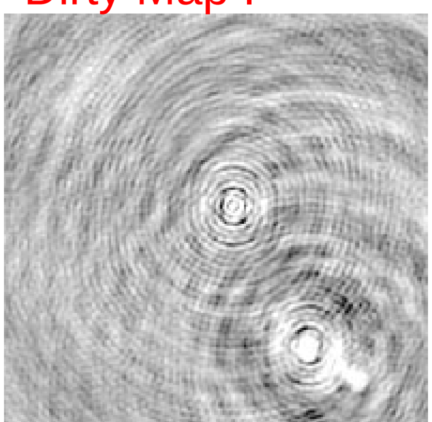


X

Sampling S



Dirty Map I’



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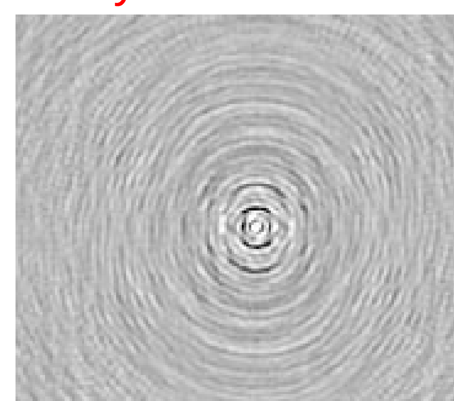
True Sky I

*

Dirty Beam B

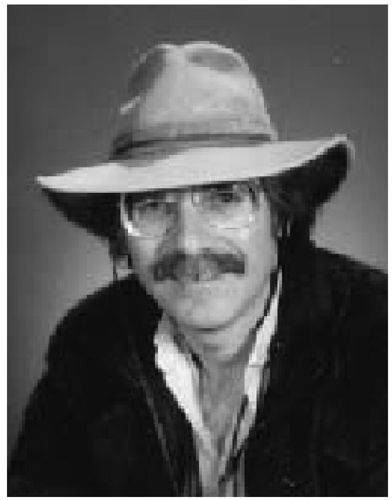


Fourier Transforms

- Information distributed across the Fourier Plane
- Sky is real, therefore uv plane is symmetric (Hermitian)
- Single point in image -> const amplitude, phase gradient in u-v plane, with slope dependent on distance from origin
  - Shift theorem
- Phase is important
- Single point in (u,v)-> sine-wave ripple in image
- Short baselines (small u,v) -> large scale smooth features
  - Smooth emission > 1/umin invisible
  - Interferometers filter out smooth emission
- Long baselines (large u,v) -> fine scale structure, sharp edges
  - Resolution is 1/umax
- Gaps in u-v plane produce sidelobes of the PSF

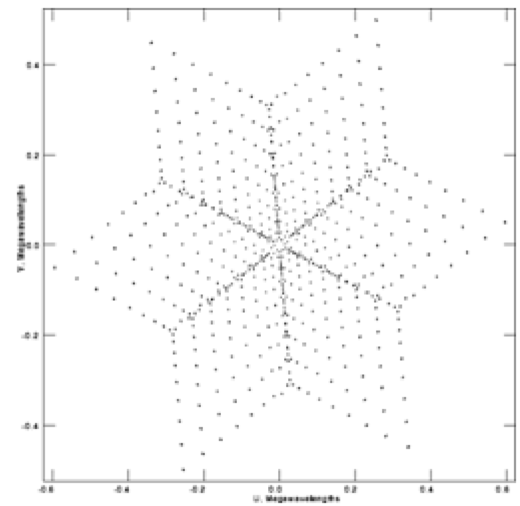
Fourier Transform Phase Party Trick

Rick *Linda*





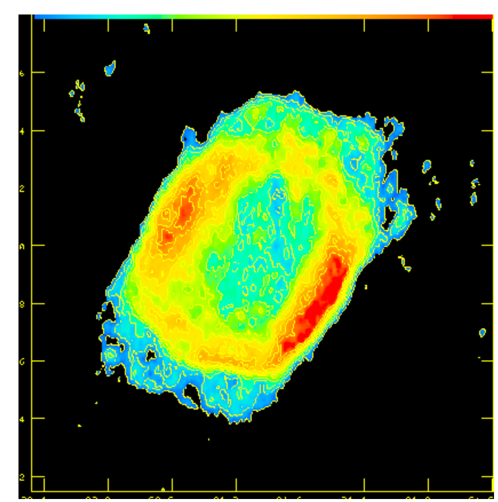
FFT and Gridding



- Fast Fourier Transform (FFT) *much* faster to compute than DFT
  - For NxN image

    DFT:few x N^4 ops

    FFT:few x N^2 logN
- Requires V’(u,v) to be interpolated on to regular grid of 2^N x 2^M points
- Automatically generates an NxM pixel image
- In practice, specify the image grid as NxM pixels with a `cellsize` approx 1/3 of the expected resolution.



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uv plane
Grid spacing: Δu
Size: u_{max}

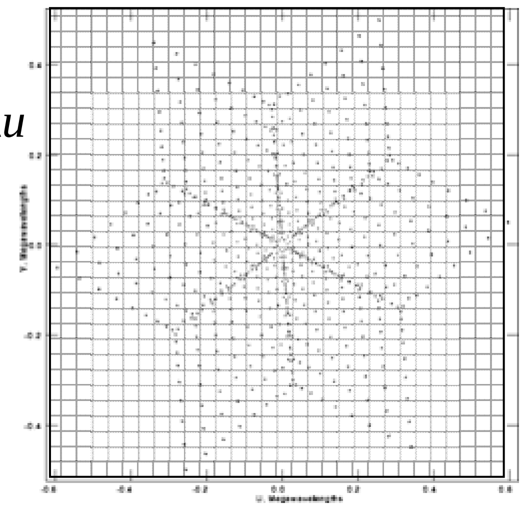
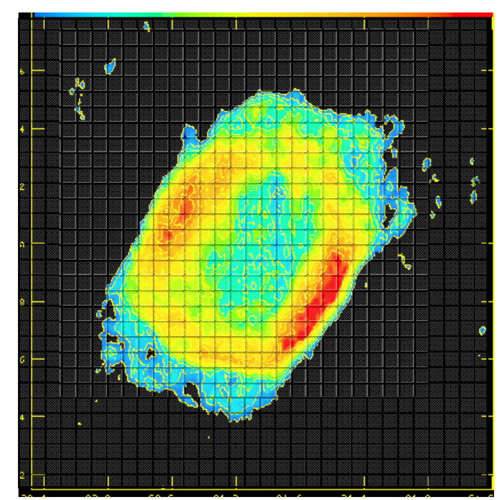


Image
Grid spacing: $1/(u_{max})$
Size: $1/(\Delta u)$



Gridding (2)

- Convolve measured points with some narrow function C (width ~ Δu), then resample on regular grid, then FFT
- In uv-plane: *convolved* with C and *multiplied* with III (series of δ-functions)

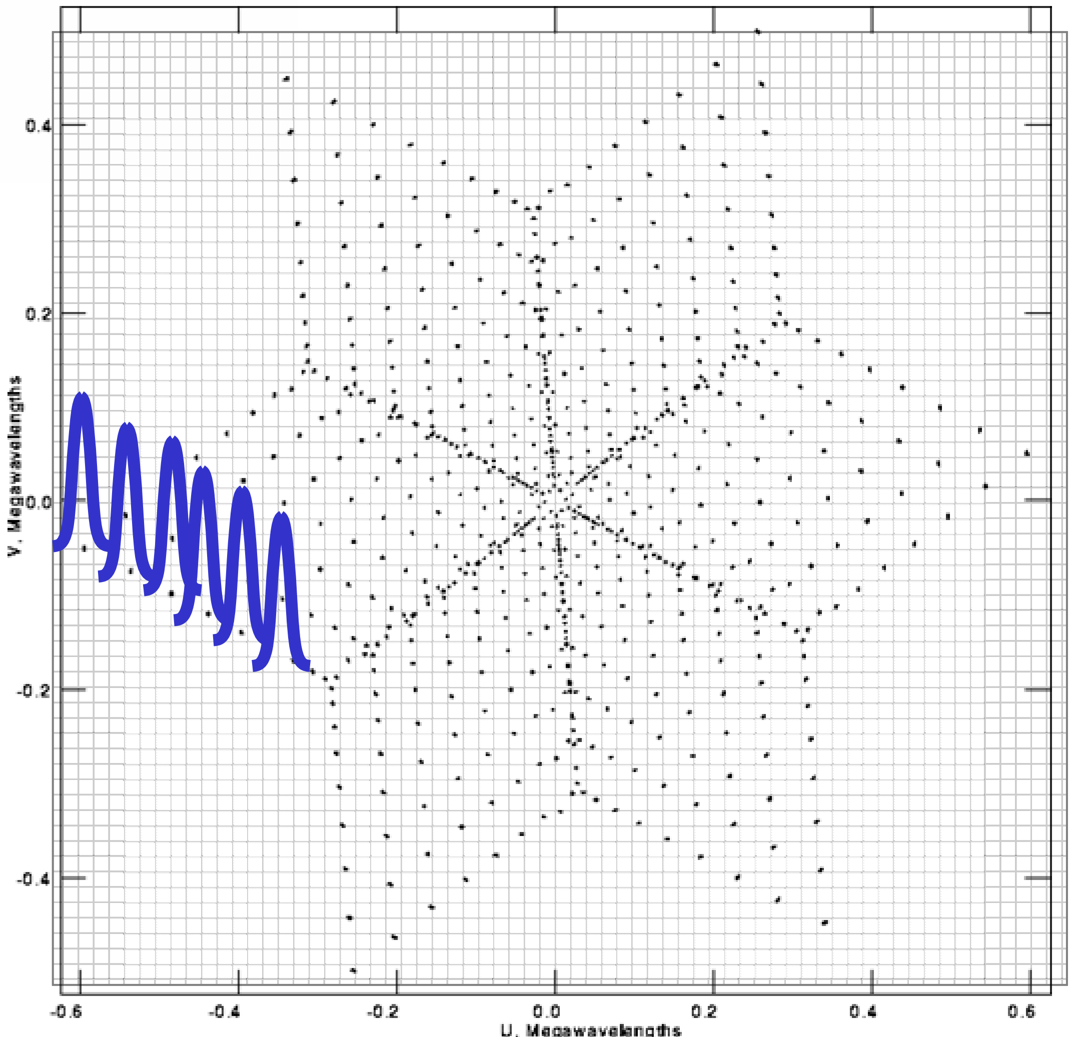
→ Image is *multiplied* with FT(C) and *convolved* with III

Multiplication → slight taper at edge of image: easily corrected by 1/[FT(C)]

Replication → aliasing: emission outside region defined by FFT of uv grid appears inside image

[Fundamentally a result of undersampling: the uv cells are too large because the image region is too small]

Gridding by convolution



Gridding and aliasing (3)

- Choice of convolution function
  - Rectangle, width Δu (cell averaging)
    - FT is sinc($\pi\Delta u l$)
  - Gaussian, width ~ Δu
    - FT is Gaussian width $1/\Delta u$
  - Ideally want rectangle in *image* plane – would remove aliasing
    - but then the convolving function would be sinc(Δu), with envelope falling as $1/(\Delta u)$ would have to evaluate at every cell.
  - Compromise: sinc x Gaussian convolving kernel
  - Optimum: spheroidal function [non-analytic, look-up table]

Gridding (4)

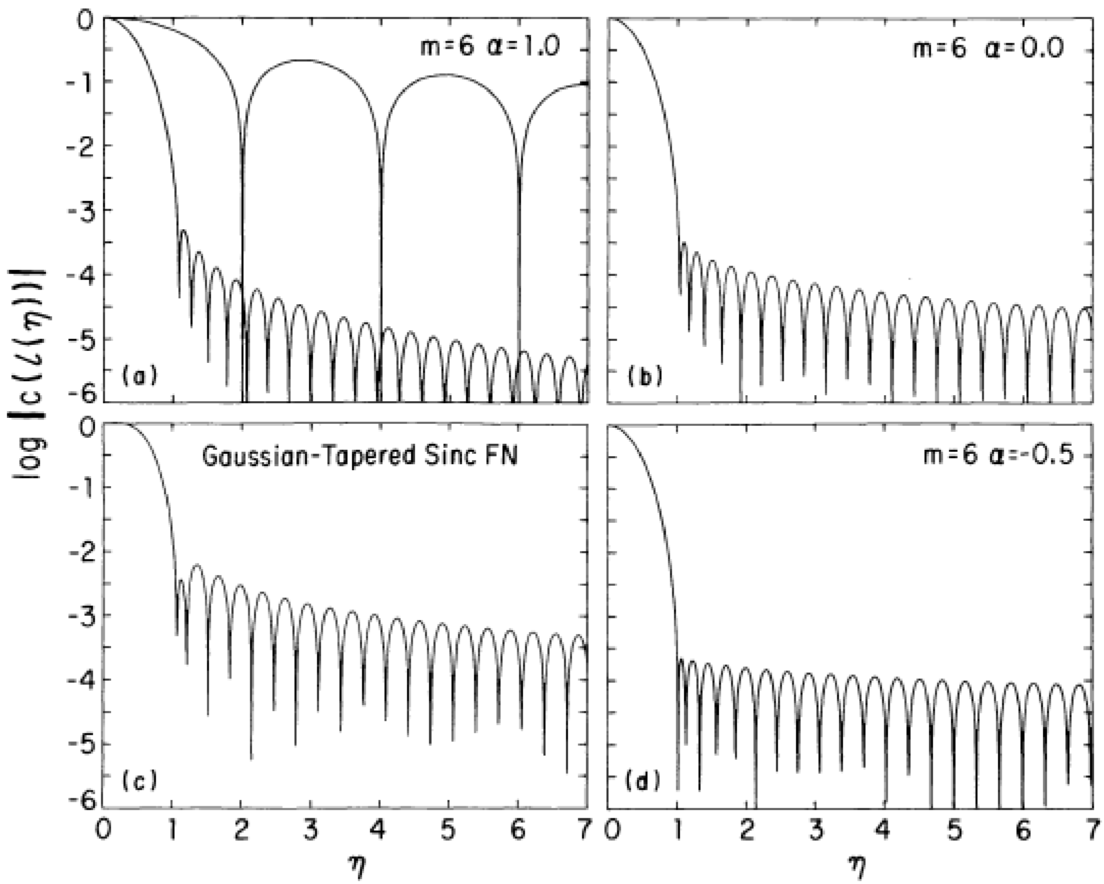


Figure 6–6. For some typical gridding convolution functions C, plots of the absolute value of the Fourier transform of C. **(a)** The spheroidal function ψ_{10}, for $m = 6$, compared with the pillbox function ($m = 1$); **(b)** the 'prolate spheroidal wave function' ψ_{00}, $m = 6$; **(c)** an optimized Gaussian-tapered sinc function, $m = 6$; **(d)** the spheroidal function $\psi_{-\frac{1}{2},0}$, $m = 6$. Adapted from Schwab (1984a).

Dirty map & dirty beam

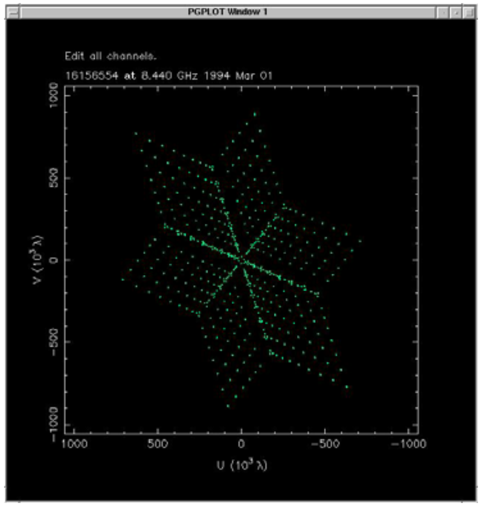
- Obtain initial image (dirty image) by gridded FFT of visibility data

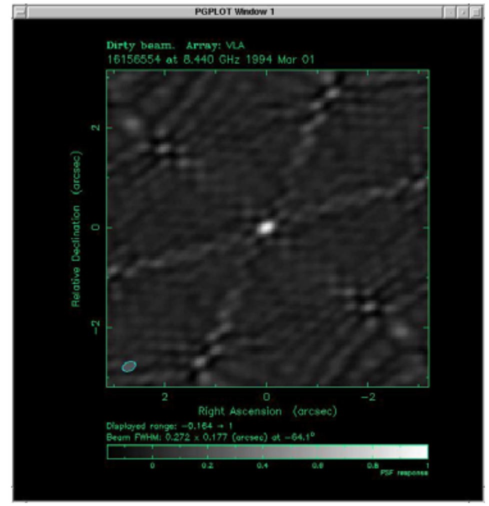
$$I'(x,y) = B(x,y) * I(x,y)$$

- Dirty image = True image * Dirty Beam (PSF)
- Properties of Dirty Beam
  - Response to a unit point source
  - FT of sampling in uv plane
  - Central maximum has width

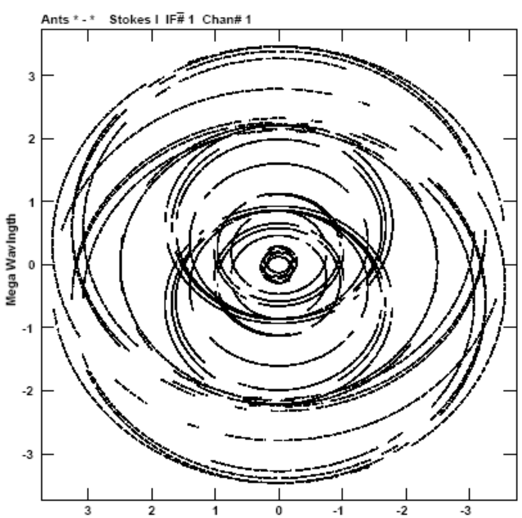
    $1/(u_{max})$ in x and $1/(v_{max})$ in y
  - Has ripples (sidelobes)
    - Rms ~ 1/N (antennas)
    - Close-in sidelobes: determined by envelope of uv points
    - Far-out sidelobes due to gaps in uv coverage

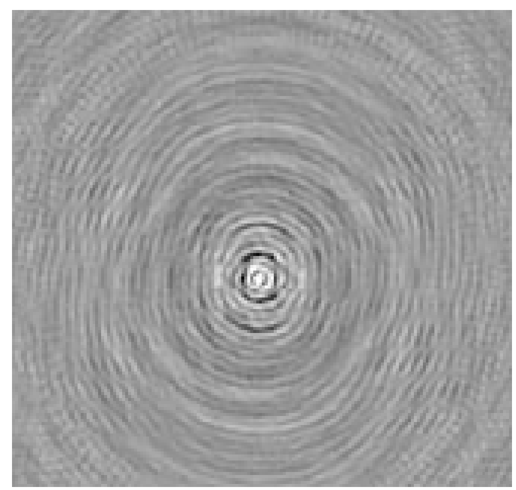
uv coverage and dirty beam





VLA
snapshot





MERLIN
track

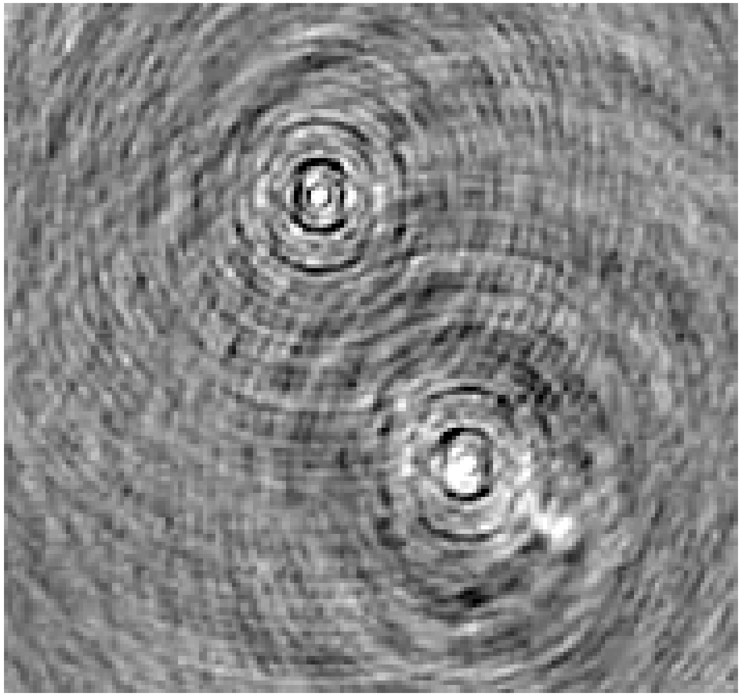
Recovering true image Deconvolution

- I′ = I*B ←→ V′=V.S
- Cannot use linear deconvolution (eg Wiener filter), because S(u,v) is zero in unsampled parts of uv plane
- Need to ‘guess’ FT of true image in these regions
- Many different images whose FTs consistent with measured points but behave differently in the gaps
- The Dirty map is just the one which is zero at all these points
- How to select the ‘right’ or ‘best’ one
- Non-linear deconvolution methods try to do this

  as a result they interpolate into the unsampled parts of the uv plane

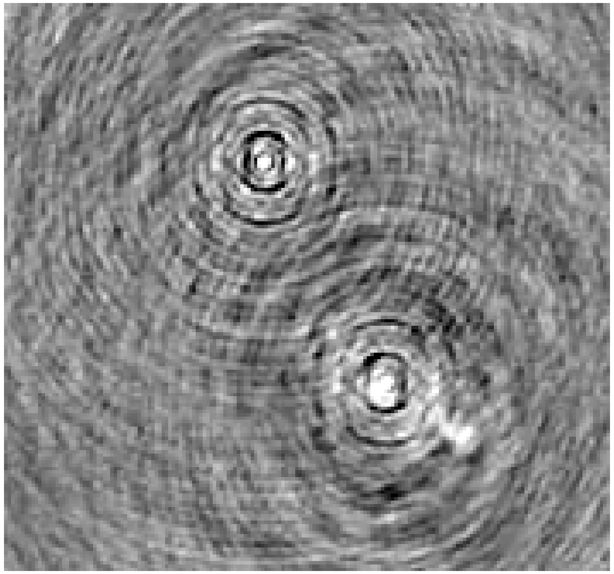
Extra information

- Choose 'best image' using *a priori* information.
  - Sky is positive
  - Sky is often mostly empty with a few localised sources
  - Individual regions of the sky may have smooth distribution of emission
- Best fit problem subject to constraints



CLEAN

- Natural response to the problem, when faced with typical early images of compact radio sources … subtract off Dirty Beam
- Procedure
  - Produce **Dirty Image, Dirty Beam**
  - Locate peak in dirty image
  - Record position and intensity
    CLEAN COMPONENTS
  - Subtract scaled & shifted dirty beam
    RESIDUAL IMAGE
  - Locate next peak …
  - Continue until residual = noise
  - Convolve clean components with clean beam (Gaussian fit to central dirty beam)
  - Add to residual map
    CLEAN IMAGE



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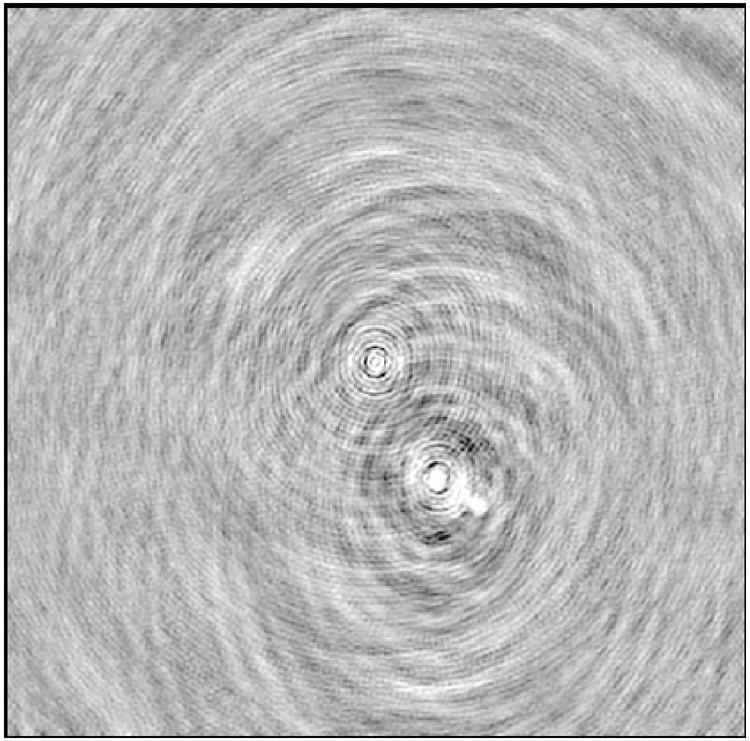
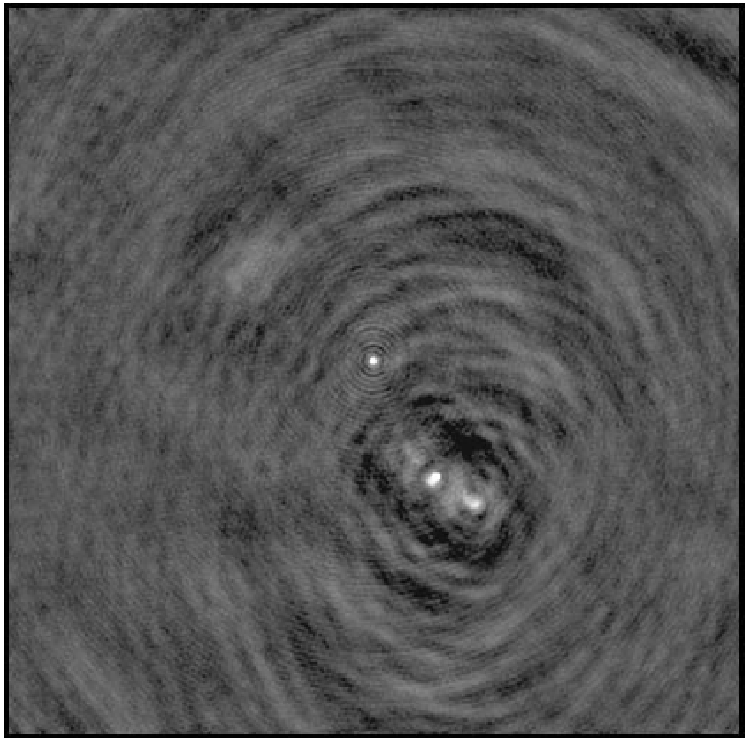
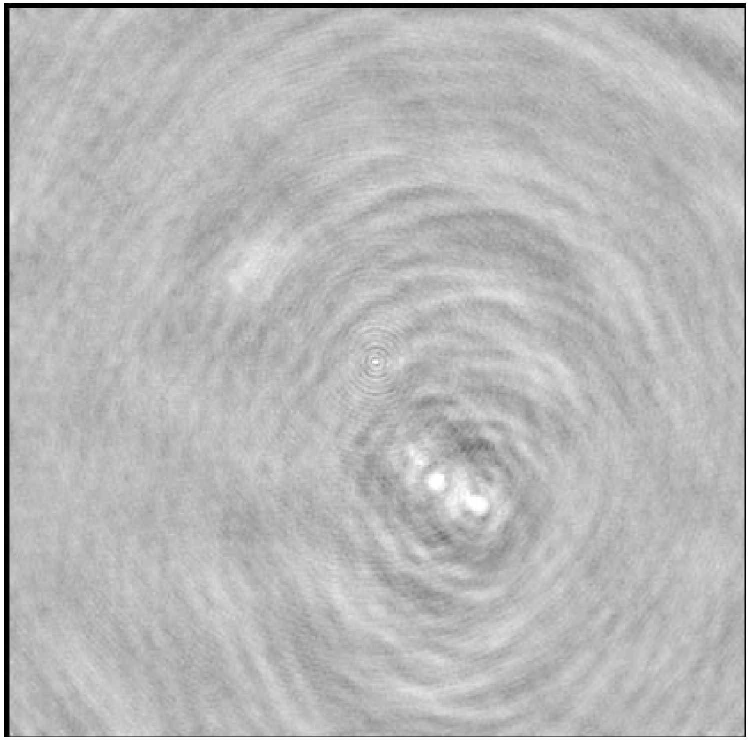
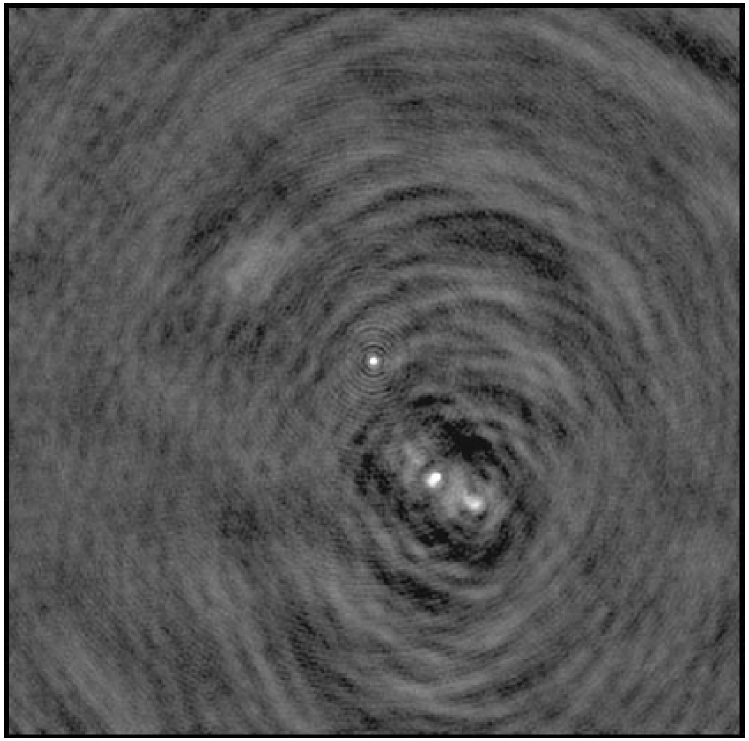
CLEAN demo



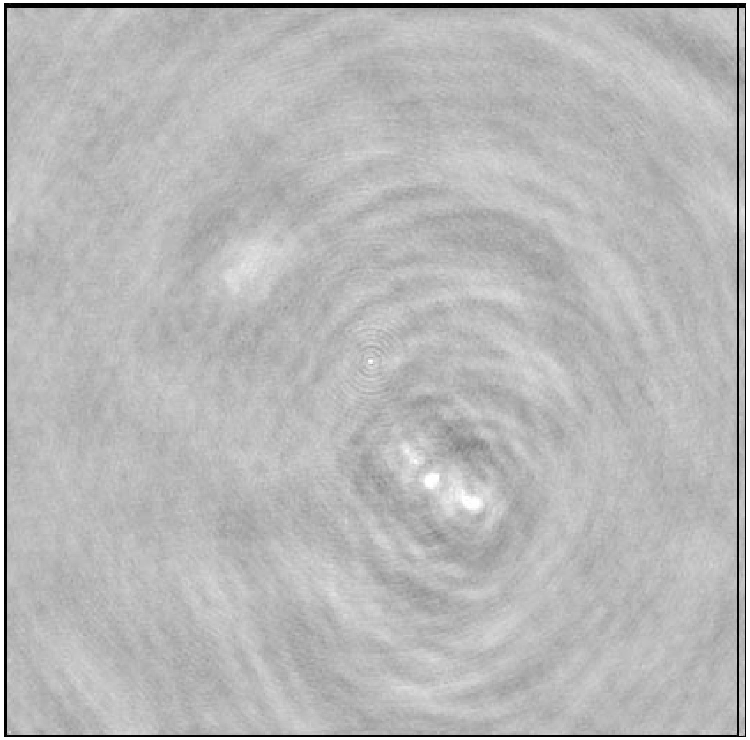


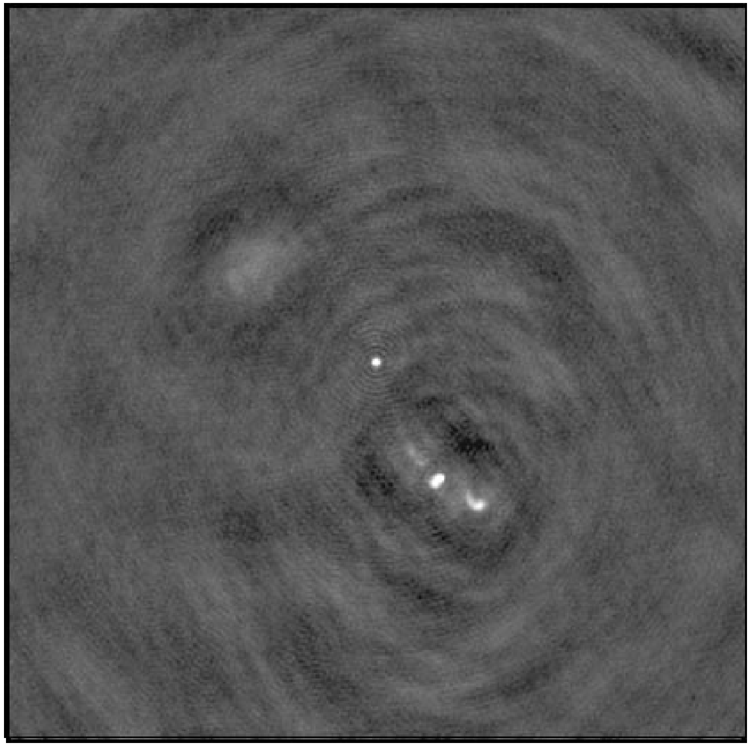
CLEAN demo



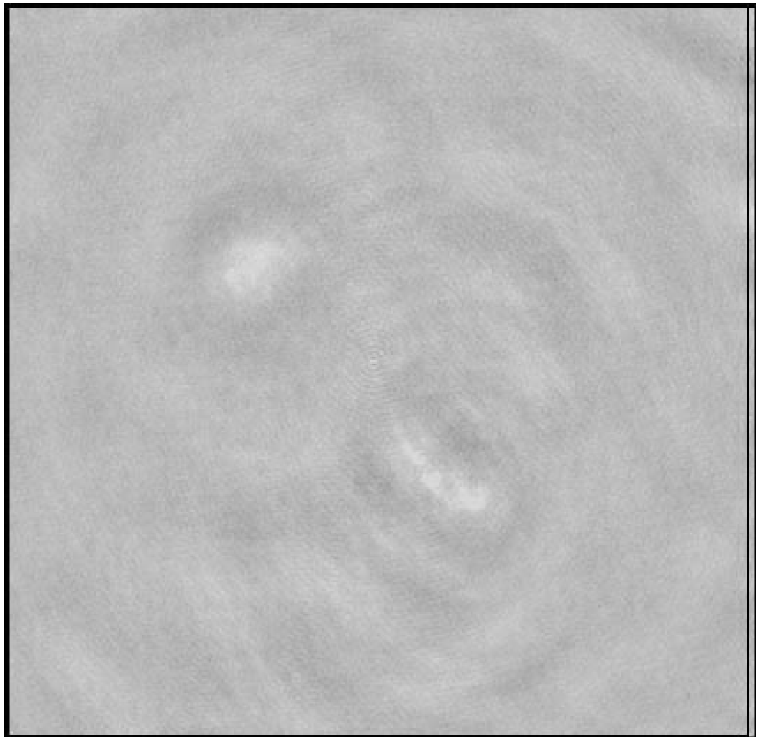


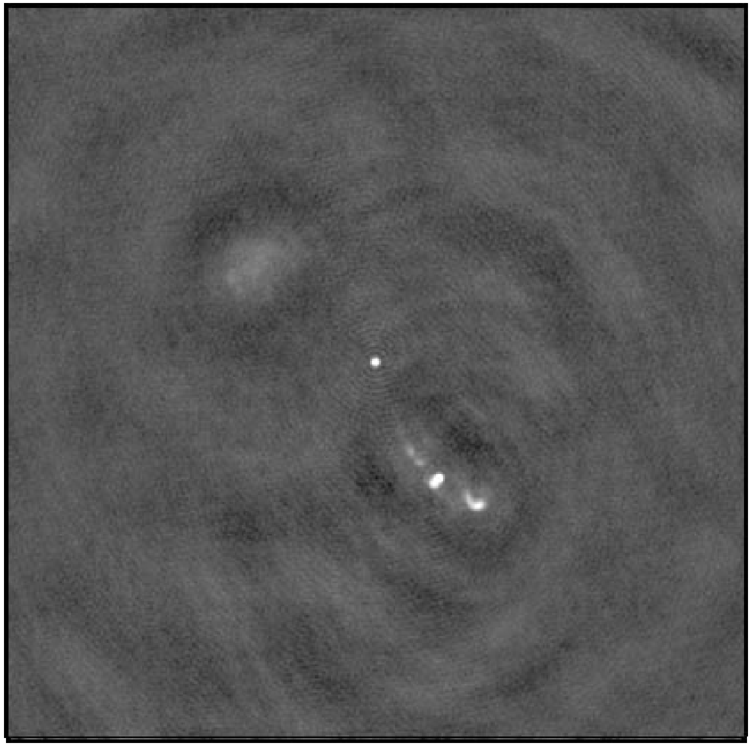
CLEAN demo





CLEAN demo





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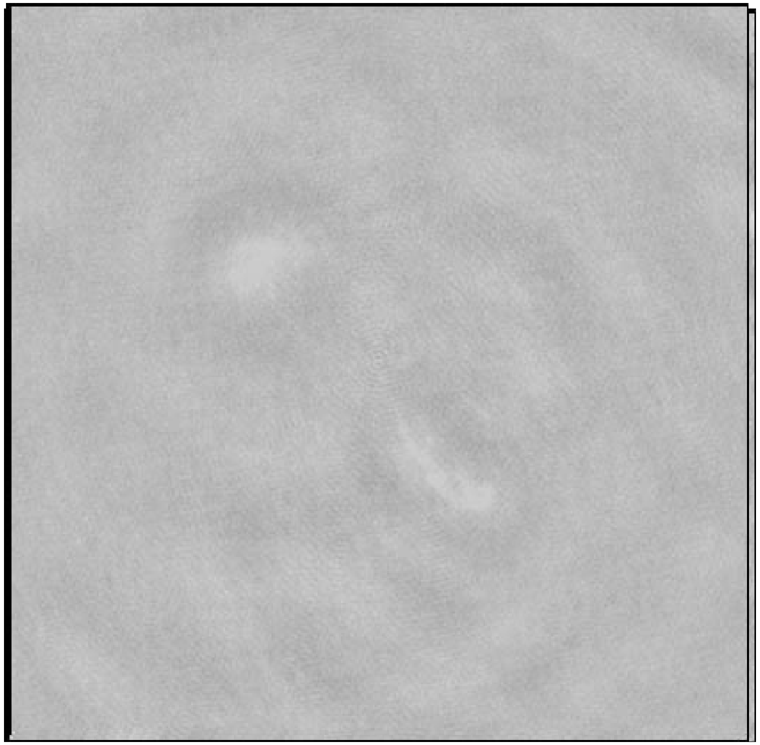
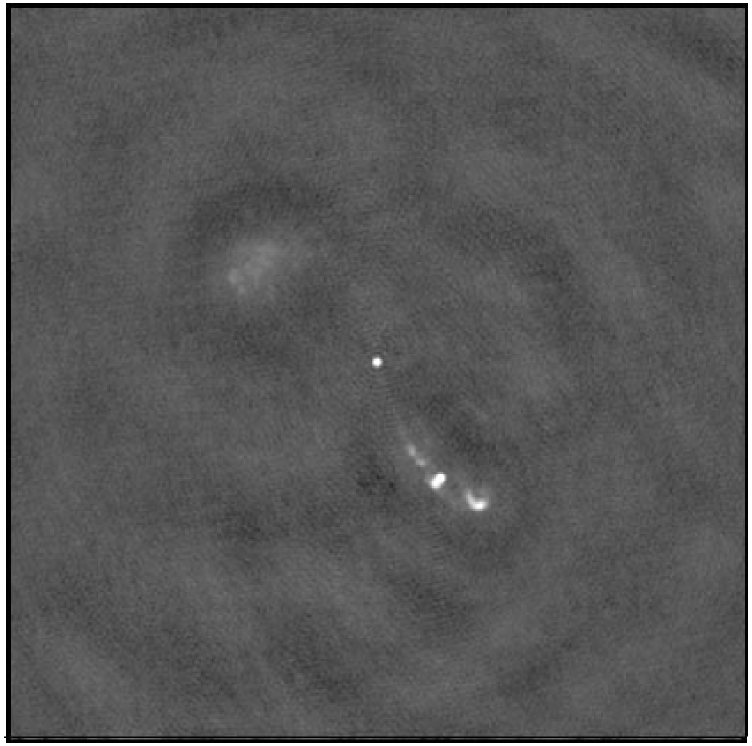
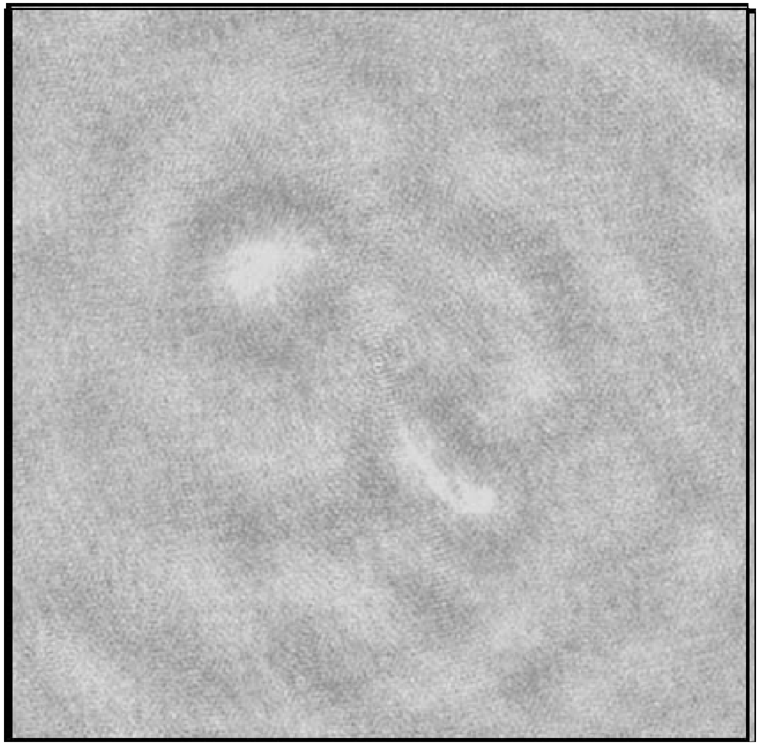
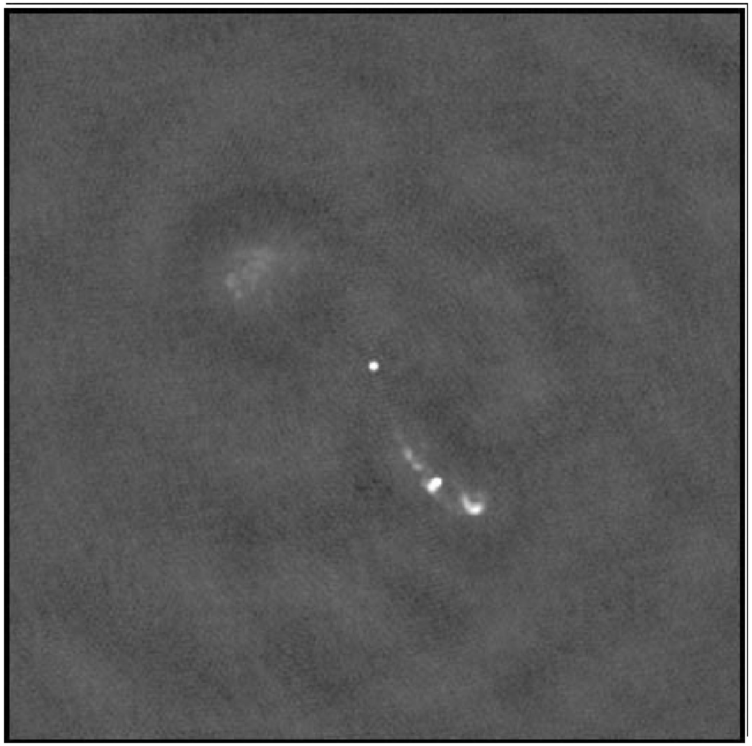




CLEAN demo





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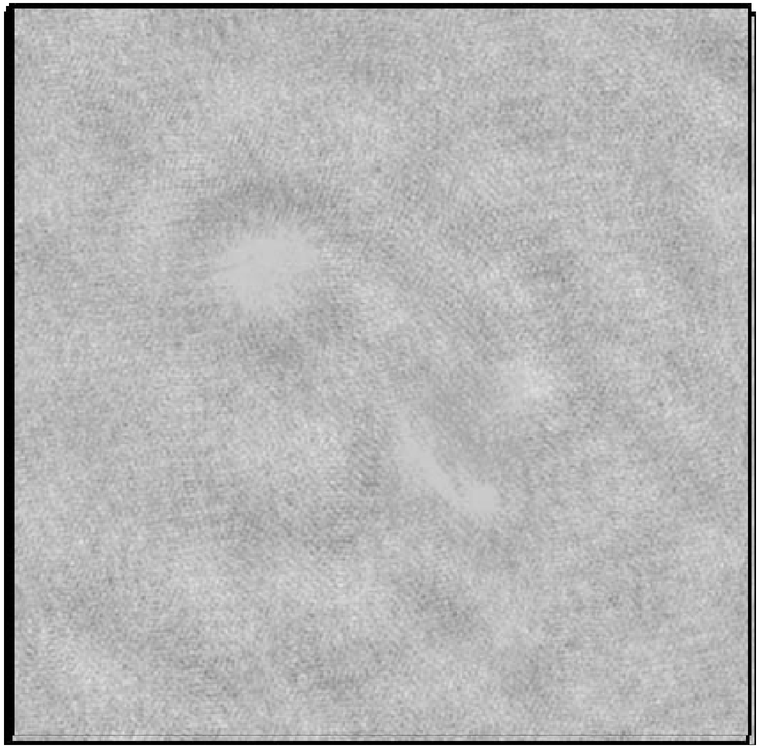

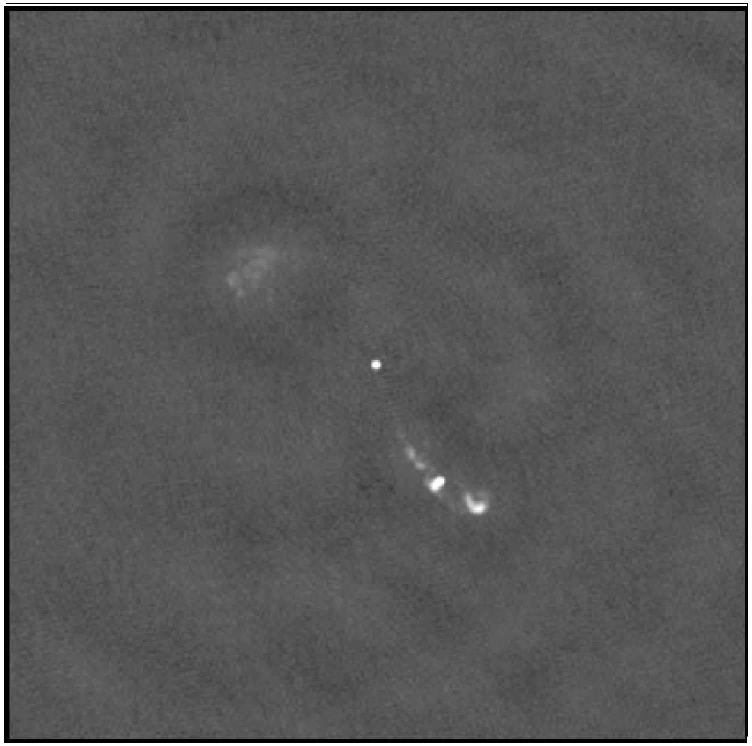

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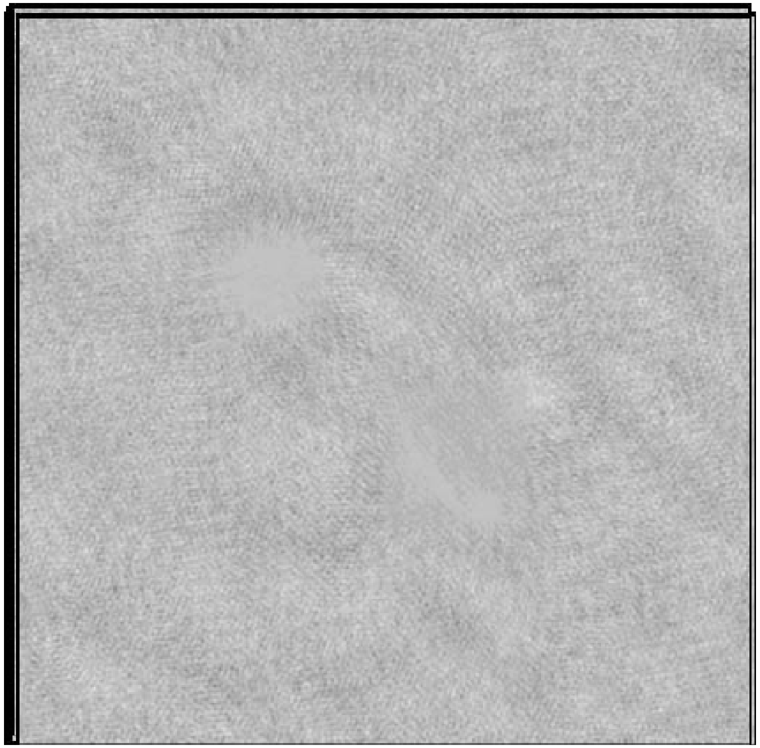
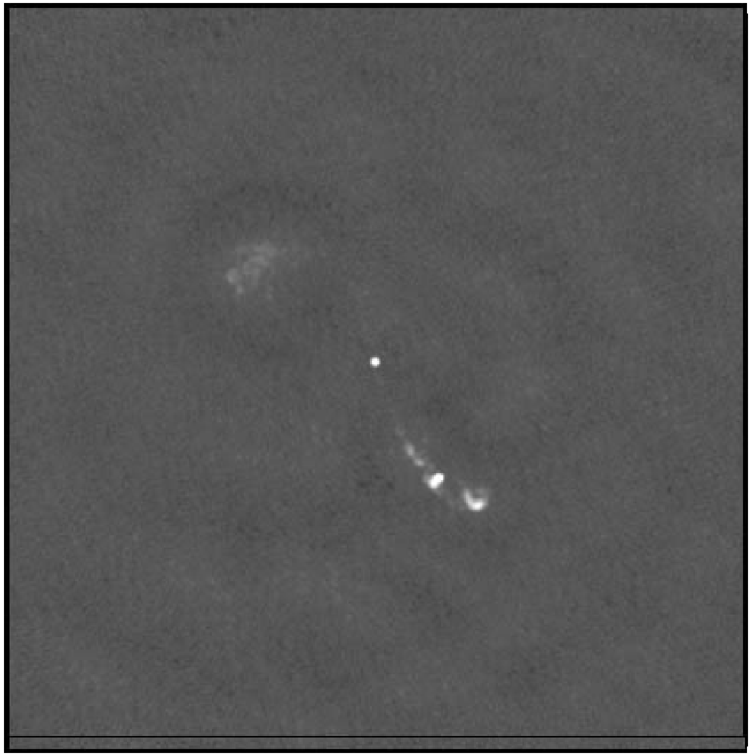
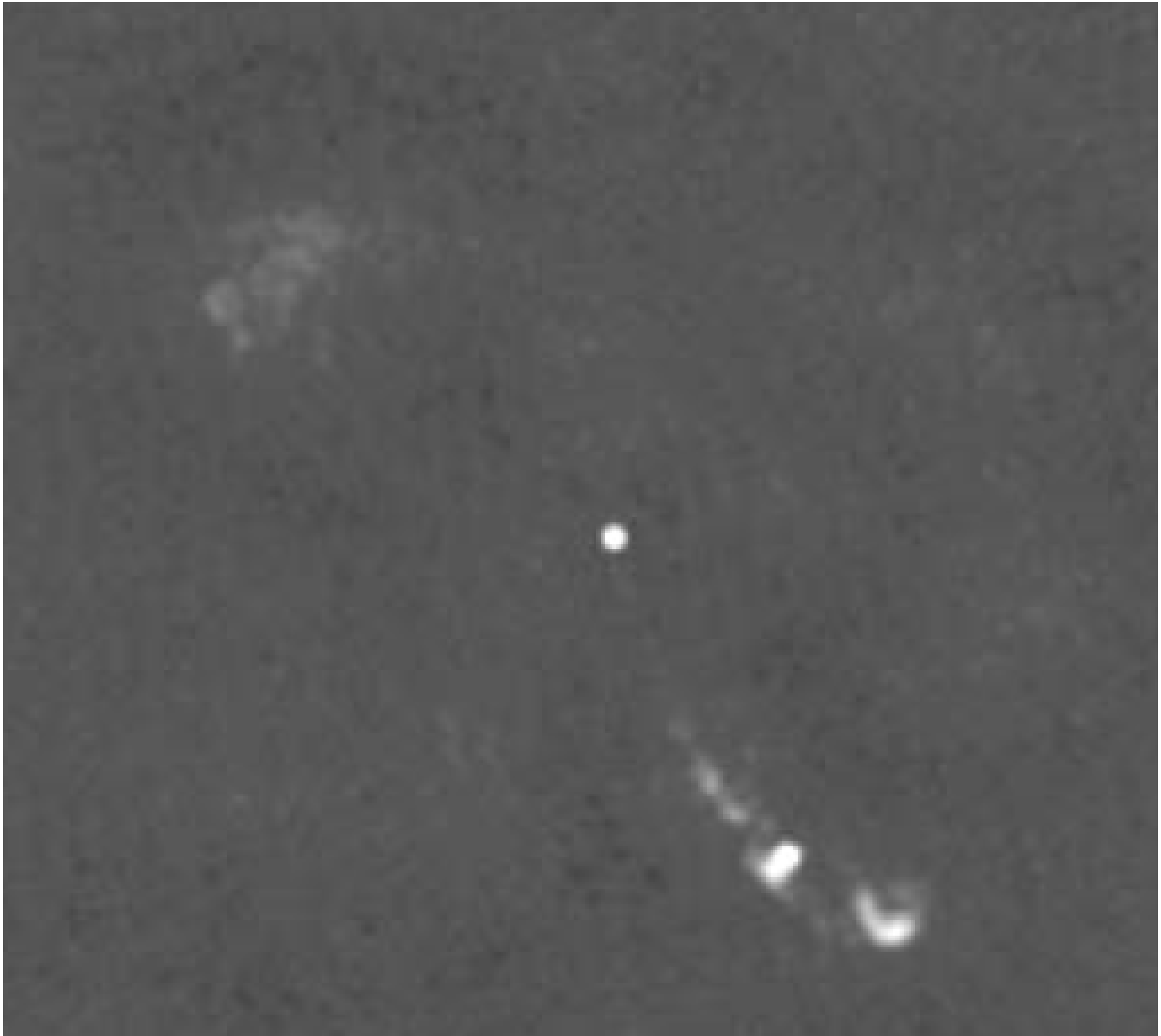
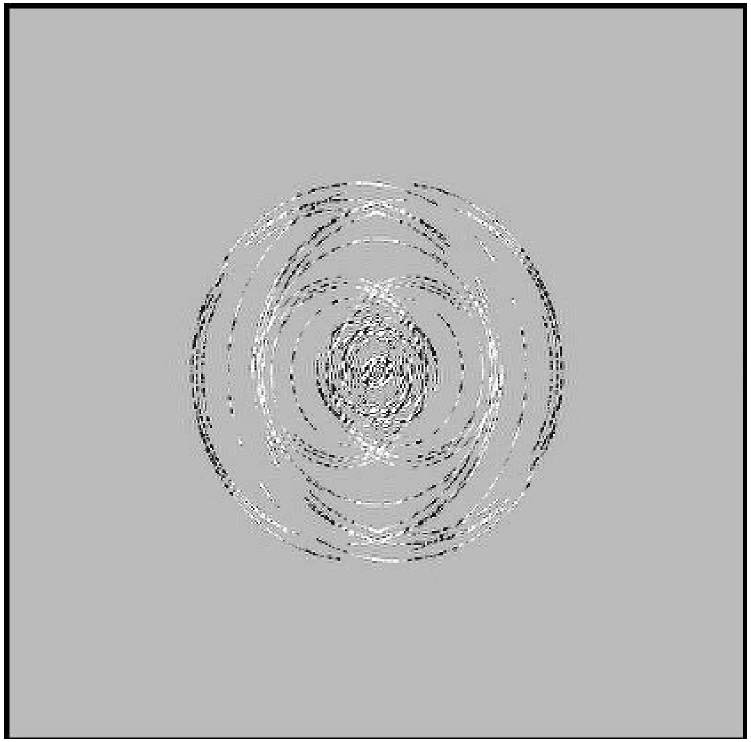






CLEAN demo in uv plane



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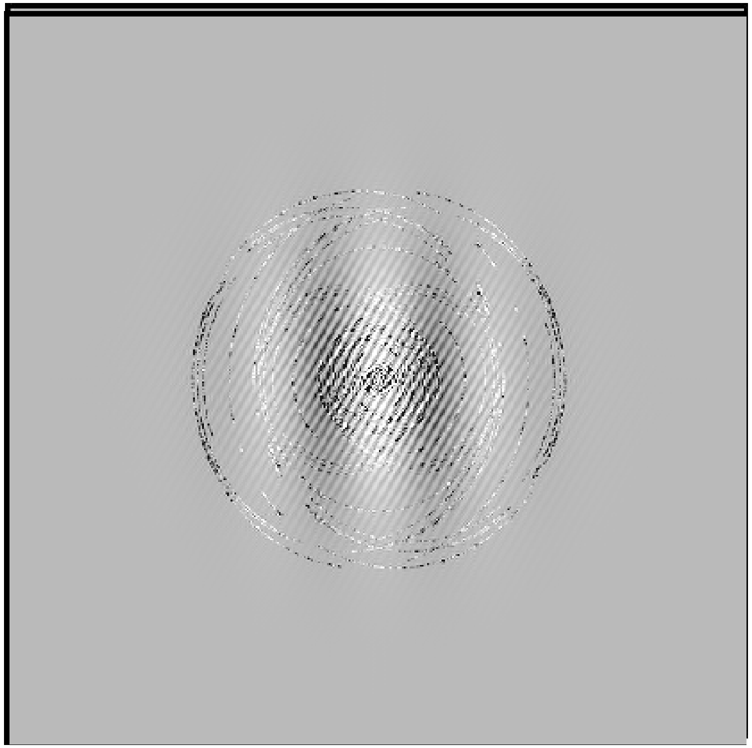

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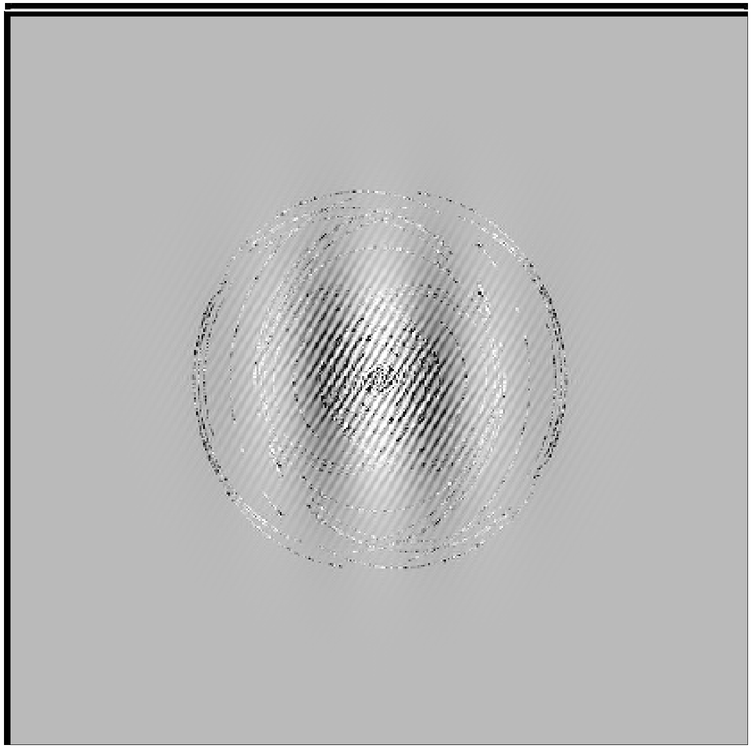

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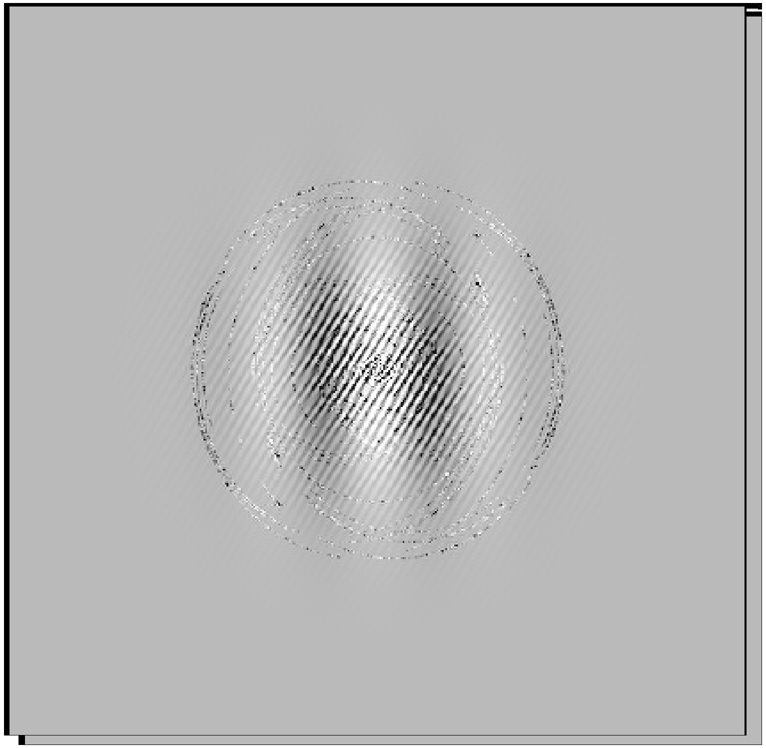

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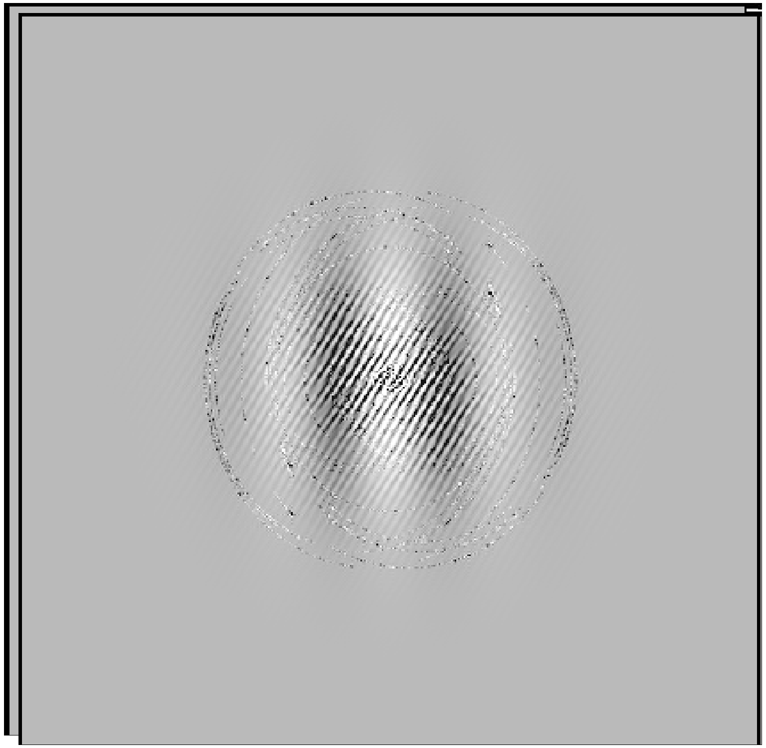

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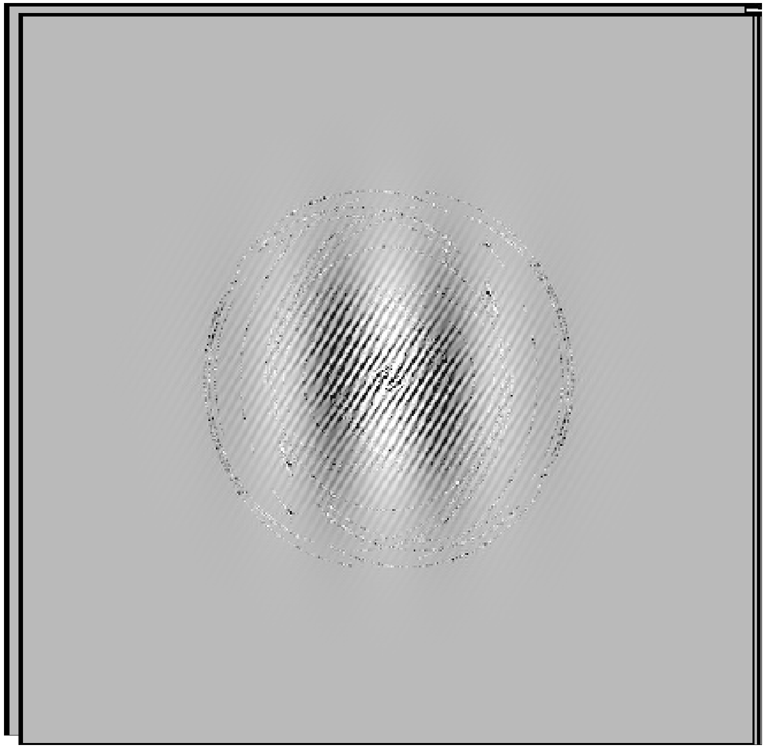

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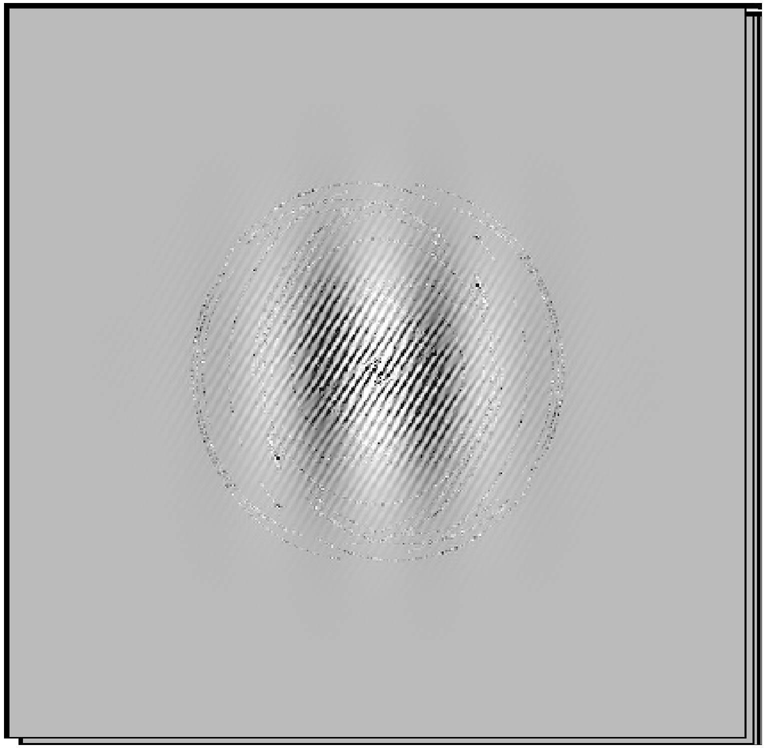

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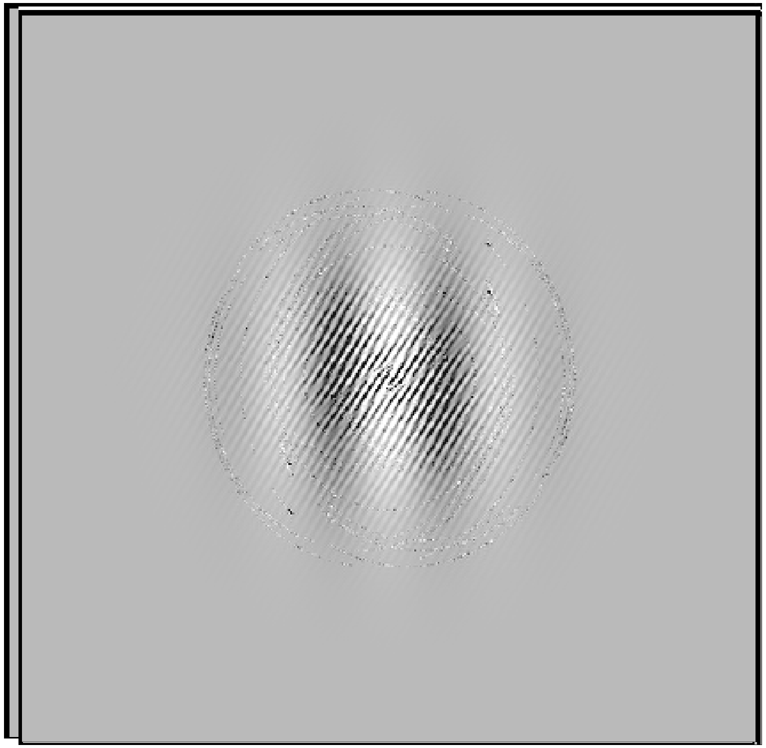

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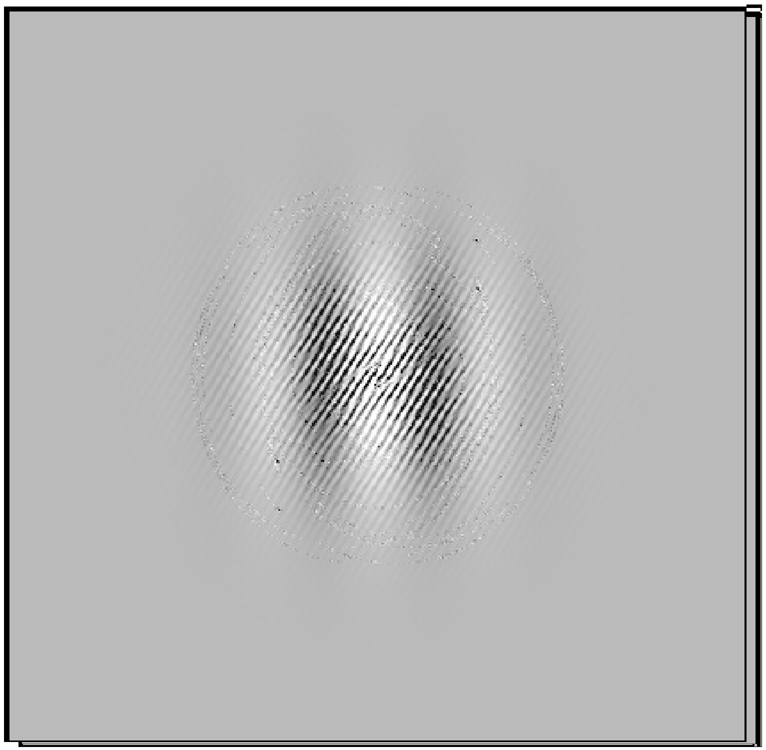
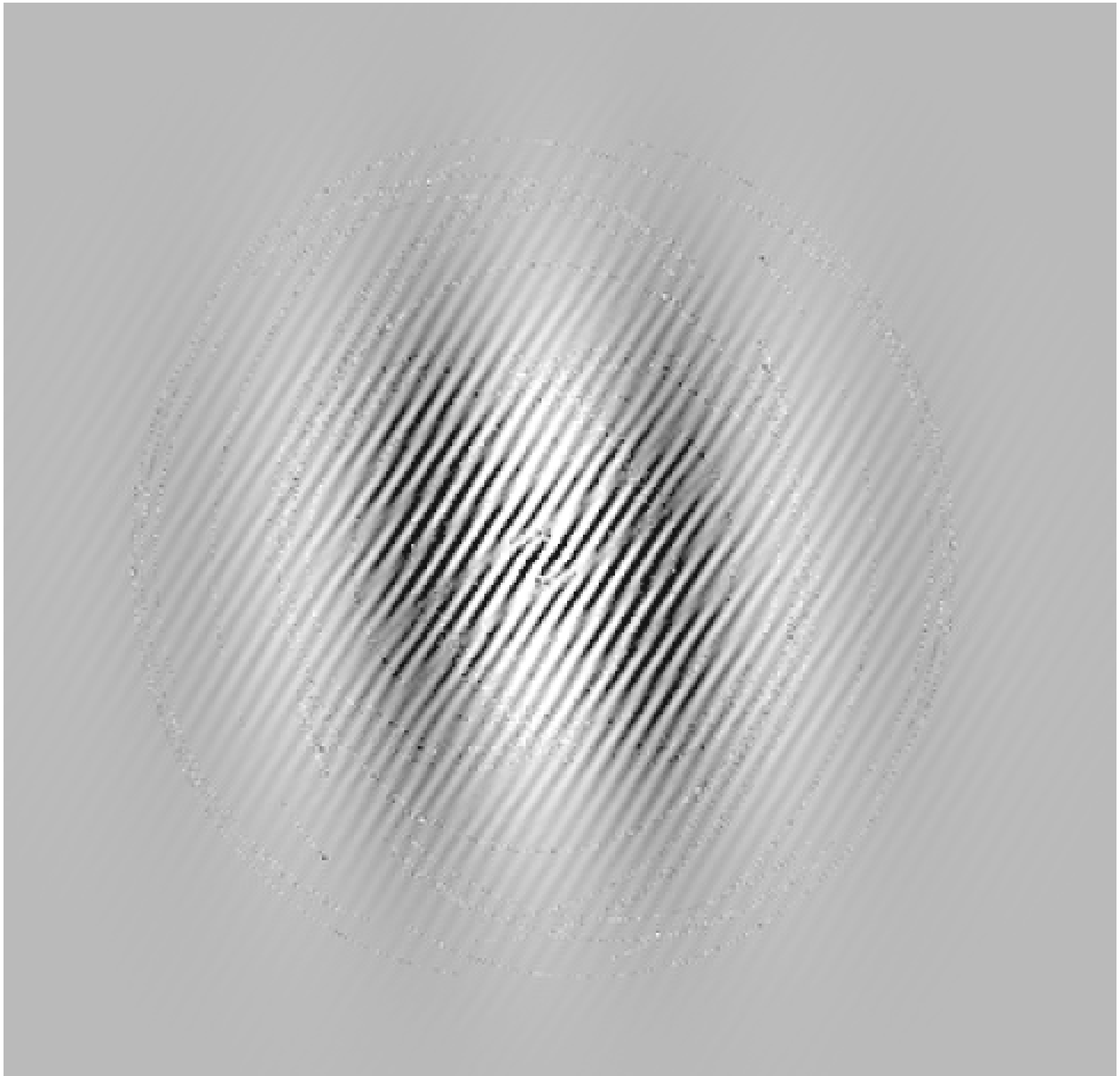




Using CLEAN

- Using windows
  - Restrict areas where clean components can be found
  - Simple way to add stronger a priori information
  - Significant impact for extended sources where uv coverage is poor
  - User bias…
- Choice of loop gain
  - Usually 0.05 -0.1
- When to stop
  - CLEAN will happily deconvolve the noise
  - Noise-only map has features (FT only non-zero on S(u,v))
  - Generally reduces apparent noise
  - CLEAN bias for VLA snapshots (FIRST survey)
- Adding zero spacing
  - Should help for extended sources; rarely used
  - NB total flux in DM = V(0,0) = 0

Variants of CLEAN

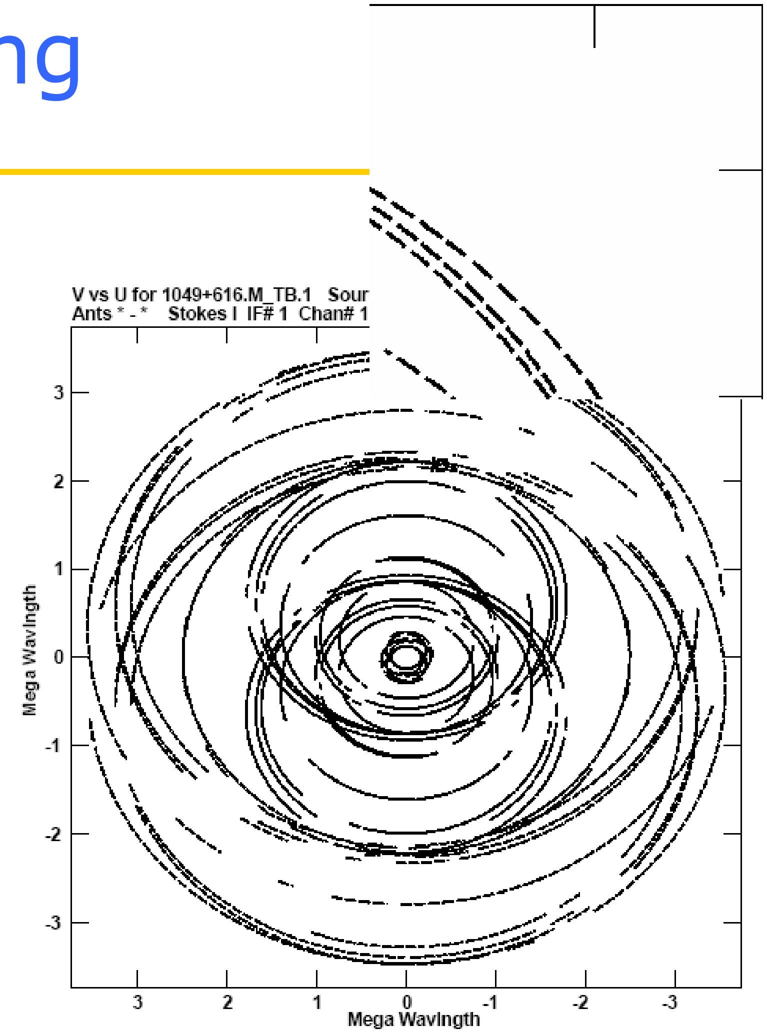
- Classic Clean due to Högbom (1974)
- Clark Clean
  - In image plane, use a restricted 'beam patch' for subtracting a number of clean components
  - Then do full subtraction of this set in visibility plane (FFT)
  - Back to image plane and locate next set
  - ~10x faster
- Cotton-Schwab
  - As above but so subtraction from un-gridded data (DFT). More accurate, can work on multiple fields at once

CLEAN problems

- Well suited to isolated, compact sources
  - Sidelobe pattern easily recognised, sources well represented by modest number of delta functions
- Might fail for very extended, smooth emission
  - Well-known ‘stripe’ instability
  - Subtraction of sidelobe pattern from smooth region generates ripples, reinforced by further subtraction

Weighting

- After gridding not all cells equal
  - Some receive many more points than others
    - For earth-rotation synthesis, rate of traversing cells scales with baseline length.
    - Long baselines can ‘clip’ cells
  - Some baselines (telescopes) may be more sensitive (VLBI, MERLIN)



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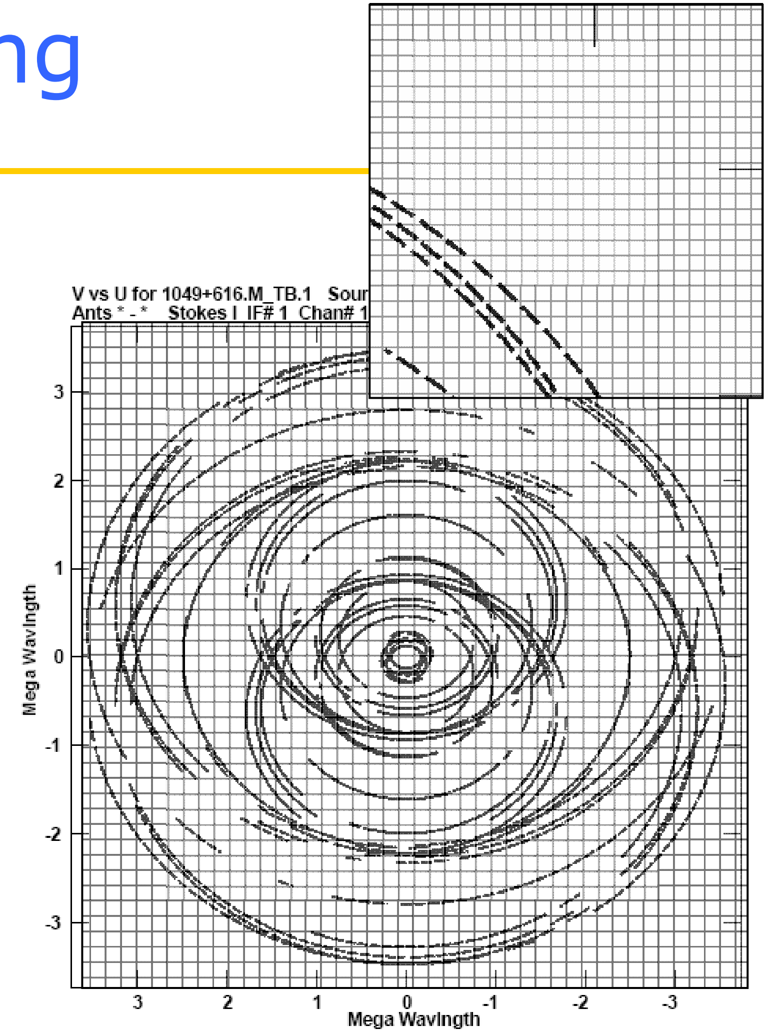
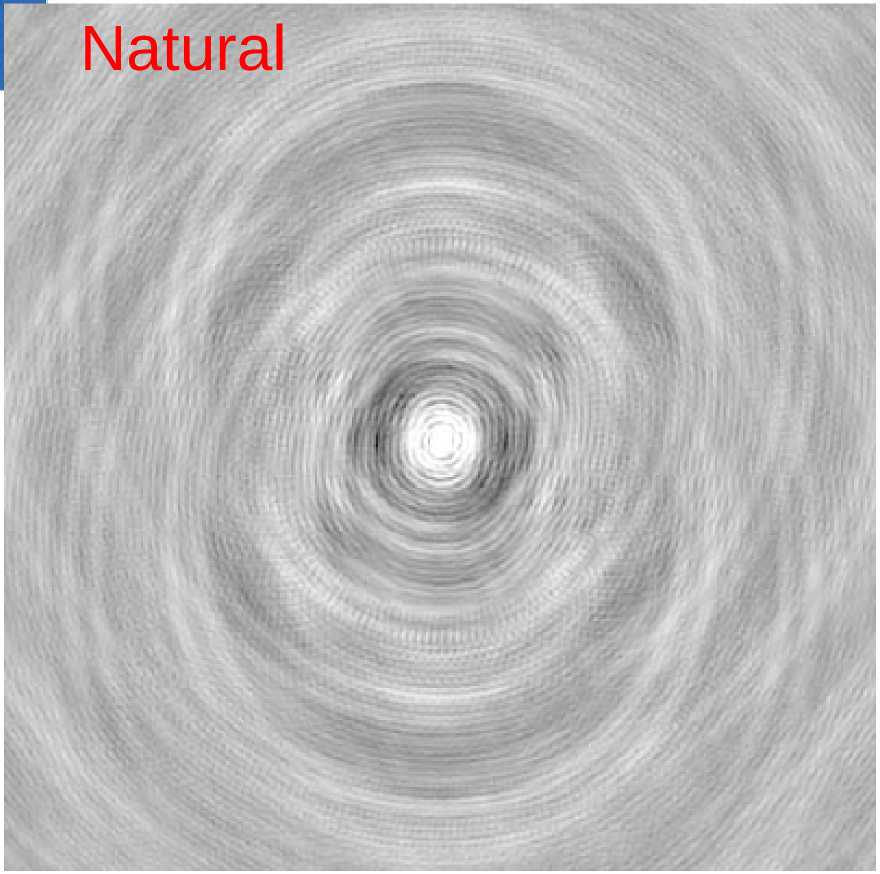
    - Long baselines can ‘clip’ cells
  - Some baselines (telescopes) may be more sensitive (VLBI, MERLIN)

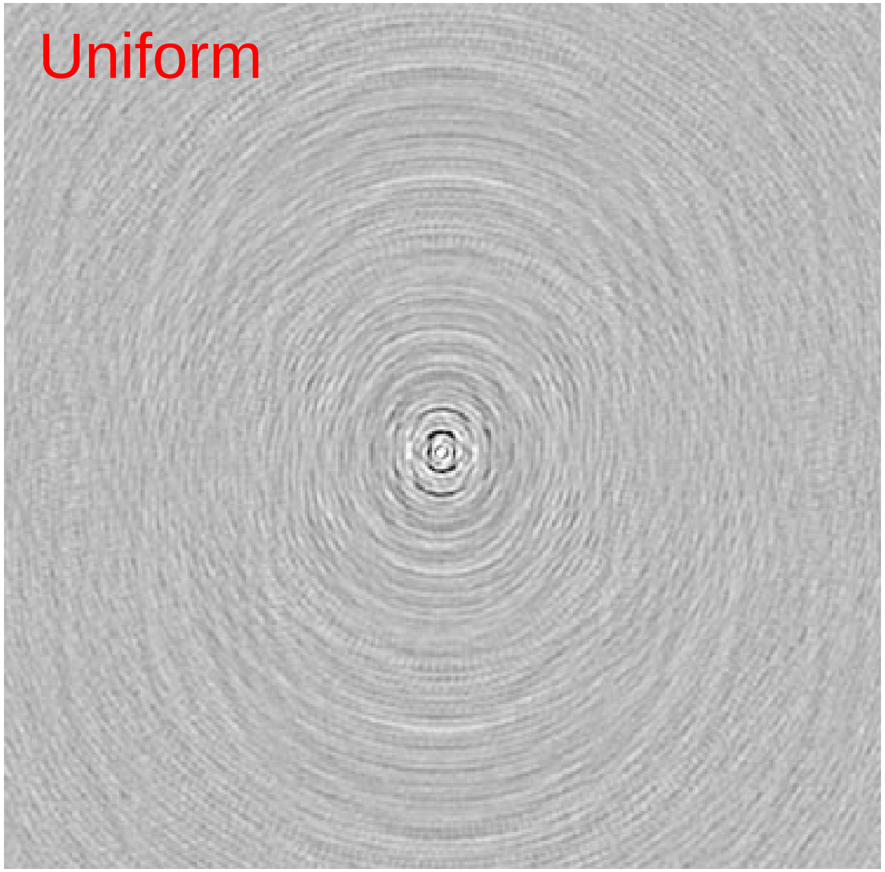


Weighting (2)

- Can optimise sensitivity
  - $W_i=1/(\sigma_i^2)$ … taking into account points/cell and individual point weights
  - 'Natural weighting'
    - highest weight on shorter baselines, so reduces resolution (makes dirty beam broader)
    - Discontinuity in weights – higher sidelobe levels
- Can minimize sidelobes
  - $W_i=1/(\rho(u,v))$
    - Optimum resolution and sidelobe level
    - Reduced sensitivity
    - 'Uniform' or inverse density weighting

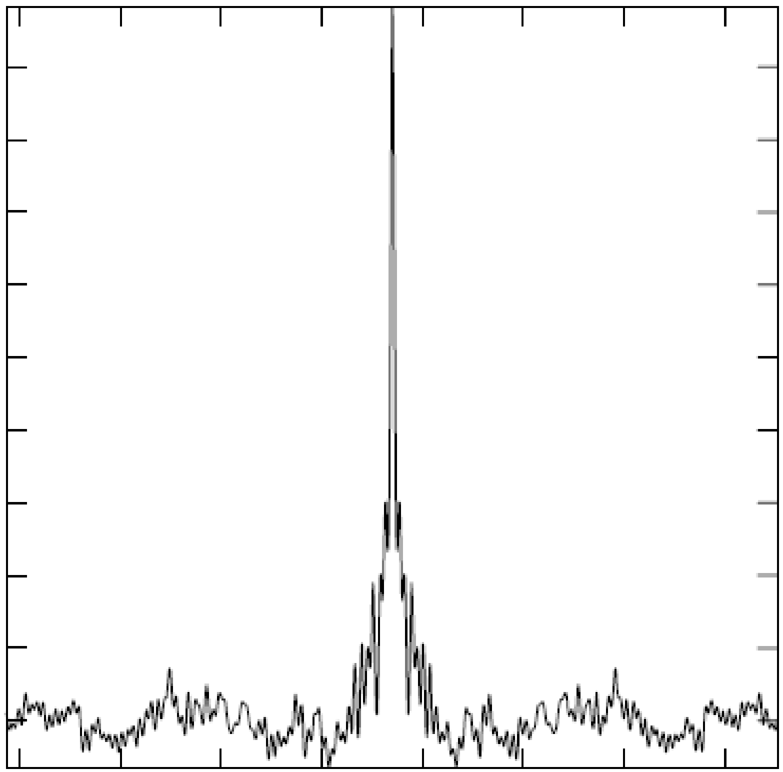
Weighting affects dirty beam



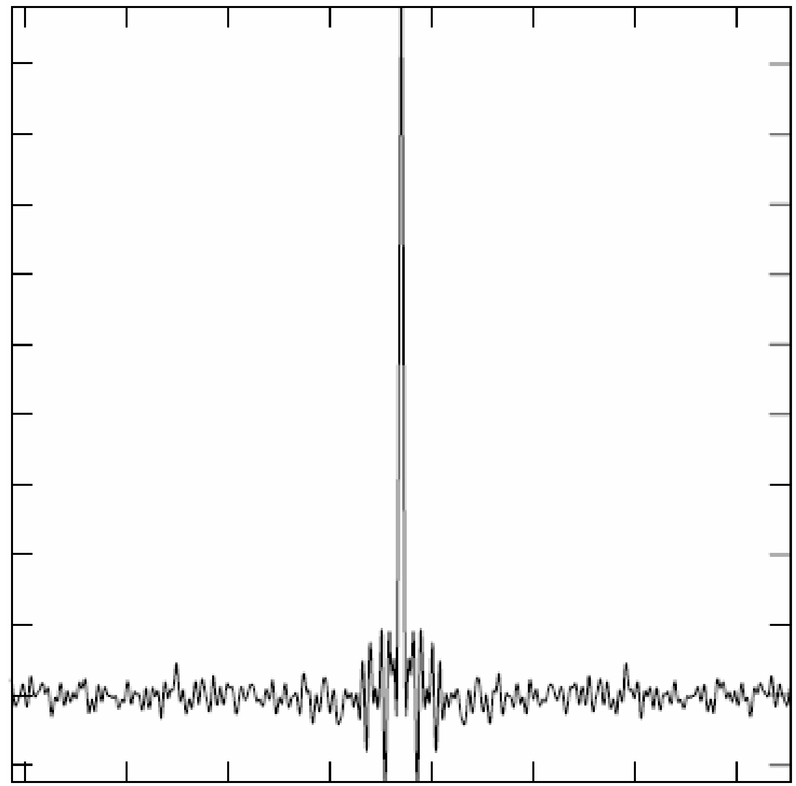


Weighting affects dirty beam

Natural



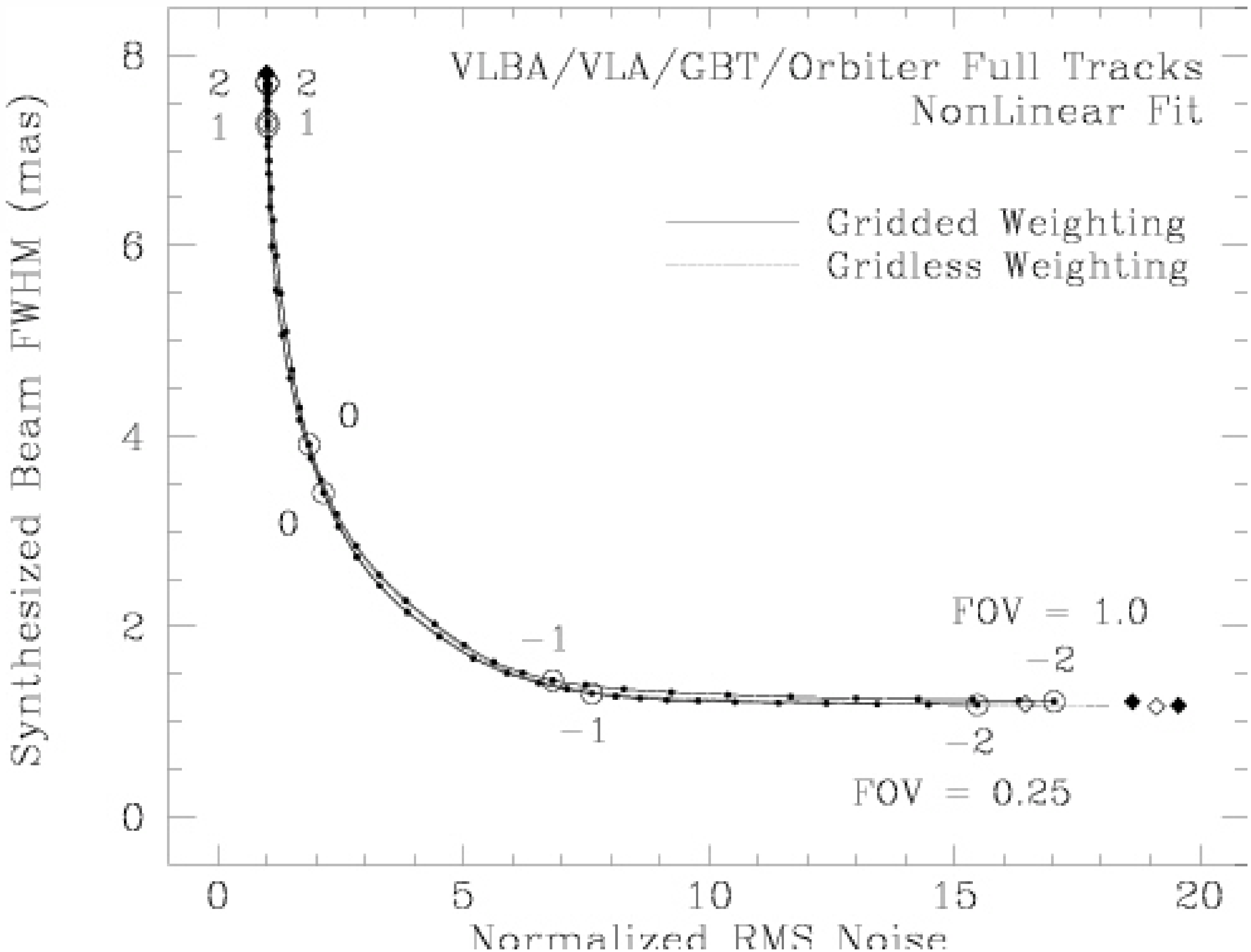
Uniform



Robust weighting

- Developed by Briggs (1995)
- Combine inverse density and noise weighting
- Possible to get (almost) best of both worlds
- Adjust using ‘robust’ parameter in IMAGR

Robust weighting



Alternative deconvolution methods

- Maximum Entropy Method (MEM)
  - Primary contraint is to maximimize the ‘Entropy’
    - Smoothest image which fits the measured data
    - Simplest (minimum information) image which fits the data
- Non-negative least squares (NNLS)
  - Primary constraint is positivity

Maximum Entropy Method

- Strong philosophical basis in information theory (Jaynes, Gull & Daniell); consistent treatment of prior information
- Maximise $$H(I) = -\sum_k I_k \cdot \log(I_k / m_k)$$
- Practically: works well for extended emission, produces smoother results than CLEAN
  - Can be faster for large images
    - Imagine single smooth component
  - Does not cope well with point sources + smooth background
  - Can use both; CLEAN to remove bright points; MEM on smoother residual image

AIPS IMAGR

- Well developed and well-trusted implementation of Cotton-Schwab CLEAN with many enhancements (multiple fields, robust weighting,…)
- Input is (calibrated) visibility file
  - Applies calibrations
  - Uses convolution to grid data, applying specified weighting
  - Produces Dirty Map, Dirty Beam
  - Performs certain number of CLEAN subtractions
    - Clean Component Table, Residual Map
  - Convolves Clean Components with Clean Beam and adds Residual Map
  - Writes out Clean Map as image file, with CC table attached

IMAGR parameters

- Simple use for single frequency, total intensity data:
- IMSIZE … size of image in pixels: 256 - 2048
- CELLSIZE … pixel size in arcsec ~ 0.3 x resolution
- NITER … can be fixed number of subtractions eg 1000; can control interactively with DOTV, or can stop when FLUX limit reached
- Options
  - RASHIFT, DECSHIFT: move the centre of the field
  - ROBUST: modify the weighting
  - CLBOX (or set in DOTV mode) windows
  - NFIELD > 1, RASHIFT, DECSHIFT … multiple fields at the same time
  - BMAJ, BMIN: set the CLEAN beam yourself
    - Often made circular ~ sqrt(Bx.By)